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CHARACTERIZING SPATIAL CROSS-CORRELATION BETWEEN GROUND-MOTION SPECTRAL ACCELERATIONS AT MULTIPLE PERIODS

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ABSTRACT

Quantifying ground-motion shaking over a spatially-distributed region rather than at just a single site is of interest for a variety of applications relating to risk of infrastructure or portfolios of properties. The risk assessment for a single structure can be easily performed using the available ground-motion models that predict the distribution of the ground-motion intensity at a single site due to a given earthquake. These models, however, do not provide information about the joint distribution of ground-motion intensities over a region, which is required to quantify the seismic hazard at multiple sites. In particular, the ground-motion models do not provide information on the correlation between the ground-motion intensities at different sites during a single event.

Researchers have previously estimated the correlations between residuals of spectral accelerations at the same spectral period at two different sites. But there is still not much knowledge about cross-correlations between residuals of spectral accelerations at different periods (or more generally between residuals of two different intensity measures) at two different sites, which becomes important, for instance, when assessing the risk of a portfolio of buildings with different fundamental periods. Spatial cross-correlations are also important when assessing the risk due to multiple ground-motion effects such as ground shaking and liquefaction, because this involves the use of multiple types of intensity measures. This manuscript summarizes recent research in ground-motion spatial cross-correlation estimation using geostatistical tools. Recorded ground-motion intensities are used to compute residuals at multiple periods, which are then used to estimate the spatial cross-correlation. These cross-correlation estimates can then be used in risk assessments of portfolios of structures with different fundamental periods, and in assessing the seismic risk under multiple ground-motion effects.

Introduction

Quantifying ground-motion shaking over a spatially-distributed region rather than at just a single site is of interest for a variety of applications relating to risk of infrastructure or portfolios of properties. For instance, the knowledge about ground-motion shaking over a region is important to predict (or estimate after an earthquake) the monetary losses associated with structures insured by an insurance company, the number of casualties in a certain area and the probability that lifeline networks for power, water, and transportation may be interrupted. The risk assessment for a single

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structure requires only the quantification of seismic hazard at a single site, which can be easily done using probabilistic seismic hazard analysis (PSHA). The hazard is typically measured in terms of an intensity measure such as the spectral acceleration corresponding to the building's fundamental period (peak response of simple single-degree-of-freedom (SDOF) oscillators with the same fundamental period of the real structure) when the damage to a building is to be estimated. Other ground-motion parameters such as the peak ground acceleration (PGA) or peak ground velocity (PGV) are used for other applications such as the prediction of liquefaction of saturated sandy soil or the response of buried pipelines. The hazard assessment procedure uses ground-motion models that have been developed to predict the distribution of the ground-motion intensity at a single site after a given earthquake. These models, however, do not provide information on the joint distribution of ground-motion intensities over a region, which is required to quantify the seismic hazard at multiple sites such as for lifeline risk assessment. In particular, the ground-motion models do not provide information on the correlation between the ground-motion intensities at different sites during a single event.

In general, the ground-motion intensities at two sites are expected to be correlated for a variety of reasons, such as a common source earthquake (whose unique properties may cause correlations in ground motions at many sites), similar locations to fault asperities, similar wave propagation paths, and similar local-site conditions. Modern ground-motion models partially account for the correlation via a specific inter-event term η_i as follows:

$$ln(S_{a_i}(T)) = ln\left(\overline{S}_{a_i}(T)\right) + \sigma_i(T)\varepsilon_i(T) + \tau_i(T)\eta_i(T) \tag{1}$$

where $S_{a_i}(T)$ denotes the spectral acceleration at period T at site i; $\overline{S}_{a_i}(T)$ denotes the predicted (by the ground-motion model) median spectral acceleration (which depends on parameters such as magnitude, distance, period and local-site conditions); $\varepsilon_i(T)$ denotes the normalized intra-event residual at site i associated with $S_{a_i}(T)$, $\eta_i(T)$ denotes the normalized inter-event residual at site i associated with $S_{a_i}(T)$. Both $\varepsilon_i(T)$ and $\eta_i(T)$ are random variables with zero mean and unit standard deviation. The standard deviations, $\sigma_i(T)$ and $\tau_i(T)$, are estimated as part of the ground-motion model and are functions of the spectral period (T) of interest, and in some models also functions of the earthquake magnitude and the distance of the site from the rupture. The term $\sigma_i(T)\varepsilon_i(T)$ is called the intra-event residual, and the term $\tau_i(T)\eta_i(T)$ is called the inter-event residual.

Though the ground-motion models partly account for the correlation via η_i, the ε_i's still show a significant amount of residual correlation. Researchers have previously estimated the correlations between residuals of spectral accelerations at the same spectral period (e.g., between $\varepsilon_i(T)$ and $\varepsilon_j(T)$) using recorded ground motions (e.g., Jayaram and Baker 2009a, Goda and Hong 2008, Wang and Takada 2005, Boore et al. 2003). These models have shown that the spatial correlation decays with site separation distance between sites i and j, and that the rate of decay is a function of the spectral period. These works, however, do not investigate the nature of the spatial cross-correlation between residuals of two different intensity measures at two different sites (e.g., between $\varepsilon_i(T_1)$ and $\varepsilon_j(T_2)$).

Considering spatial correlation in risk analysis is important because correlation between residuals can lead to large ground-motion intensities over a spatially-extended area. Recent research has shown that ignoring spatial correlations can significantly underestimate the seismic risk of portfolios of buildings and of other lifelines such as transportation networks (Jayaram and Baker 2009b, Park et al. 2007). For instance, Fig. 1 shows the exceedance rates of earthquakeinduced travel-time delays in the San Francisco Bay Area transportation network estimated by Jayaram and Baker (2009b) while considering/ignoring spatial correlation. This figure shows that the likelihood of observing large delays gets significantly underestimated when spatial correlations are ignored. Spatial cross-correlations are equally important when multiple intensity measures are used for assessing the system risk. This arises, for instance, when predicting damage to a portfolio of structures whose individual damage states are predicted using spectral accelerations at multiple periods. Spatial cross-correlations are also important when secondary effects such as landslides and liquefaction are considered apart from ground shaking. For instance, according to HAZUS (1997), the susceptibility of soil to liquefy is a function of the peak ground acceleration (i.e., $S_a(0)$) at the site, which might be different from the primary intensity measure ($S_a(T)$) of interest.

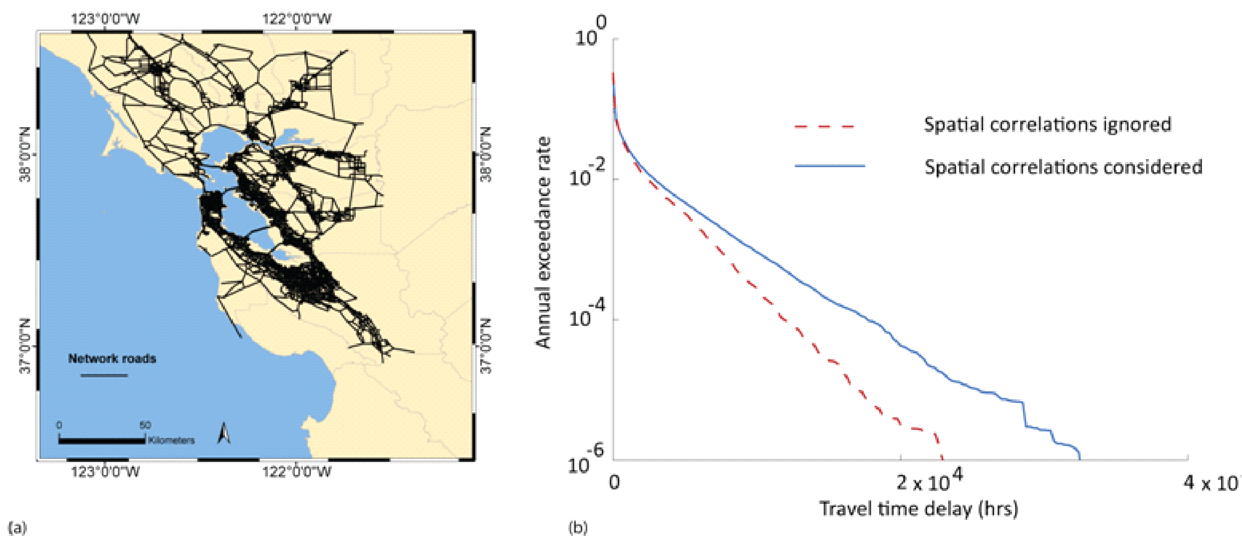

Figure 1. (a) The San Francisco Bay Area transportation network, and (b) Annual exceedance rates of various travel time delays on that network (results from Jayaram and Baker 2009b).

This manuscript summarizes recent research in ground-motion spatial cross-correlation estimation using geostatistical tools. Recorded ground-motion intensities are used to compute residuals at multiple periods, which are then used to estimate the spatial cross-correlation. These cross-correlation estimates can then be used in risk assessments of portfolios of structures with different fundamental periods, and in assessing the seismic risk under multiple earthquake effects.

Statistical Estimation of Spatial Cross-Correlation

In this study, geostatistical tools are used to estimate the spatial cross-correlations using recorded ground-motion data from the Pacific Earthquake Engineering Research (PEER) Center's Next Generation Attenuation (NGA) ground-motion library.

The first step involved in developing an empirical cross-correlation model using recorded ground-motion time histories is to use the time histories to compute the corresponding ground-motion intensities ($\boldsymbol{S_a}(T_1)$, $\boldsymbol{S_a}(T_2)$,…, $\boldsymbol{S_a}(T_m)$) and the associated normalized residuals ($\boldsymbol{\varepsilon}(T_1)$, $\boldsymbol{\varepsilon}(T_2)$,…, $\boldsymbol{\varepsilon}(T_m)$) using a ground-motion model. The cross-correlation structure of the residuals can then be represented by a `cross-semivariogram', which is a measure of the average dissimilarity between the data (Goovaerts 1997). Let u and u' denote two sites separated by $\boldsymbol{h}$. The cross-semivariogram ($\gamma(u,u')$) is defined as follows:

$$\gamma(u,u') = \frac{1}{2} E\left[\left\{\varepsilon_u(T_1) - \varepsilon_{u'}(T_1)\right\}\left\{\varepsilon_u(T_2) - \varepsilon_{u'}(T_2)\right\}\right] \tag{2}$$

The cross-semivariogram defined in Eq. 2 is location-dependent and its inference requires repetitive realizations of $\varepsilon(T_1)$ and $\varepsilon(T_2)$ at locations u and u'. Such repetitive measurements are, however, never available in practice (e.g., in the current application, one would need repeated observations of ground-motion intensities at every pair of sites of interest). Hence, it is typically assumed that the cross-semivariogram does not depend on site locations u and u', but only on their separation $\boldsymbol{h}$ to obtain a stationary cross-semivariogram. The stationary cross-semivariogram ($\gamma(\boldsymbol{h})$) can then be estimated as follows:

$$\gamma(\mathbf{h}) = \frac{1}{2} E\left[\left\{\varepsilon_u(T_1) - \varepsilon_{u+\mathbf{h}}(T_1)\right\}\left\{\varepsilon_u(T_2) - \varepsilon_{u+\mathbf{h}}(T_2)\right\}\right] \tag{3}$$

A stationary cross-semivariogram is said to be isotropic if it is a function of the separation distance ($h = |\boldsymbol{h}|$) rather than the separation vector $\boldsymbol{h}$. An isotropic, stationary cross-semivariogram can be empirically estimated from a data set as follows:

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} \left\{\varepsilon_{u_\alpha}(T_1) - \varepsilon_{u_\alpha+h}(T_1)\right\}\left\{\varepsilon_{u_\alpha}(T_2) - \varepsilon_{u_\alpha+h}(T_2)\right\} \tag{4}$$

where $\hat{\gamma}$(h) is the experimental stationary cross-semivariogram (estimated from a data set); $N(h)$ denotes the number of pairs of sites separated by h; and $\left\{\varepsilon_{u_\alpha}(T_1), \varepsilon_{u_\alpha+h}(T_2)\right\}$ denotes the α'th such pair. It can be theoretically shown that the following relationship can be used to estimate the cross-correlations from the cross-semivariograms:

$$\gamma(h) = \rho_{12}(0) - \rho_{12}(h) \tag{5}$$

where $\rho_{12}(0)$ denotes the cross-correlation between $\boldsymbol{\varepsilon}(T_1)$ and $\boldsymbol{\varepsilon}(T_2)$ at the same site u and $\rho_{12}(h)$ denotes the cross-correlation between $\varepsilon_u(T_1)$ and $\varepsilon_{u+h}(T_2)$. Therefore, it would suffice to estimate the cross-semivariogram of the residuals in order to determine their cross-correlations. The correlation term $\rho_{12}(0)$ has been estimated in the past (e.g., Baker and Jayaram 2008, Baker and Cornell 2006), and this work extends these results to include the effects of differing locations as well.

Once the cross-semivariogram values are obtained at discrete values of h, they are fitted using a continuous function of h for prediction purposes. In this work, we fit the discrete cross-semivariogram values with an exponential function which has the following form:

$$\gamma(h) = S\left(1 - e^{-\frac{3h}{R}}\right) \tag{6}$$

where S and R denote the sill and the range of the cross-semivariogram respectively. The value of the sill equals $\rho_{12}(0)$ (from Eq. 5 and 6), and the range denotes the separation distance at which the cross-correlation decays to less than 0.05S. Since the values of $\rho_{12}(0)$ have been previously computed (Baker and Jayaram 2008), it will suffice to estimate the range R to quantify the extent of spatial cross-correlation.

Sample Results and Discussion

This section discusses some sample cross-correlation estimates obtained using recorded time histories from the 1999 Chi-Chi earthquake. In particular, spatial cross-correlation estimates are computed for the 1 second and the 2 second spectral acceleration residuals from the Chi-Chi earthquake ground motions using the geostatistical procedure described in the earlier section. These residuals are first computed from the recorded ground motions using the Boore and Atkinson (2008) ground-motion model, and are shown in Fig. 2a-b. Visually, the presence of spatial cross-correlation is indicated by the similarity between the nearby residuals across Fig. 2a-b.

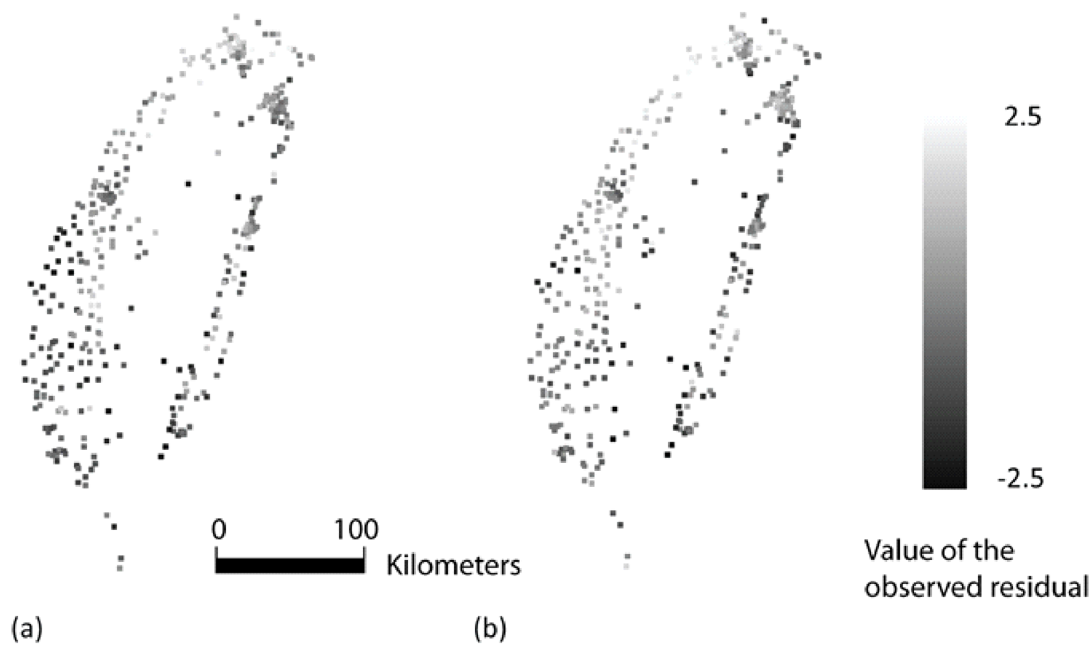

Figure 2. Chi-Chi earthquake residuals computed using spectral accelerations at (a) 1 second, and (b) 2 seconds.

Fig. 3 shows the cross-semivariogram estimated using the above-mentioned residuals. An exponential function is then fitted to the discrete cross-semivariogram values, the sill of which equals 0.7490 (which is the $\rho_{12}(0)$ value obtained from Baker and Jayaram 2008). The range of the cross-semivariogram equals 47km, and has been chosen to provide a good fit at short separation distances, although compromising on the quality of the fit at larger separation distances. This is because it is more important to model the cross-semivariogram structure well at short separation distances since large separation distances are associated with low correlations, which thus have relatively little effect on joint distributions of ground motion intensities. In addition to having low correlation, widely separated sites also have little impact on each other due to an effective 'shielding' of their influence by more closely-located sites (Goovaerts 1997). A more detailed discussion on the importance of fitting well at short separation distances can be found in Jayaram and Baker (2009a).

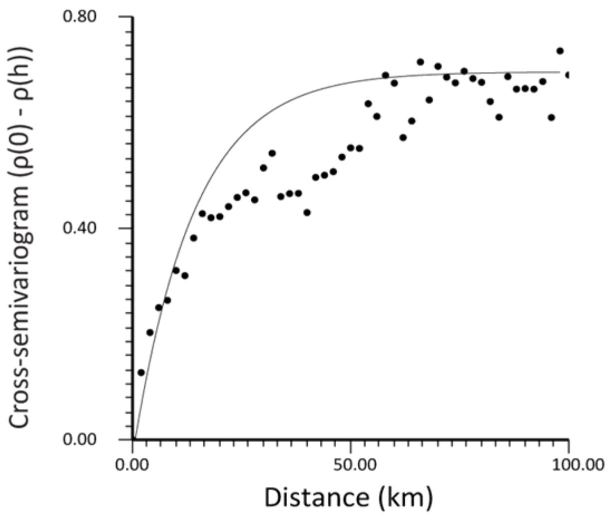

Figure 3. Cross-semivariogram estimated using the 1s and 2s Chi-Chi earthquake residuals.

The sample semivariogram in Fig. 3 shows that the extent of spatial cross-correlation is reasonably significant. For instance, the value of the cross-correlation equals 0.4 for sites separated by 10km and increases up to 0.75 for sites that are very close to each another. As a result, it will likely be important to consider spatial cross-correlations while studying multiple types of intensity measures distributed over a region.

Currently, the authors are in the process of developing a spatial cross-correlation model considering the residuals from multiple intensity measures using recordings from multiple earthquakes.

Conclusions

This manuscript summarized recent research in ground-motion spatial cross-correlation estimation using geostatistical tools. Spatial cross-correlations become important while quantifying the distribution of different types of ground-motion intensity measures over a region. This work used cross-semivariograms to model the cross-correlation structure. A cross-semivariogram is a measure of dissimilarity between the data, whose functional form (e.g., exponential function), sill and range uniquely identify the ground-motion cross-correlation as a function of separation distance.

In this work, recorded ground-motion spectral accelerations were used to compute residuals at multiple periods, which are then used to estimate the spatial cross-correlation. The manuscript showed sample cross-correlation estimates obtained using the 1s and 2s Chi-Chi earthquake residuals. The extent of the cross-correlation was found to be fairly significant, and hence, it will likely be important to consider spatial cross-correlations while studying the distribution of multiple types of intensity measures over a region. Currently, the authors are in the process of developing a spatial cross-correlation model considering the residuals from multiple intensity measures using recordings from multiple earthquakes. Once developed, these cross-correlation estimates can be used in risk assessments of portfolios of structures with different fundamental periods, and in assessing the seismic risk under multiple ground-motion effects.

References

Baker, J. W. and C. A. Cornell, 2006. Correlation of response spectral values for multicomponent ground motions, *Bulletin of the Seismological Society of America* 96(1), 215–227.

Baker, J.W. and N. Jayaram, 2008. Correlation of response spectral values from NGA models, *Earthquake Spectra* 24(1), 299–317.

Boore, D. M. and G. M. Atkinson, 2008. Ground-motion prediction equations for the average horizontal component of PGA, PGV and 5% damped SA at spectral periods between 0.01s and 10.0s, *Earthquake Spectra* 24(1), 99–138.

Boore, D. M., J. F. Gibbs, W. B. Joyner, J. C. Tinsley, and D. J. Ponti, 2003. Estimated ground motion from the 1994 Northridge, California, earthquake at the site of the Interstate 10 and La Cienega Boulevard bridge collapse, West Los Angeles, California, *Bulletin of the Seismological Society of America* 93(6), 2737–2751.

Goda, K. and H. P. Hong, 2008. Spatial correlation of peak ground motions and response spectra, *Bulletin of the Seismological Society of America* 98(1), 354–365.

Goovaerts, P., 1997. *Geostatistics for Natural Resources Evaluation*, Oxford, New York: Oxford University Press.

HAZUS, 1997. Earthquake loss estimation methodology, *Technical manual,* Prepared by the National Institute of Building Sciences for Federal Emergency Management Agency.

Jayaram, N. and J. W. Baker, 2009a. Correlation model for spatially-distributed ground-motion intensities, *Earthquake Engineering and Structural Dynamics,* in press.

Jayaram, N. and J. W. Baker, 2009b. Efficient sampling techniques for seismic risk assessment of lifelines, In *10th International Conference on Structural Safety and Reliability (ICOSSAR09)*, Kobe, Japan.

Park, J., P. Bazzurro, and J. W. Baker, 2007. Modeling spatial correlation of ground motion intensity measures for regional seismic hazard and portfolio loss estimation, In *10th International Conference on Application of Statistics and Probability in Civil Engineering (ICASP10)*, Tokyo, Japan.

Wang, M. and T. Takada, 2005. Macrospatial correlation model of seismic ground motions, *Earthquake Spectra* 21(4), 1137–1156.