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THE EFFECT OF FOUNDATION FLEXIBILITY ON RESPONSE REDUCTION FACTOR OF R/C FRAME STRUCTURES

A. M. Halabian[1] and M. Erfani[2]

ABSTRACT

The seismic codes anticipate that structures will undergo inelastic deformations under strong seismic events; therefore, such inelastic behaviour is usually incorporated into the design by dividing the elastic spectra by a factor, R, which reduces the spectrum from its original elastic demand level to a design level. The most important factors determining response reduction factors are the structural ductility and overstrength capacity. For a structure supporting on flexible foundation, as SSI extends the elastic period and also increases damping of the structure-foundation elastic system, the structural ductility could be affected by frequency dependent foundation-soil compliances. For inelastic systems supporting on flexible foundations, the inelastic spectra ordinates are greater than for elastic systems when presented in terms of flexible-base structure's period. This implies that the reduction factors, which are currently not affected by SSI effect, could be altered; therefore, the objective of this research is to evaluate the significance of foundation flexibility on force reduction factors of R/C frame structures. In this research, by developing some generic R/C frame models supporting on flexible foundations subjected to a set of artificial earthquake records, effects of stiffness and strength of the structure on force reduction factors are evaluated for different relative stiffnesses between the structure and the supporting soil. The difference between inelastic and elastic resistance in terms of displacement ductility capacity factors has been quantified. The results indicated that the foundation flexibility could significantly change the ductility of the system and neglecting this phenomenon may lead to erroneous conclusions in the prediction of seismic performance of flexibly-supported R/C frame structures.

Introduction

In the current force-based design procedures adopted by most seismic design codes, the designer is allowed to utilize the ductile capacity of the structure and therefore, the seismic design of building structures is based on static or dynamic analyses of elastic models of the structure using elastic design spectra. Force-based design procedures are likely to remain as the primary seismic design method for some time since performance-base design methods are still in the development phase. In these procedures, the elastic design strengths are substantially reduced on the provision of adequate ductility capacity of the structure, to sustain a targeted amount of plastic deformation under a maximum credible earthquake condition. The factor, R, which reduces the spectrum from its original elastic demand level to a design level, is related to the

[1]Professor, Dept. of Civil Engineering, Isfahan University of Technology, Isfahan, Iran.
[2]Graduate Research Assistant, Dept. of Civil Engineering, Isfahan University of Technology, Isfahan, Iran.

overall performance of different types of buildings during actual earthquakes. Despite the fact that the reduction factor, R, serves the same function in all seismic codes, is called behaviour factor (q) in EC8 and response modification factor (R) in NEHRP, while is termed the structural quality factor or the system performance factor following recommendations of the SEAOC Committee. In the current study, the term response reduction factor is adopted since it offers a clearer indication of the nature of this factor, which plays a paramount role in seismic design.

Differences in the numerical values of the response reduction factors specified in different seismic codes for the same type of structure can be quite noticeable, reflecting to this fact that the values could be based on judgment, experience and observed ductile performance of buildings during past earthquakes. The key role of apparent inconsistencies in the way these response reduction factors are defined in different seismic codes have highlighted that the investigation of different parameters affecting on inelastic spectra and the ensuing response reduction factors is needed. As it was mentioned, it is almost generally accepted in various seismic codes that the response reduction factor of structures may be defined as the ratio of their elastic strength demand to their inelastic strength demand within the selected ductility capacity limits. Accordingly, the value of the response reduction factor is known to mainly depend on the inherent ductility of the structure, on the overstrength of individual members and on the (effective) damping of the structure assuming the structure is supported by fixed-base foundation. In fact, seismic analyses of buildings are often based on the assumption that the foundation flexibility has no effect on foundation-structure interacting forces. The first studies of SSI for elastic systems showed that soil structure interaction phenomenon could affect the dynamic characteristics of structures; resulting an increase to the fundamental natural period as well as the inherent structural damping. Veletsos and Verbic (1974), investigated the transient elastoplastic response of a flexible-supported structure supporting on an elastic half-space suggesting that yielding decreases the effect of SSI on structures. Whereas in studies conducted by Avilés and Pérez-Rocha (2003) it was shown that SSI effects for inelastic systems with different order could be as important as for elastic systems.

The current SSI provisions based on linear structural models may not be directly applicable to seismic design of typical buildings, expected to deform considerably beyond the yield limit during severe earthquakes. So that Stewart et al. (2003), recognized the SSI provisions in the ATC and NEHRP codes have a significant shortcoming expressing no link between the response reduction factors and the effects of foundation flexibility. Investigations of SSI effect on the response reduction factors of structures are scarce. Avilés and Pérez-Rocha (2005) evaluated effect of SSI on strength-reduction, R_μ, and displacement- reduction, C_μ, factors for a SDOF elastoplastic structure subjected to Michoacon earthquake (1985). The result of the research noted that these factors could be altered by the SSI phenomenon depending on the supporting soil-structure stiffness ratio. Halabian and Kabiri (2006) evaluated the effect of foundation flexibility on ductility of R/C stake-like structures. The results showed that for these types of structures, the SSI could decrease the ductility demand and consequently response reduction factor. The present study is aimed to evaluate the R factors of R/C frame structures considering the foundation flexibility effect using refined definition of response reduction factors employed by most modern seismic codes.

Response reduction factor

Having this fact that parameters affecting the response reduction factors such as ductility, structural overstrength and structure's redundancy have long been recognized and assuming no supplemental damping devices, an appropriate definition of the response reduction factor can be expressed in the form of

$$R = R_{\mu}\Omega_{d}\, R_{R} \tag{1}$$

where R_{μ} is the ductility reduction factor, Ω_d is the overstrength factor and R_R is the redundancy factor which quantifies the improved reliability of seismic framing systems that use multiple lines of vertical seismic framing in each principle direction of a building. Since the overstrength factor implicitly accounts for redundancy through redistribution of actions, the overstrength and redundancy parameters can be considered as one factor and therefore, the response reduction factor is simply given as:

$$R = R_{\mu}\Omega_{d} \tag{2}$$

It is understood that seismic nonlinear responses and then the ductility demand of building structures resting on flexible foundations could be changed, due to the foundation flexibility (Halabian and Emami (2009)). Since the concept of response reduction factor expressed by Eq. 1 does not explicitly take into account the foundation flexibility, the Eq. 2 can be furthermore modified as:

$$R = R_{\mu}\Omega_{d}R_{S} \tag{3}$$

in which R_S in this study is so called as the foundation factor that depends on the foundation-structure stiffness ratio and the structural strength as well. If a proper calibration of response reduction factors is to be made, each of the major components contributing to it should be precisely investigated. In the following, the ductility dependent factor (product of R_{μ} and R_S) is evaluated on the basis of some repeated non-linear analyses of flexible-base structures for selected earthquake records. There are some different methods based on force-displacement relationship resulted from inelastic analyses of the structure to evaluate the response reduction factors according to Eq. 3. However, the current study is not intended to review and discuss all these approaches. In the following taking into account the ground motion dependence of the response reduction factor, a more rigorous approach for proper calibration of these factors on the basis of repeated non-linear analyses is described. The factor in various modern seismic codes reduces the elastic base shear (V_e), obtained from the elastic acceleration spectrum $(S_a)^{in}$ in the first natural period of the structure, to the design base shear level (V_d), evaluated from spectrum used in design $(S_a)^{in}$ again corresponding to the first natural period of the structure. Thus,

$$R_{code} = (S_a)^{el} / (S_a)^{in} \tag{4}$$

Since collapse is normally anticipated under the effect of an earthquake with a spectrum higher than the elastic spectrum, therefore, for a particular structure under a specific accelerogram, the following formula evaluates an ultimate value of the response reduction factor:

$$R_{c,dy} = (S_a)_c^{el} / (S_a)^{in} \tag{5}$$

where the subscripts 'c' refer to collapse and 'dy' refer to the yield level assumed in design. Having this fact that structure is mainly designed for forces consistent with its yield limit state, Elnashai and Broderick (1996) used a definition that utilises the spectral acceleration causing actual yield in the denominator, as given in the following equation:

$$R_{c,ay} = (S_a)_c^{el} / (S_a)_y^{el} \tag{6}$$

In the above equation, the subscript 'ay' refers to the actual yield. Assuming the response spectra of the yield and collapse earthquakes have constant dynamic amplification (the ratio of the peak ground acceleration to the peak response acceleration), Eq.6 can be rewritten as:

$$R_{c,ay} = a_{g\,(collapse)} / a_{g\,(actual\ yield)} \tag{7}$$

In the above equation, $a_{g\,(collapse)}$ is the peak ground accelerations of the collapse and $a_{g\,(actual\ yield)}$ is the PGA corresponding to the first yielding in system. In fact, structures designed using modern seismic codes usually exhibit some considerable level of overstrength that leads to significant differences between the PGA causing the first global yield ($a_{g\,(actual\ yield)}$) and the yield intensity implied by the design ($a_{g\,(design\ yield)}$ = design PGA / R_{code}) (Fig. 1). Therefore, Eq.7 should be modified adding an overstrength factor, Ω_d, as,

$$R`_{c,ay} = R_{c,ay} \cdot \Omega_d = \left(a_{g\,(collapse)} / a_{g\,(actual\ yield)}\right) \cdot \Omega_d \tag{8}$$

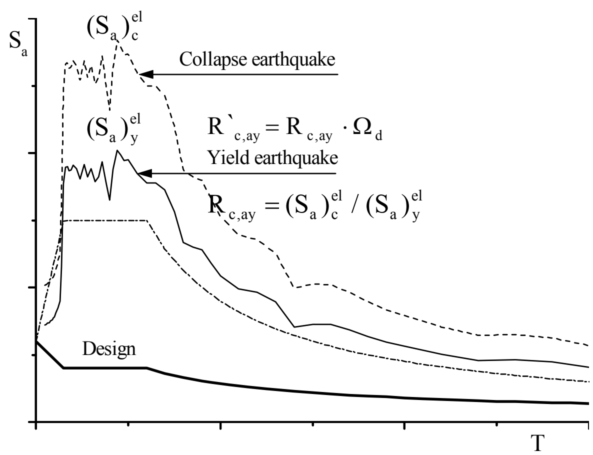

Figure 1. Evaluation of the force reduction factor by definition of $R_{c,ay}$ and Ω_d [Wafi and Elnashai, 2002]

Overstrength factor accounts for the reverse strength between the actual yield and design levels and can be employed to reduce the seismic forces used in the design, hence leading to more economical structures. The main sources of overstrength include: the difference between the actual and the design material strength; conservation of the design procedure; load factors and multiple load cases; serviceability limit state provisions, participation of nonstructural elements and structure's redundancy. To evaluate the peak ground accelerations causing collapse and first yielding, the incremental nonlinear dynamic analysis is performed by progressively scaling and applying each of the employed set of earthquake records, starting from a relative low intensity, and terminating with the intensity at which all yield and collapse definitions are achieved. This method allows evaluating the performance of the structure at different levels of excitations. Hence, the peak ground accelerations causing yield and collapse can be identified according to the performance criteria adopted for structural elements. It is mentionable, since a generally applicable and precise estimation of overstrength is difficult due to many factors contributing to it, therefore, in this study the overstrength factor is not taken into account in the evaluation process of response reduction factor.

Generic models supporting of flexible foundation

Since, a large number of parameters generally influence the seismic demands of structures, it was shown that generic models could be an appropriate approach for assessing the influence of these parameters in seismic demands of R/C frame buildings. In a general view, generic models adopted in the current study consist of 2D single-bay R/C moment-resisting frames reflecting different stiffness and story strength. To take into account the structure's height effect, three types of generic structures representing short, medium and high-rise R/C frames were developed. Only the results for low-rise and mid-rise frames are reported in this paper. The stiffness of generic structures are tuned so that the structure's deflected shape under a given design load pattern become a straight line. The lateral load pattern is selected based on SRSS modal superposition method. Employing UBC'97, the SRSS lateral load distribution is obtained using story shear forces calculated from the SRSS combination of modal responses throughout of the linear response spectrum analysis. In the generic structure, inelastic deformations are permitted only at the ends of the beam in each story and at the base of the columns. Thus, the basic plastic hinge mechanism under lateral loads represents structures complying with the weak beam–strong column requirement. Assuming that the overall mass matrix of the generic model is constant, a generic structure with specific fundamental period is produced by changing all element stiffnesses in every story, simultaneously. In this research, the fundamental elastic period of the 5-story and 10-story generic frame structures are assumed to be T=0.5 sec and T=1.0 sec. Seismic design of frame structures are performed according to back-calculated base shear strength and a design lateral load distribution. The base shear strength is varied in relation to specific purposes of the analysis. Given this quantity, the individual story shear strengths are arranged to the story shear forces obtained from the design load pattern.

Generally, the base shear strength V_{base} is estimated according to:

$$V_{base} = C.W \tag{9}$$

In this equation, W is the effective weight of structure and C represents the base shear strength factor. Assuming different values for C, every generic frame includes a set of idealized models with the same stiffness but different strengths against lateral load. In this study to examine the strength effect on response reduction factors, the generic frames with the same stiffness, designed for two strength levels, C=0.1 and 0.2 against lateral loading, based on ACI 318-05 code.

In order to SSI analysis, the substructure method is employed for dynamic analysis of generic structures supporting on flexible foundation. Fundamental step in the substructure method is to evaluate the foundation impedance functions. The dynamic stiffnesses are complex-valued functions depending on the geometry of the foundation and the characteristic of the soil, that generally can be expressed as:

$$[S(\omega)] = [K_{st}].[k(a_0) + ia_0c(a_0)] \tag{10}$$

in which $[S(\omega)]$ and $[K_{ST}]$ are the dynamic and static stiffness matrix, respectively. Moreover, k and c are dimensionless coefficients depending on Poisson's ratio ν and the dimensionless frequency parameter $a_0 = \omega.r / V_S$, where r is the equivalent radius of the footing and V_S is the shear wave velocity of soil halfspace. In this study, the stiffness and damping constants of the foundation are obtained using the theory due to Veletsos and Wei (1971) and Veletsos and Verbic (1973). Using this method, effect of exciting frequency, foundation type and size, and dynamic properties of the soil layers can be also included in estimation of foundation impedance functions. The generic frames are assumed to be supported by a homogeneous viscoelastic halfspace soil having shear wave velocity $50m/\sec \leq V_s \leq 500m/\sec$ and the Poisson's ratio equal to 0.3.

Nonlinear modeling of R/C frames and equation of motions

The generic models are idealized as 2D frames using beam and column elements. In this study, with the purpose to investigate the effect of inelastic behaviour of the R/C frame structural elements two types of model accounting different types non-linear behaviour were adapted. The nonlinear behaviour of beam elements is modeled by means of moment-curvature theory, and in column elements, multi-axial spring model based on material stress-strain relation, called "fiber model", is used to represent the interaction among axial force and bending. In the current study, the M-ϕ relations in monotonic loading (backbone curve) are taken tri-linear (Fig.2a), and element response for cyclic loading is governed by a set of parameters which are correlated to strength and stiffness degradation (Fig. 2b).

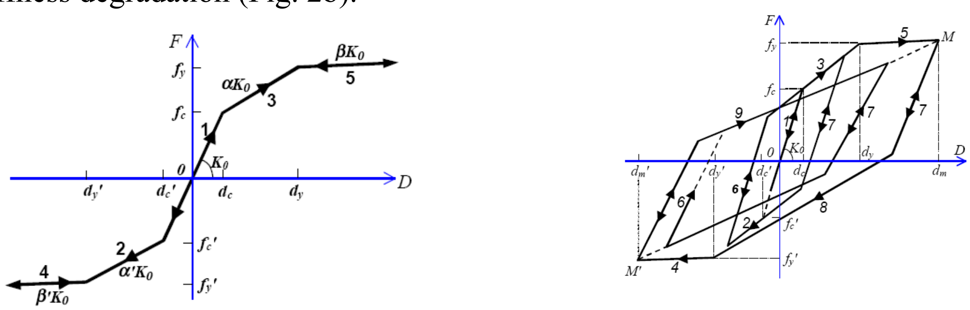

Figure 2. Non-linear behaviour of RC structural elements: a) backbone curve b) hysteretic loop

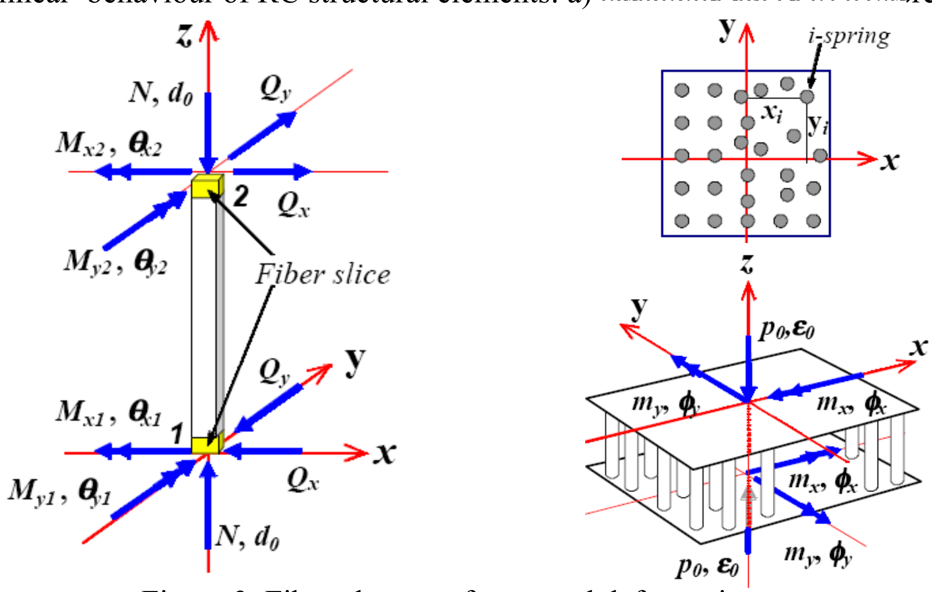

Figure 3. Fiber elements forces and deformations

In the Fiber Element Method for modeling non-linear behaviour of columns, the cross-section of each column element is subdivided into some spring elements (Fig.3). Each spring is subjected to axial load, given by the combination of axial force and bending moment acting on the section. The nonlinear governing equation of motion for the generic structure can be written as:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=[M]\{I\}\{\ddot{u}_g\} \tag{11}$$

where [M], [C] and [K] are mass and damping and stiffness matrix of soil-structure system. The Rayliegh's damping is used to model the structural damping and having taken into account the damping coming from the impedance functions resulting from flexibility of foundation. Thus the total damping including non-classical damping is expressed by:

$$[C]=a_m\times[M]+a_k\times[K_0]+[C_v] \tag{12}$$

The coefficient a_m, a_k are damping factor proportional to mass matrix and initial stiffness. To solve the differential equation of motion (Eq. 8), the time step integration method is adapted in this study. Therefore, at the time t+Δt, the equation of motion can be expressed by integration:

$$[M]\int_0^{t+\Delta t}\{d\ddot{u}\}+(a_m[M]+a_k[K_0])\int_0^{t+\Delta t}\{d\dot{u}\}+\int_0^{t+\Delta t}[C_V]\{d\dot{u}\}+\int_0^{t+\Delta t}[K]\{du\}=-[M]\int_0^{t+\Delta t}d\ddot{u}_g \tag{13}$$

A numerical integration procedure known as Newmark' β method is used to express the differential relationship of the time function. To satisfy the equilibrium at each time step, the iteration are performed using the Newton-Raphson scheme within each time step.

Numerical results

To get insight the effect of SSI phenomenon on ductility reduction factor of R/C frame structures supporting on flexible foundation a repeated non-linear time history analysis approach based on refined definition of the response reduction factor described above was employed. Five artificially-generated records compatible with UBC-97 response spectrum having peak ground accelerations approximately equal to 1.0g (Halabian and Kabiri (2006)) are selected as ground excitations in non-linear SSI dynamic analyses.

Using assumed set of artificial earthquake records, repeated non-linear analyses were performed by gradually increasing the intensity of acceleration time histories to a level, where first yielding of steel is occurred. The values of PGA for which the 5-story and 10-story generic frames supporting on different flexible foundations reaches its yielding state of response are given in Table 1. The yield limit state is defined when the strain in the main tensile reinforcement exceeds the design yield strain of steel. The definition of collapse for R/C frames as a whole is quite subjective and depends on engineering judgment. In this study, the collapse limit state is assumed corresponding to the formation of a lateral mechanism in structure. The values of PGA for which the 5-story and 10-story generic frames supporting on different flexible foundations reaches its collapse state are also given in Table 2.

Using Eq. 7 the ductility reduction factor can be quantified by subdividing the peak ground accelerations of the collapse to the PGA corresponding to the first yielding in system. The variation of ductility reduction factors with respect to soil shear wave velocities are shown in Figs. 5 for each particular acceleration time history and for two levels of strength at the stiffness level corresponding to period equal to 0.5. Figs.6 show the same information but for the different strength levels at T=1. The calculated response reduction factors for the fixed-base generic models having different levels of strengths are also shown in Figs. 5 and 6.

Comparison of Figs. 5 and 6 draws back some important conclusions:

- The results show that the effect of SSI on response reduction factor for R/C frames supporting on very soft soils could be detrimental. In this case, excessive flexible base could probably act as a figurative soft story causing noticeable decrease (up to 50 percent) in response reduction factor.
- For typical stiff soils with $300 < V_s \leq 500 m/ses$, the effect of foundation flexibility on ductility

Table 1. Ground acceleration at yield limit state for generic frames

$a_{g(actual\ yield)}$						
art. Records	fixed base	V_s=500	V_s=300	V_s=200	V_s=100	V_s=50
T=0.5 , C=0.1						
ACC1	0.37	0.36	0.34	0.33	0.36	0.35
ACC2	0.39	0.38	0.37	0.37	0.35	0.33
ACC3	0.35	0.37	0.4	0.39	0.37	0.3
ACC4	0.34	0.34	0.34	0.34	0.29	0.26
ACC5	0.41	0.4	0.36	0.35	0.3	0.3
T=0.5sec , C=0.2						
ACC1	0.59	0.59	0.56	0.56	0.56	0.55
ACC2	0.54	0.56	0.54	0.55	0.52	0.53
ACC3	0.49	0.5	0.51	0.51	0.51	0.41
ACC4	0.52	0.52	0.53	0.52	0.52	0.5
ACC5	0.59	0.58	0.59	0.58	0.59	0.5
T=1sec , C=0.05						
ACC1	0.36	0.36	0.36	0.37	0.38	0.38
ACC2	0.36	0.36	0.36	0.36	0.39	0.4
ACC3	0.39	0.39	0.38	0.37	0.4	0.45
ACC4	0.32	0.32	0.32	0.31	0.3	0.45
ACC5	0.4	0.4	0.38	0.4	0.36	0.43
T=1sec , C=0.1						
ACC1	0.48	0.48	0.48	0.47	0.44	0.5
ACC2	0.52	0.52	0.52	0.52	0.53	0.5
ACC3	0.47	0.47	0.48	0.46	0.45	0.47
ACC4	0.48	0.48	0.48	0.46	0.44	0.55
ACC5	0.46	0.46	0.47	0.45	0.5	0.45

Table 2. Ground acceleration at collapse limit state for generic frames

$a_{g(actual\ yield)}$						
art. Records	fixed base	V_s=500	V_s=300	V_s=200	V_s=100	V_s=50
T=0.5 , C=0.1						
ACC1	1.98	1.95	1.93	1.89	1.9	0.95
ACC2	2.09	2.01	1.91	1.9	1.87	1.05
ACC3	1.87	1.96	1.95	1.87	1.9	1.1
ACC4	2.32	2.31	2.33	2.28	2.19	0.95
ACC5	1.93	1.9	1.92	1.93	2.17	1.1
T=0.5sec , C=0.2						
ACC1	2.2	2.3	2.19	2.1	2.2	1.3
ACC2	2.1	2.15	2.02	2	1.75	1.7
ACC3	1.95	1.95	1.95	1.91	1.87	1.13
ACC4	2.2	2.19	2.18	2.05	1.97	1.3
ACC5	2.2	2.12	2.11	2.03	2.01	1.35
T=1sec , C=0.05						
ACC1	1.34	1.35	1.35	1.36	1.37	1.16
ACC2	1.48	1.47	1.48	1.44	1.36	1.2
ACC3	1.39	1.39	1.36	1.32	1.44	1.49
ACC4	1.94	1.95	1.99	1.99	2	1.69
ACC5	1.91	1.97	1.83	2.05	1.96	1.83
T=1 sec , C=0.1						
ACC1	1.79	1.79	1.78	1.91	1.9	1.43
ACC2	2.19	2.19	2.1	2	1.69	1.49
ACC3	1.8	1.8	1.79	1.76	1.72	1.1
ACC4	1.79	1.78	1.76	1.7	1.74	1.58
ACC5	1.64	1.65	1.66	1.67	1.6	1.3

reduction factors could be beneficial. However, for short period structures this result should be used with caution.

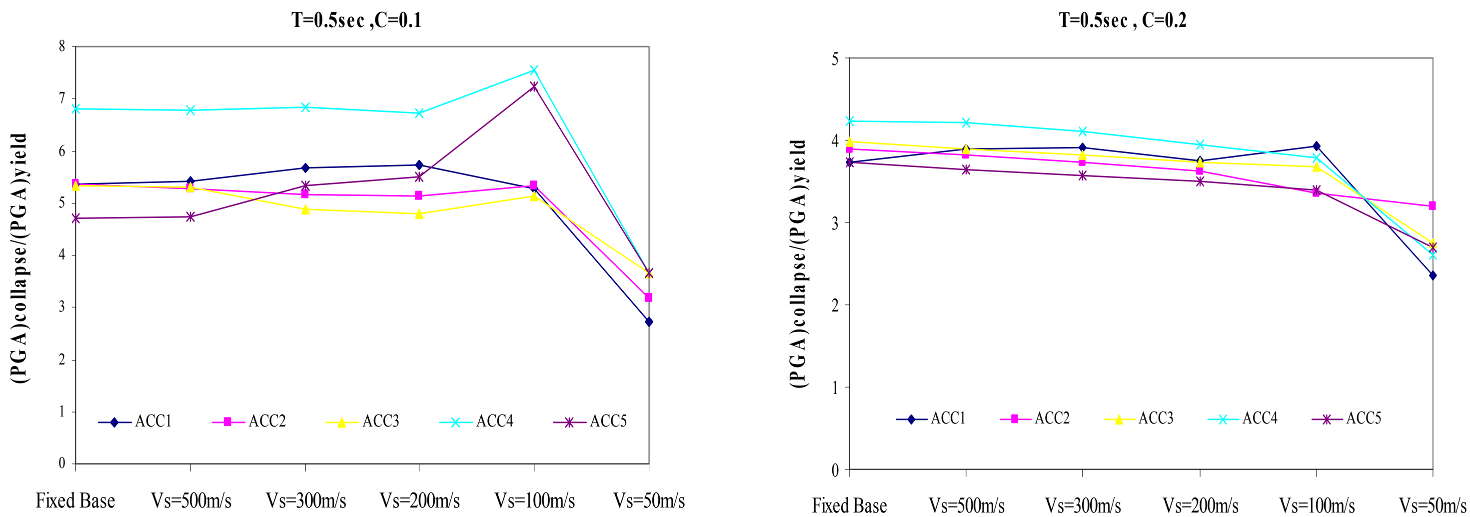

Figure 5. Effect of foundation flexibility on response reduction factor of 5-story generic model a)T= 0.5 sec, C=0.1 ,b) T= 0.5 sec, C=0.2

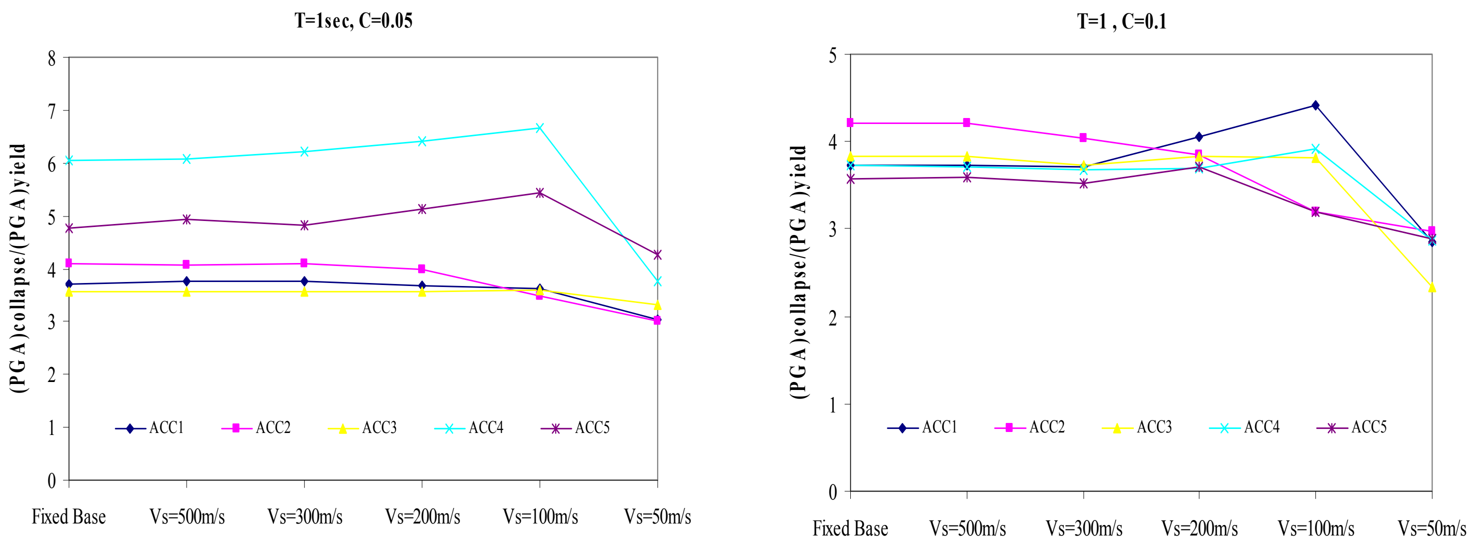

Figure 6. Effect of foundation flexibility on force reduction factor of 5-story generic model a)T= 1 sec, C=0.05 ,b) T= 1 sec, C=0.1

- In the case of typical soft soils with $100 \le V_s \le 300 m/\sec$, it can be noted that depending on the strength level of the structure, the effect of foundation flexibility on ductility reduction factor of R/C frames could be beneficial or detrimental. The results demonstrate that 5-story frames with lower strength show up to 50% increase in ductility reduction factor; while for 5-story frames with higher strength outcomes show a decrease about 15% in the response reduction factor. For 10-story frames with lower strength the effect of foundation flexibility for most of seismic excitation depends on frequency content is less than 10%, while for the frames with higher strength the results demonstrate the response reduction factor could be changed up to 25% in the side of detrimental or beneficial. In fact, it is indicated that the frames with same stiffness but various strengths could demand different ductility when considering SSI effect.

Conclusions

In this study, to determine SSI effect on force reduction factor of R/C frame structures, 2D generic frames, were developed. The incremental time history analysis using some artificial earthquake records was employed to assess the response reduction factor. The results showed the whenever the ductile behavior of R/C frame structures is considered in design, the ductility reduction factor of the structure is sensitive to the structure stiffness, strength of the structure and the foundation flexibility. The effect of soil structure interaction on response reduction factors for typical soft soils changes with the strength of frame structures. The results also demonstrated that as the strength of short period frame structures increases, the effect of foundation flexibility on the response reduction factor decreases, while the reverse trend was observed for mid-rise structures. For R/C frame structures resting on very soft soil with V_s=50m/s, effect of foundation flexibility considerably decreases the ductility reduction factor, whereas for typical stiff soil with $300 < V_s \leq 500 m/\sec$, effect of foundation flexibility for R/C frame structures is approximately negligible.

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