



FUZZY MODEL FOR THE LIFE-CYCLE ANALYSIS OF BUILDING IN SEISMIC REGIONS

S. Tesfamariam¹ and M. Sanchez-Silva²

ABSTRACT

Over 50 years design life, buildings are exposed to different magnitude and frequency of earthquakes that requires consideration of life-cycle cost (LCC). The LCC considers the building performance under seismic load and investments throughout the structures' life. Traditional LCC utilizes probability of being in different damage states. However, these probabilities are not readily available for buildings that have different deficiencies and irregularities. In this paper, this shortcoming is handled through a systems theory and the corresponding possibility of being in different damage states are quantified using fuzzy set theory. The proposed method is illustrated with a six-storey reinforced concrete (RC) building for seismicity of Vancouver, Canada.

Introduction

Although structural design codes are calibrated for fixed design life, e.g. 50 years for commercial buildings (Bartlett et al. 2003) and 75 years for infrastructure components, the useful life of buildings is much longer. Decisions about design specifications and operation of infrastructure and buildings should be made by considering life-cycle and expected investments during that period. The life-cycle is defined by the time window required to achieve the functional or economic objectives, for which the project was intended. Whereas, the life-cycle cost (LCC) of a project is defined as the distribution of total cost that is incurred, or may be incurred, in all stages of the project life. The LCC analysis provides a framework to support decisions about resource allocation related to the design, construction and operation of infrastructure systems (Sanchez-Silva et al. 2009). Within this context, key infrastructure and structures located in seismic regions should undergo a LCC analysis given the possibility of sustaining damage during their lifetime.

The major considerations in a LCC analysis is the proper treatment of uncertainties in the demand and capacity, and cost incurred due to unsatisfactory performance (Wen 2000). Several formulations of the life-cycle performance of buildings and infrastructure systems are reported in the literature (e.g., Sánchez-Silva *et al.* 2009, Wen 2001). They all describe a stochastic representation of the structural performance linked with the costs associated to any intervention

¹ Assistant Professor, School of Engineering, The University of British Columbia - Okanagan, 3333 University way, Kelowna, BC, Canada, V1V 1V7, Tel: (250)-807-8185, E-mail: Solomon.Tesfamariam@ubc.ca, corresponding author

² Associate Professor, Dept. of Civil and Environmental Engineering, Universidad de los Andes, Carrera 1 No. 18A-70, Edificio W, Piso 3, Bogota', Colombia. E-mail: msanchez@uniandes.edu.co

(maintenance or reconstruction). In earthquake engineering, structural performance is commonly modeled by a renewal process in which shocks (earthquakes) may or may not cause the system failure. Depending upon the complexity of the model, the uncertainties about the shock sizes, shock occurrence times, the threshold value that defines damage, the condition after the structure has been repaired, etc., may be included. The classic life-cycle approach requires quantification of probability of different damage states in order to compute expected costs. However, vertical or plan irregularities, for example, have shown to be critical for the structural performance, but the damage probabilities are not readily available. A major issue in damage assessment is the need to evaluate variables that differ in nature and that can only be evaluated linguistically (e.g., construction quality), which necessitates consideration of different uncertainties. Blockley (1995) classifies the sources of uncertainty into lack of a pattern (randomness), incompleteness (what is unknown) and fuzziness (difficulty in defining boundaries between categories). The latter is particularly relevant for damage assessment mainly because there is not a well defined measure of structural damage or of the contributing factors.

This paper will present a model that incorporates, in the life-cycle cost of a structure, concepts of fuzzy logic to evaluate information coming from different sources arranged hierarchically to better represent the processes leading to building damageability. Based on these considerations, the objectives of the paper are:

1. Propose a life-cycle model of structures based on a systems approach.
2. Extend existing approaches to life-cycle cost analysis to include a detailed methodology of damage assessment and quantification.

This paper is organized as follows: section 2 will describe the life cycle cost analysis. Section 3 will focus on structural performance model, and section 4 discussed the fuzzy model. The fuzzy LCC evaluation procedure is described in section 5; and an illustrative example will be discussed in section 6.

Life cycle cost analysis

Formulation of the life cycle cost

The structural life-cycle cost model can be described as: $Z(\mathbf{X})=B(\mathbf{X})-C(\mathbf{X})-D(\mathbf{X})$, where $B(\mathbf{X})$ is the benefit, $C(\mathbf{X})$ is the construction cost and $D(\mathbf{X})$ the cost of losses. The vector parameter \mathbf{X} describes any mechanical property of the structure (e.g., design peak ground acceleration). It might also include aspects such as: plan irregularity, vertical irregularity, year of construction, construction quality. According to classic decision theory, the life-cycle cost should be evaluated in terms of the expected cost. In most cases, estimating the benefits is a difficult task and, therefore, the analysis becomes a cost minimization problem. The model presented in this paper is based on Wen (2001) LCC model, but any other model can be used as the basis for the LCC analysis. The expected total cost in which the owner will incur during the project life t can then be expressed as a function of time and the building performance modifier vectors \mathbf{X} (Wen 2001):

$$E[C(t, X)] = C_o(X) + E \left[\sum_{i=1}^{N(t)} \sum_{j=1}^k C_j e^{-\lambda_i t} P_{ij}(X_i, t_i) + \int_0^t C_m(X) e^{-\lambda_i \tau} d\tau \right] \quad (1)$$

where $E[\cdot]$ is the expected value; C_o is the initial cost for new construction or retrofitting; \mathbf{X} is a vector depicting building performance modifiers; i is the number of severe loading occurrences (e.g. live, wind, seismic loads), t_i is the loading occurrence time; a random variable; $N(t)$ is the total number of severe loading occurrences in t ; C_j is the cost in present dollar value of j th limit state being reached at time of the loading occurrence, which include costs of damage, repair, loss of service, and deaths and injuries; $e^{-\lambda t}$ is the discounted factor over time; λ is the constant discount rate per year; P_{ij} is the probability of j th limit states being exceeded given i th occurrence of a single hazard or joint occurrence of different hazards; k the total number of limit states under consideration; and C_m is the operation and maintenance cost per year.

With the assumption of Poisson frequency of hazard occurrence ν and for a single hazard, Equation (2) can be simplified to (Wen 2001):

$$E[C(t, X)] = C_o + [C_1 P_1(X) + C_2 P_2(X) + \dots + C_k P_k(X)] \frac{\nu}{\lambda} (1 - e^{-\lambda t}) + \frac{C_m}{\lambda} (1 - e^{-\lambda t}) \quad (2)$$

Computing the probability P_j of j th limit states being exceeded is a daunting task, which becomes more difficult if a set of building performance variables need to be included in the assessment. Moreover, damage estimation models cannot easily be incorporated due to the complexity of damage evaluation process. This considerations support the need for (1) using a systems approach that uses a hierarchical representation of the structural performance leading to a robust damage assessment; and (2) including assessment aspects that are usually not taken into account in the structural LCC evaluations.

Structural performance model

Developing a complex mathematical formulation of each building in the LCC assessment is not feasible. The building performance is affected by topology, code design consideration and quality of construction. Thus, the proposed building vulnerability model should be versatile to incorporate the different irregularities. The complex problem incorporating different building irregularities can be handled through a hierarchical structure. The hierarchical structure follows a logical order where the causal relationship for each supporting argument is further subdivided into specific contributors.

In this paper, the model proposed by Tesfamariam and Saatcioglu (2008) is further modified to quantify the building damageability. A four-level hierarchical structure is proposed to model the building damageability (Figure 1). Level 1 of the hierarchy is the overall goal of the analysis, i.e., *building damageability*. The building damageability is computed by integrating the parameters at level 2, *site seismic hazard* and *building vulnerability*. Levels 3 and 4 of the hierarchy are the five building performance modifiers. In this paper, the building performance modifiers considered are: i) building type, ii) vertical irregularity, iii) plan irregularity, iv) year of construction, and v) construction quality. Level 3 of the hierarchy is topology, structural system and construction features. The topology is quantified through aggregation of vertical and plan irregularities (Level 4). Furthermore, at level 4, the construction features are modelled through construction quality and year of construction. The year of construction is used to infer the type of building code considered and the corresponding seismic design consideration.

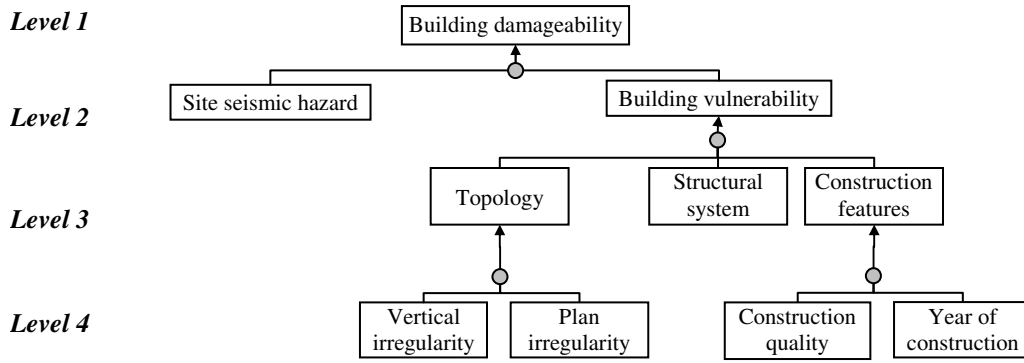


Figure 1. Hierarchical earthquake risk assessment of reinforced concrete buildings

Fuzzy Based Modelling

The basic theory of fuzzy sets was first introduced by Zadeh (1965) to deal with the difficulties in quantifying the uncertainty where human intervention was significant. A fuzzy set describes the relationship between an uncertain quantity x and a membership function μ_x , which ranges between 0 and 1. A fuzzy set is an extension of the traditional set theory (in which x is either a member of set A or not) in that an x can be a member of set A with a certain degree of membership μ_x . In this paper, a triangular fuzzy number is used for its simplicity. Fuzzy logic has been used extensively to handle the difficulties in defining limits. Fuzzy logic maps qualitative judgement into numerical reasoning. The strength of fuzzy logic is that it can integrate descriptive (linguistic) judgement and numerical data and use approximate reasoning algorithms to propagate the uncertainties (Zadeh 1973). Three steps process of fuzzy inference system is outlined below for quantifying the topology (T) given the vertical (VI) and plan (PI) irregularities.

Step 1 of the analysis is the fuzzification of the input parameters. Given the presence of VI and PI, the corresponding fuzzifications can be shown to be $(\mu_L^{VI}, \mu_M^{VI}, \mu_H^{VI}) = (0, 0.40, 0.60)$ and $(\mu_L^{PI}, \mu_M^{PI}, \mu_H^{PI}) = (0, 0.40, 0.60)$, respectively.

Step 2 entails inferencing using the fuzzy rule base. For linguistic consequent parameters, Mamdani type inferencing can be used (Mamdani 1977). Mamdani's inference mechanism consists of three connectives: the aggregation of antecedents in each rule (AND connectives), implication (i.e., IF-THEN connectives), and aggregation of the rules (ALSO connectives). The IF-THEN rules can be established as:

$$R_i: \text{IF } x_1 \text{ is } A_{i1} \text{ AND } x_2 \text{ is } A_{i2} \text{ THEN } y \text{ is } B_i, \quad i = 1, \dots, n \quad (3)$$

Thus, using this rule, the T is computed to be $(\mu_L^T, \mu_M^T, \mu_H^T) = (0, 0.40, 0.60)$.

Step 3 entails the defuzzification process using a simple weighted average method, where the T = 0.80. This will be used as an input for building vulnerability.

Evaluation of the fuzzy life-cycle cost

Once the five building performance modifiers are obtained through a *walk down* survey and are fuzzified, inferencing is done through the FRB modelling. The building vulnerability coupled with the site seismic hazard is used to quantify the building damageability, given as a five-tuple membership values $(\mu_N^{BD}, \mu_L^{BD}, \mu_M^{BD}, \mu_H^{BD}, \mu_C^{BD})$. Each membership value, respectively, is associated with five discrete damage states, *none-slight* (N), *light* (L), *moderate* (M), *heavy* (H) and *major-destroyed (collapse)* (C). The building damageability membership functions μ_j can be viewed as the possibility of the structure being in *j*th limit state, for given a seismic hazard magnitude and building vulnerability. Furthermore, the building vulnerability is assumed to be time invariant.

The fuzzy membership μ_j values can be used to replace the P_i values given in Equation (2) leading to:

$$E[C(t, X)] = C_o + (C_1\mu_{N-S}^{BD} + C_2\mu_L^{BD} + \dots + C_k\mu_{M-D}^{BD}) \frac{v}{\lambda} (1 - e^{-\lambda t}) + \frac{C_m}{\lambda} (1 - e^{-\lambda t}) \quad (4)$$

Case Study with the statewide School and emergency facility retrofit program

A six-storey RC moment resisting frame building constructed in 2004 and located in Vancouver, Canada, is used to illustrate the proposed method. This building is classified as building type = {C1} and YC = 2004. The height of the six-storey building is 21.9 m, and the corresponding fundamental period T_1 can be estimated to be $T_1 = 0.76$ sec.

The objective of this analysis is to highlight impact of different performance modifiers on the building LCC. The performance modifiers and linguistic parameters are summarized in Table 1; vertical irregularity (VI) {yes, no}, plan irregularity (PI) {yes, no} and construction quality (CQ) {poor, good}. Based on possible combination of these parameters, eight scenarios were compared and analyzed (Table 1).

Table 1: Scenarios for the LCC illustrative example

Scenario	VI	PI	CQ	YC	$p(D > 20)^{\$}$
1	No	No	Good	2004	0.15
2	No	Yes	Good	2004	0.42
3	Yes	No	Good	2004	0.44
4	Yes	Yes	Good	2004	0.44
5	No	No	Poor	2004	0.68
6	No	Yes	Poor	2004	0.64
7	Yes	No	Poor	2004	0.65
8	Yes	Yes	Poor	2004	0.67

The parameters considered in the calculation of the LCC given in Equation 5 are:

- yearly rate of earthquake occurrence $v = 3/$ year,
- discount rate $\lambda = 2\%$, and
- simulation time window $t = 50$ years.

Earthquake hazard

The building damageability (Figure 1) requires quantifying seismicity at a given site. For the RC building situated in Vancouver, in agreement with the current Canadian building code (Adams and Halchuk 2003; Atkinson 2004), the spectral acceleration is calculated as a function of earthquake magnitude M and epicentral distance R . The attenuation law relating the peak spectral acceleration with the earthquake magnitude and the epicentral distance is (Adams and Halchuk 2003):

$$\ln S_A(T_n, \xi) = (b_1 + b_2(M - 6) + b_3(M - 6)^2 + b_5 \log_{10} r + b_6 + \varepsilon) \ln(10) \quad (5)$$

where $S_A(T_n, \xi)$ represents the peak spectral acceleration (PSA) in centimetres per second squared of a linear elastic single degree of freedom (SDOF) system on firm soil sites with the natural vibration period T_n and damping ratio $\xi = 5\%$; b_i ($i = 1, \dots, 6$), are the model parameter that depend on T_n and ξ ; M is the moment magnitude of the earthquake; $r = (r_{epi}^2 + h^2)^{0.5}$, r_{epi} (km) is the epicentral distance; h (km) represent a fictitious depth, and ε is the uncertain error term that is modelled as a normal variate with zero mean and standard deviation represented by σ_ε .

The parameters for Equation (5) are provided in Adams and Halchuk (2003), and the conditions considered to derive these parameters are: magnitude M range from 5.0 to 7.7, and r_{epi} is less than or equal to 100 km (Hong and Goda 2006). In Adams and Halchuk (2003), b_1 is provided as [lower, best, upper] values, and each having corresponding probabilities of [0.30, 0.40, 0.30], respectively. However, for simplicity of illustrating the proposed model, in this paper, only the best (median) value of b_1 is considered. For $T_1 = 0.76$ sec, values of b_i ($i = 1, \dots, 6$) are interpolated between $T_n = 0.5$ and $T_n = 1.0$ to be: $b_1 = 2.74184$; $b_2 = 0.41832$; $b_3 = -0.026$; $b_4 = -0.0012148$; $b_5 = -0.82104$; $b_6 = 0.2972$.

The earthquake occurrence for a given source zone is often modelled as a homogeneous Poisson process with an annual occurrence rate of the earthquake of magnitudes, and expressed in terms of the cumulative probability distribution function $F_M(m)$:

$$F_M(m) = \frac{1 - \exp(-\beta(m - M_{min}))}{1 - \exp(-\beta(M_{max} - M_{min}))} \quad (6)$$

where β is a magnitude-recurrence parameter, and M_{min} and M_{max} are the minimum and maximum magnitudes, respectively. The magnitude-recurrence parameter β is treated as an epistemic uncertainty and is provided for different earthquake sources (Hong and Goda 2006). In this paper, only one western Canada source zone (JDFN) is considered and the best estimate of β for source zone JDFN is $\beta = 2.07$ and maximum magnitude $M_{max} = 7.3$ (Hong and Goda 2006). Results of the probability of exceedence magnitudes (Equation 6) and peak spectral acceleration (Equation 5) for distance = 100 km are calculated and plotted in Figures 2a and 2b, respectively.

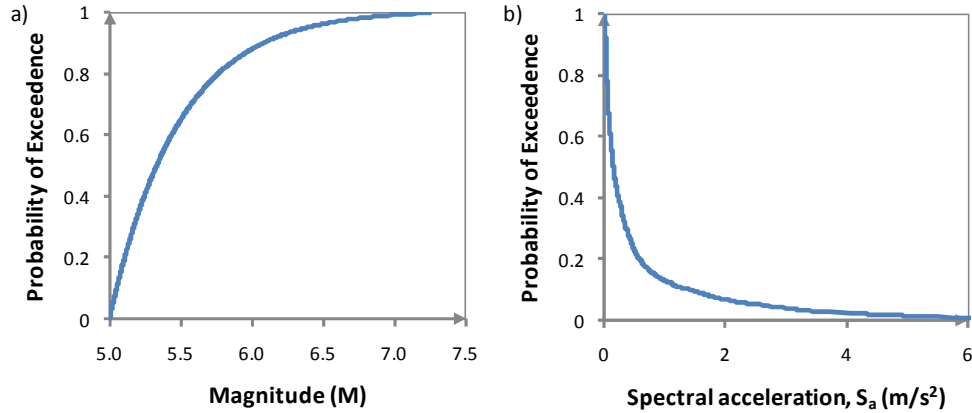


Figure 2: a) Probability of exceedence magnitudes, $M_{min} = 5$, $M_{max} = 7.3$ and $\beta = 2.07$
b) Probability of exceedence of spectral acceleration

Life-cycle cost analysis

For the LCC calculation (Equation 4), the regular maintenance cost C_m and initial cost C_o are set to zero. This will not have a bearing on the overall LCC since the objective of this example is to highlight the relative sensitivity of the building performance modifiers. The costs of the five possible damage states considered are provided in Table 2. The total square footage for the six-storey building is 1008 m^2 and the replacement cost is $C_R = \$1,301,997$.

Table 2: Direct cost corresponding to the four damage levels

Damage level	Cost (C_i)	Comments
None	$C_1 = (c_1 \times C_R) + (0 \times \text{ICAF})$	$c_1 = 0$
Light	$C_2 = (c_2 \times C_R) + (0 \times \text{ICAF})$	$c_2 = 10\%$
Moderate	$C_3 = (c_3 \times C_R) + (0 \times \text{ICAF})$	$c_3 = 20\%$
Major	$C_4 = (c_4 \times C_R) + (0 \times \text{ICAF}) + \text{Demo}$	$c_4 = 50\%$
Collapse	$C_5 = (c_5 \times C_R) + (N_L \times \text{ICAF}) + \text{Demo}$	$c_5 = 100\%$

N_L = number of lives lost, C_R = Replacement building cost, ICAF = Implied cost of averting fatality,
 c_i ($i = 1, \dots, 5$) = Percentage of damage cost, Demo = Demolition and debris removal cost (assumed to be 15% of C_R)

There has always been a great deal of discussion about the cost of human life, but despite the moral and ethical considerations, economic values are still assigned mainly by insurance companies. For instance, FEMA (1992) reports that, for the United States, the cost of injury can be taken as US\$1,000/person and US\$10,000/person for minor and serious injury, respectively. The Implied Cost of Averting a Fatality (ICAF) (Rackwitz *et al.* 2005) as an alternative to estimating the cost of saving lives was proposed by Rackwitz based on Life Quality Index (LQI) (Nathwani *et al.* 1997). The ICAF is derived from changes in mortality by changes in safety-related measures implemented in a regulation, code, or standard by the public. Therefore, in an exposed group of technical projects, with N_L potential fatalities, the ‘‘life-saving cost’’ is $C_5 = \text{ICAF} \cdot k \cdot N_L$, where k ($0 < k < 1$) is a constant that relates changes in mortality to changes in the failure rate and can be interpreted as the probability of actually being killed in the case of failure. Including the life-saving cost implies that incremental investments into structural safety should be undertaken as long as one can ‘‘buy’’ additional life years (Sánchez-Silva and Rackwitz 2004). In this study, the building is assumed to be an office building with occupancy of 4 persons/1000 ft^2 . Furthermore, in case of total collapse, the fatality is assumed to be a fraction of

1 in 5 people (ATC 1985), i.e., $k=1/5$; and for Canada ICAF= 2.07×10^6 (US\$).

The procedure to calculate the fuzzy LCC is outlined below (*Scenario 5*, Table 1). Assume that after a *walk down* survey, the following information about the building characteristics is obtained: building type = {C1}, and VI = {No}, PI = {No}, CQ = {Poor}, and YC = {2004}. Using these values, the *building vulnerability* ($\mu_N^{BV}, \mu_L^{BV}, \mu_M^{BV}, \mu_H^{BV}, \mu_C^{BV}$) can be computed, and since this value time invariant and this calculation has to be done only once.

Monte Carlo simulation (MCS) can be implemented to generate N realizations describing potential damage cost scenarios. For each simulation, random earthquake magnitudes are computed from Equation 6 by the inverse transformation method, i.e., $m=F^{-1}(u)$, $u \in [0,1]$. Then, m and b_i are used to calculate spectral acceleration $S_A(T_n, \xi)$ from Equation 5. The $S_A(T_n, \xi)$ and building vulnerability aggregated to get the *building damageability* ($\mu_N^{BD}, \mu_L^{BD}, \mu_M^{BD}, \mu_H^{BD}, \mu_C^{BD}$) = (0, 0, 0, 0.62, 0.04), which is further normalized to (0, 0, 0, 0.94, 0.06). This value is then used to calculate the life-cycle expected cost $E[C(t, X)]$ (Equation 4) using the damage cost summarized in Table 2. The result after N simulations is the distribution of expected costs in which the owner of the building will incur during the life-cycle of the structure. In this study, a total of 1000 simulations (possible earthquake scenarios) were carried out and the statistics of the data were analyzed. Expected costs were arranged in an ascending order and assign the plotting positions to each iteration using mean rank formula, $k/(N + 1)$; the results are depicted in Figure 3 (see scenario 5).

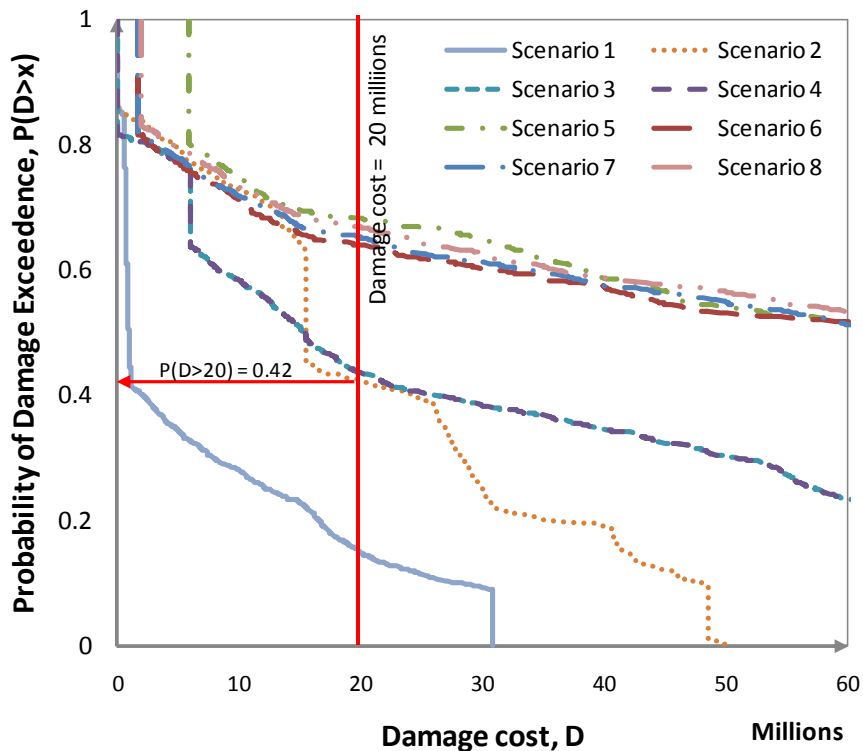


Figure 3. Life cycle cost analysis for the six-storey RC building, results of Scenarios 1 to 8.

Results of the MCS for Scenarios 1 to 8 are plotted in Figure 3. For example, for Scenario 2, the probability that damage will exceed 20 million $p(D > 20) = 0.42$ (Figure 3). Similarly, for the eight scenarios, $p(D > 20)$ are calculated and summarized in Table 1. In general, from the results depicted in Figure 3 and summarized in Table 1, it can be observed that, as expected, with the presence irregularity, the probability of damage exceedence increases. Table 1 shows that the RC building without vertical and plan irregularity (Scenario 1), $p(D > 20) = 0.15$, and for a building with plan and vertical irregularity and poor quality construction (worst possible case – Scenario 8), $p(D > 20) = 0.67$. For a building with good construction quality and YC = 2004, the vertical irregularity shows slightly higher impact than the plan irregularity shown as Scenarios 3 and 2, respectively. Whereas, the presence of both VI and PI showed no marked difference from the presence of VI only, where for both cases the $p(D > 20) \cong 0.44$. The maximum damage D_{max} observed, i.e. $p(D > D_{max}) = 0$, for Scenarios 1 to 8, respectively, are 30.8, 50.2, \$27,940, \$27,940, \$45,690, \$45,690, \$45,690, and \$45,690 million. Results of Table 1 and Figure 3 highlight that poor construction quality dominates value of the building damageability; consequently, incorporation of other building performance modifiers showed less sensitivity. The higher maximum damage D_{max} is as a result of loss of lives.

Conclusions

The life-cycle cost of a project is defined as the distribution of total cost that is incurred, or may be incurred, in all stages of the project life. The LCC requires quantification of damage probabilities under each possible earthquake scenario. In this paper, it is highlighted that buildings damage potential depends highly on aspects such as construction and topology, which are difficult to quantify. Thus, the damage estimations are prone to vagueness uncertainty that cannot be handled using traditional probabilistic methods. Based on this consideration, in this paper, a life-cycle model of structures based on a systems approach is proposed.

A heuristic based hierarchical structure is considered to quantify building vulnerability subject to different performance modifiers. Furthermore, the proposed method is extended to fuzzy life-cycle cost analysis to include a detailed methodology of damage assessment and quantification. The fuzzy life cycle cost model is proposed and highlighted with an example. The impact of different performance modifiers on the overall LCC is quantified and illustrated. Sensitivity of the performance modifiers shows that, the construction quality and plan irregularity have the most and least impact on the overall LCC, respectively.

Since the decision to retrofit existing buildings is complex and expensive undertaking, the proposed method is a tool for retrofitting prioritization of individual buildings or defining strategies for prevention and mitigation of large number of buildings. The merit of the proposed approach are: (1) it provides a framework that allows to take into account, in seismic risk assessments, variables that differ in nature (construction quality, building age, topological characteristics); (2) the model is easy to implement and use in practice (individual buildings and urban centers); (3) it can be used to gather evidence about the proneness to failure, of a building or a city, which can be later used for prevention and mitigation purposes.

Finally, this model introduces the concept of LCC analysis to a set of buildings, but it can

also include many other aspects of an urban centre. Within this context LCC is a valuable tool for long term planning and contributes to decisions regarding urban sustainability. The authors are currently extending the proposed model for the LCC optimization of spatially distributed buildings. Also, the authors are extending this model to incorporate different seismic sources and epistemic uncertainty of the seismic hazard.

References

- Adams, J. and Halchuk, S. 2003. Fourth Generation Seismic Hazard Maps of Canada: Values for over 650 Canadian Localities Intended for the 2005 National Building Code of Canada. Open-File 4459, Geological Survey of Canada, Ottawa, Ontario, Canada.
- ATC. 1985. Earthquake Damage Evaluation Data for California. Applied Technology Council, ATC-13 Report, Redwood City, California.
- Atkinson, G. 2004. An overview of developments in seismic hazard analysis. Keynote paper (5001), Proc. 13th World Conf. Earthquake Eng., Vancouver, B.C., Aug. 2-6.
- Bartlett, F.M., Hong, H.P., Zhou, W. 2003. Load factor calibration for the proposed 2005 edition of the National Building Code of Canada: Companion-action load combinations. Canadian Journal of Civil Engineering, 30, 440-448.
- Blockley, D.I. 1995. Engineering Safety. McGraw Hill, London.
- Hong, H.P. and Goda, K. 2006. A comparison of seismic-hazard and risk deaggregation. Bulletin of the Seismological Society of America, 96(6), 2021-2039.
- Mamdani, E.H. 1977. Application of fuzzy logic to approximate reasoning using linguistic synthesis. IEEE Transactions on Computers, 26(12): 1182-1191.
- Nathwani, J. S., Lind, N. C., and Pandey, M. D. (1997). Affordable safety by choice: The life quality method, Institute for Risk Research, Univ. of Waterloo, Ontario, Canada.
- Rackwitz R., Lentz, A. and Faber, M.H. 2005. Socio-economically sustainable civil engineering infrastructures by optimization. Structural Safety, 27, 187-229.
- Sánchez-Silva, M. and Rackwitz, R. 2004. Implications of the Life Quality Index in the design of optimum structures to withstand earthquakes. ASCE Journal of Structural engineering, 130(6): 969-977.
- Sánchez-Silva, M., Klutke, G-A and Rosowsky, D. 2009. Life-cycle performance of structures subject to multiple deterioration mechanisms. (Under review).
- Tesfamariam, S. and Saatcioglu, M. 2008. Risk-based seismic evaluation of reinforced concrete buildings. Earthquake Spectra, 24(3), 795-821.
- Wen, Y.K. 2001. Minimum lifecycle cost design under multiple hazards. Reliability Engineering and System Safety, 73, 223-231.
- Zadeh, L.A. 1965. Fuzzy sets. Information Control, 8, 338-353.
- Zadeh, L.A. 1973. Outline of a new approach to the analysis of complex systems and decision processes. IEEE Transactions on Systems, Man, and Cybernetics, 3, 28-44.