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A CRITICAL REVIEW OF NUMERICALLY PREDICTED ACCELERATIONS IN NONLINEAR HYSTERETIC SYSTEMS

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ABSTRACT

When evaluating the seismic behaviour of a structural system, peak floor accelerations must be considered because they determine the design forces for floor diaphragms and also affect the performance of nonstructural elements. Some studies of new self-centering systems, as well as of base isolation systems, have suggested that structures that include these systems may be subject to relatively large accelerations following abrupt stiffness changes. This paper examines a simplified two-degree-of-freedom system to gain insight into the accelerations that occur following abrupt stiffness changes in a dynamic system. These accelerations can be particularly large following a stiffness increase occurring at a high velocity, and they are exacerbated when an element has a very large initial stiffness. Both of these situations are common in analytical models of self-centering systems. Significantly smaller peak accelerations may be calculated for systems that are initially less rigid, even though these changes may have little effect on other seismic response parameters.

Introduction

Over the last 20 years, many innovations have been proposed to improve the seismic response of structures. These innovations include systems that use unbonded post-tensioning or new materials to achieve self-centering behaviour, as well as various base-isolation systems (Christopoulos and Filiatrault 2006). At the same time, structural engineering research has benefitted significantly from substantial improvements in computing capabilities.

Peak floor accelerations are an important aspect of the seismic response of structures because they determine the forces that floor diaphragms must be able to carry safely to the seismic force-resisting system, and because excessive floor accelerations can cause significant damage to acceleration-sensitive nonstructural components, such as fire suppression systems and electronic equipment. When examining friction base-isolated systems, both Kelly (1982) and Dolce and Cardone (2003) identified very large acceleration spikes that they attributed to a force discontinuity when the isolator reversed velocity. Similarly large acceleration spikes were found by Rodríguez et al. (2002) in self-centering systems that were modelled using an origin-centered

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hysteresis rule. The authors wrote that the large accelerations they calculated were because that rule had an abrupt change of stiffness at the origin, and this change would be more gradual in real systems. Tremblay et al. (2008) discussed a representative acceleration spike in a system with self-centering axial braces, showing that the large acceleration was caused by a force imbalance between two adjacent levels. This force imbalance developed quickly because of the high elastic stiffness and sharp stiffness transitions, so the authors believed that more gradual stiffness changes would reduce the peak accelerations. Wiebe and Christopoulos (2009a) suggested that abrupt stiffness increases cause much larger acceleration spikes than abrupt stiffness decreases, and that the magnitude of acceleration spikes is also related to the velocity of the system at the time that the stiffness changes. While all of these studies have shown a connection between acceleration spikes and modelling decisions, the mechanism of this relationship has not been thoroughly investigated and explained.

This paper explores how abrupt stiffness changes can cause acceleration spikes. It suggests that the accelerations that have been computed have a physical basis but may be magnified by numerical modelling assumptions. First, a simplified system is studied in order to identify the parameters that can lead to large acceleration spikes. These parameters are then examined, showing how modelling decisions can affect the computed acceleration response. The observations that are made for simplified systems are shown to apply also to a larger, more complex example.

Acceleration Spikes in Two-Degree-of-Freedom Systems

The acceleration spikes that have been observed following stiffness changes in multi-degree-of-freedom (MDOF) systems can be understood by considering a simple two-degree-of-freedom (2DOF) system. While a single-degree-of-freedom (SDOF) system would be simpler, the acceleration spikes that occur in an SDOF system due to abrupt stiffness changes will generally be small relative to the accelerations that would occur during the seismic response of a linear SDOF system (Wiebe and Christopoulos 2009b). The 2DOF system that is considered in this section is not intended to be a direct simplification of any particular real MDOF system, but it does represent an MDOF system at a time when the acceleration response of a mass is dominated by two modes. For example, a base-rocking structure may be represented by two degrees of freedom, one to describe the rocking joint, and the other to describe the total response of the rest of the structure above the rocking joint.

The system shown in Fig. 1 (a) is considered for a unit total mass, and the mass ratio is defined as:

$$\mu = m_1 / m_2 \tag{1}$$

where m_1 and m_2 are the masses of degrees of freedom (DOFs) 1 and 2, respectively. The mass ratio is taken as 0.01 for this example, which is based on a typical rocking system, where most of the mass is located away from the rocking joint. The first spring is assigned the self-centring hysteretic loop shown in Fig. 1 (b). Spring 2 is linear elastic, and its stiffness, k_2, is related to the initial stiffness of spring 1, k_1, by the initial spring stiffness ratio:

$$\kappa = k_1 / k_2 \tag{2}$$

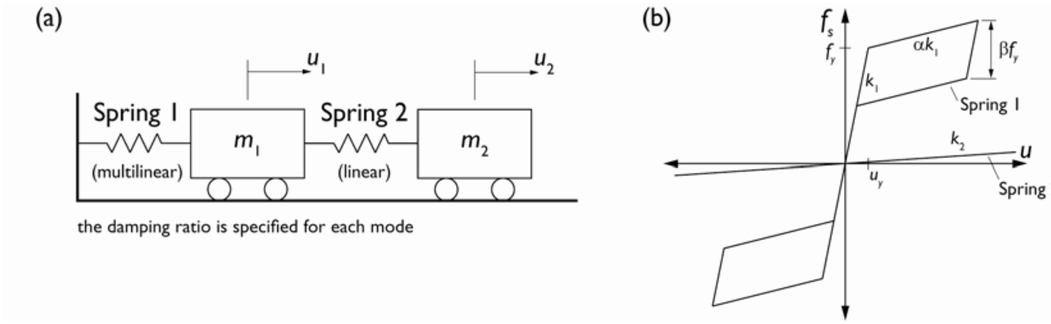

Figure 1. (a) Schematic of 2DOF system; (b) Spring force-deformation relationships

This ratio is taken as 100 for this example. This is based on a rocking system, where the stiffness of the rocking joint is initially very large relative to the lateral stiffness of the superstructure. The yield force of spring 1, f_y, is taken as 10% of the total weight of the system:

$$f_y = 0.10(m_1 + m_2)g \tag{3}$$

The nonlinear stiffness ratio of spring 1 is taken as $\alpha = 0.05$ and the energy dissipation parameter as $\beta = 0.0$ (see Fig. 1 (b)) because these choices highlight the typical response of many nonlinear systems. It will be seen that the relative values of stiffness have the greatest influence on the acceleration spikes, not the energy dissipation ratio, so the observations are also relevant for other force-deformation relationships. The period of the first mode is specified as 1.0s, and this results in a second mode period of 0.01s. The constant damping matrix is defined to give 3% viscous damping in both modes.

A static force is applied at DOF2 that causes a displacement there of 1.3 times the displacement of DOF2 when spring 1 yields ($u_{2,0} = 1.3u_{2,y}$), resulting in a deformation in spring 1 that is 6.1 times its yield deformation ($u_{1,0} = 6.1u_y$). Fig. 2 shows the response of the system after the force is released, computed using an iterative Newmark-Beta integration scheme to integrate the equations of motion. Sensitivity analyses showed that a timestep of 0.0002s was needed to ensure convergence (Wiebe and Christopoulos 2009b).

The upper left corner of Fig. 2 plots the force-deformation relationship of each spring. Aligned to the right of this is the force time-history for both springs, as well as the damping forces computed at each DOF. Below the force time-history are the time-histories of the accelerations of both masses, the velocities within both springs, and the displacements of both masses. For all plots, all black lines refer to spring and mass 1, while all red lines relate to spring and mass 2. The velocity within the second spring is normalized by the nonlinear spring stiffness ratio, $\alpha\kappa$, and the displacement of DOF2 is normalized by the initial spring stiffness ratio, κ. This choice of scaling parameters will be explained subsequently. Beyond the first 1.25 cycles of response that are shown, the hysteretic response of spring 1 is linear.

The damping forces (upper right corner of Fig. 2) are very small, particularly at DOF1, even at the times that the peak accelerations are reached. Therefore, they are neglected in the explanation that follows; the response is expected to be similar for any reasonable assumption of the amount and type of damping.

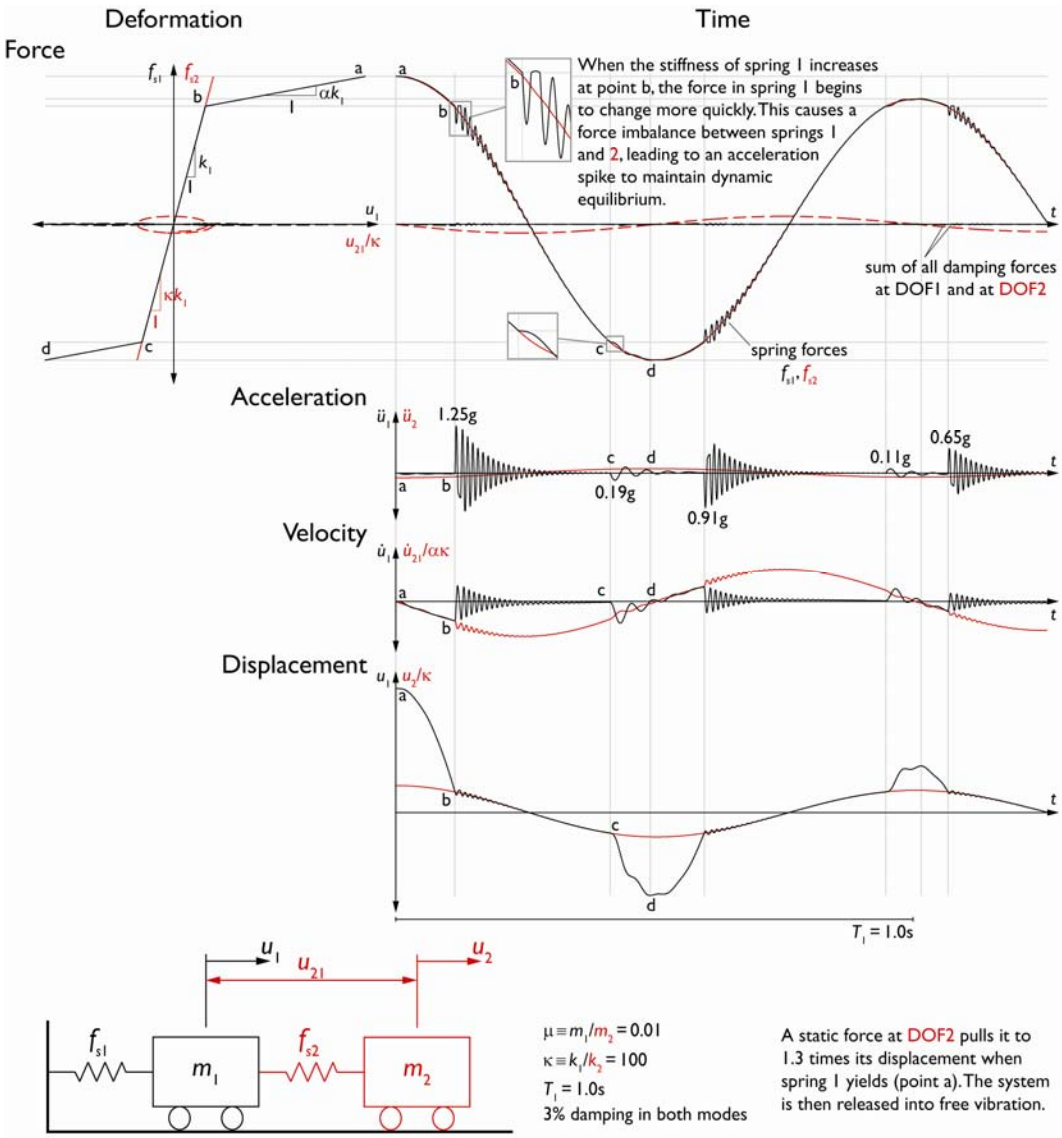

Figure 2. Response of 2DOF system in free vibration

Point a to point b

The force that had been applied at DOF2 is released at point a. At this moment, the forces in springs 1 and 2 are equal (see force time-history at top right of Fig. 2), and the velocity within each spring is zero (see velocity time-history at the lower middle right of Fig. 2).

As the system moves from point a to point b, the forces in the two springs remain similar to each other (see force time-history in the upper right corner of Fig. 2). This is because, at any time that the two forces are not equal to each other, mass 1 will accelerate in the direction that reduces the difference in force. In order for these two forces to stay equal, the velocities within the two springs must be related by the ratio of the tangent stiffness of each spring:

$$\dot{u}_1 = \dot{u}_{21} / \alpha\kappa \quad (4)$$

where $\dot{u}_1$ and $\dot{u}_{21}$ are the velocities within springs 1 and 2, respectively. Thus, when the velocity of spring 2 is normalized by the nonlinear spring stiffness ratio, $\alpha\kappa$, it corresponds to the velocity of spring 1 in the velocity time-history (lower middle right of Fig. 2).

Point b to point c

At point b, the stiffness of spring 1 suddenly increases by a factor of 20. The velocity within each spring, however, cannot change instantaneously. Spring 2 (red line), which has the same stiffness after b as before and a similar velocity, continues to change force at a similar rate. This is seen in the first detail within the force time-history at the top right corner of Fig. 2. Spring 1 (black line), which has a much higher stiffness after point b but a similar velocity, now changes force approximately 20 times as quickly as before. The increased rate of change of force within spring 1 causes a force imbalance between the two springs, with the force in spring 1 becoming much less than the force in spring 2. Therefore, mass 1 must accelerate. Because m_1 is small, the acceleration is large, 1.25g, as seen in the acceleration time-history of Fig. 2.

The acceleration is sustained long enough to change the sign of the velocity of DOF 1 (the black line in the velocity-time plot), so spring 1 begins to elongate. The force in that spring overshoots the force in spring 2, as seen in the first detail within the force time-history at the top right corner of Fig. 2, and the result is an oscillatory response at DOF1. In fact, the force overshoots enough that the stiffness of spring 1 reduces back to the post-yield stiffness, thus limiting the force. The oscillatory component of the response damps out over time, and the forces in the two springs become similar once again, with a new relationship between the velocities of the two springs.

Point c to point d

When the stiffness of spring 1 changes again at point c in Fig. 2, a force imbalance develops again. Whereas the previous imbalance was because the force in spring 1 suddenly began to change 20 times as quickly as before, here it suddenly begins to change 20 times more slowly than before. Therefore, the imbalance develops more slowly and does not become as large, so the acceleration spike in the acceleration time-history of Fig. 2, while present, is significantly smaller (0.19g).

After point d

The system reaches its maximum displacement in the negative direction at point d of Fig. 2. It then reverses direction, exhibiting the same response characteristics as before at each change in the stiffness of spring 1.

Modal influences

The influence of the second mode is most apparent in the acceleration and velocity response of DOF1. The second mode is also responsible for high-frequency oscillations that occur immediately after each stiffness change in the force time-history of DOF1, which is otherwise controlled mostly by the first mode. The response of DOF2 is dominated by the first mode, apart from some minor velocity oscillations at the times when DOF1 has an acceleration spike.

Effect of System Properties on Acceleration Spikes in 2DOF Systems

Wiebe and Christopoulos (2009b) considered the coupled nonlinear second order ordinary differential equations that describe response of a 2DOF system like the one considered above. By considering the response beginning immediately after a change in the stiffness of spring 1, assuming a constant value for that stiffness, they derived the following approximation:

$$\ddot{u}_{1,\text{peak}} = -\frac{\dot{u}_2(t^*)\overline{\omega}_2^{\text{after}}}{\eta\lambda+1}\left(\frac{1}{\Psi}\right)\left(\frac{\lambda}{\lambda+1}\right)\left(\frac{\Psi+1}{2}-\eta+\frac{\Psi-1-\mu}{2\lambda}\right) \tag{5}$$

where $\dot{u}_2(t^*)$ is the velocity of DOF2 immediately before the change in stiffness of spring 1, $\overline{\omega}_2^{\text{after}}$ is the instantaneous natural frequency of the second mode after the stiffness change, λ is the ratio of the stiffness of spring 1 after it changes stiffness to the stiffness of spring 2, η is the ratio of the stiffness of spring 1 before it changes stiffness to the stiffness of spring 1 after it changes stiffness, and Ψ is defined as:

$$\Psi = \frac{\sqrt{(\lambda+\mu+1)^2-4\lambda\mu}}{\lambda+1} \tag{6}$$

Eq. 5 assumes that the forces in the two springs are equal before spring 1 changes stiffness, and it also assumes that the mass ratio is small and that the stiffness ratio is large. The assumptions made in deriving Eq. 5 are discussed more fully by Wiebe and Christopoulos (2009b), and the equation is validated there using time-history analyses of several 2DOF systems with varying properties. The first term in Eq. 5 is a velocity term, showing that systems that move with higher velocity through a stiffness change will generally have larger acceleration spikes due to that stiffness change. The second term is the instantaneous natural frequency of the second mode, which may be written as a linear function of the initial system natural frequency, Ω:

$$\Omega = \sqrt{\frac{\left(1/k_1+1/k_2\right)^{-1}}{m_1+m_2}} \tag{7}$$

Therefore, relatively large accelerations due to stiffness changes should be expected in systems with high natural frequencies. Normalizing Eq. 5 by $\dot{u}_2(t^*)$ and by Ω, the remaining terms of Eq. 5 can be written in terms of three parameters: the mass ratio μ, the initial spring stiffness ratio κ, and the nonlinear spring stiffness ratio $\alpha\kappa$. The parameter η is equal to α in cases where the stiffness increases, and to $1/\alpha$ when the stiffness decreases. Fig. 3 shows how the peak normalized acceleration spike varies with these parameters. Each point in this figure represents the peak normalized acceleration in a unique 2DOF system. Each subplot within the total array of 25 subplots contains two lines: the solid line shows the peak normalized acceleration as a function of mass ratio for abrupt stiffness increases, and the dotted line shows the peak normalized acceleration as a function of mass ratio for abrupt stiffness decreases. For every subplot, the normalized acceleration is consistently much larger at stiffness increases than at stiffness decreases. Stiffness increases in traditional systems occur only when the velocity is zero, so they do not result in acceleration spikes. In self-centering systems, however, the stiffness increases at a high velocity, potentially resulting in very large accelerations. This effect was seen in the 2DOF system that was discussed earlier. The peak normalized acceleration decreases as mass 1 becomes larger relative to mass 2.

Moving from the top row of subplots in Fig. 3 toward the bottom row, the initial stiffness of spring 1 becomes larger relative to the stiffness of spring 2. Moving from the left to the right column within Fig. 3, the nonlinear stiffness of spring 1 becomes smaller relative to the stiffness of spring 2. The acceleration spikes are largest for systems where spring 1 is initially much stiffer than spring 2 (bottom row), but where the stiffness of spring 1 becomes much less than that of spring 2 (right column).

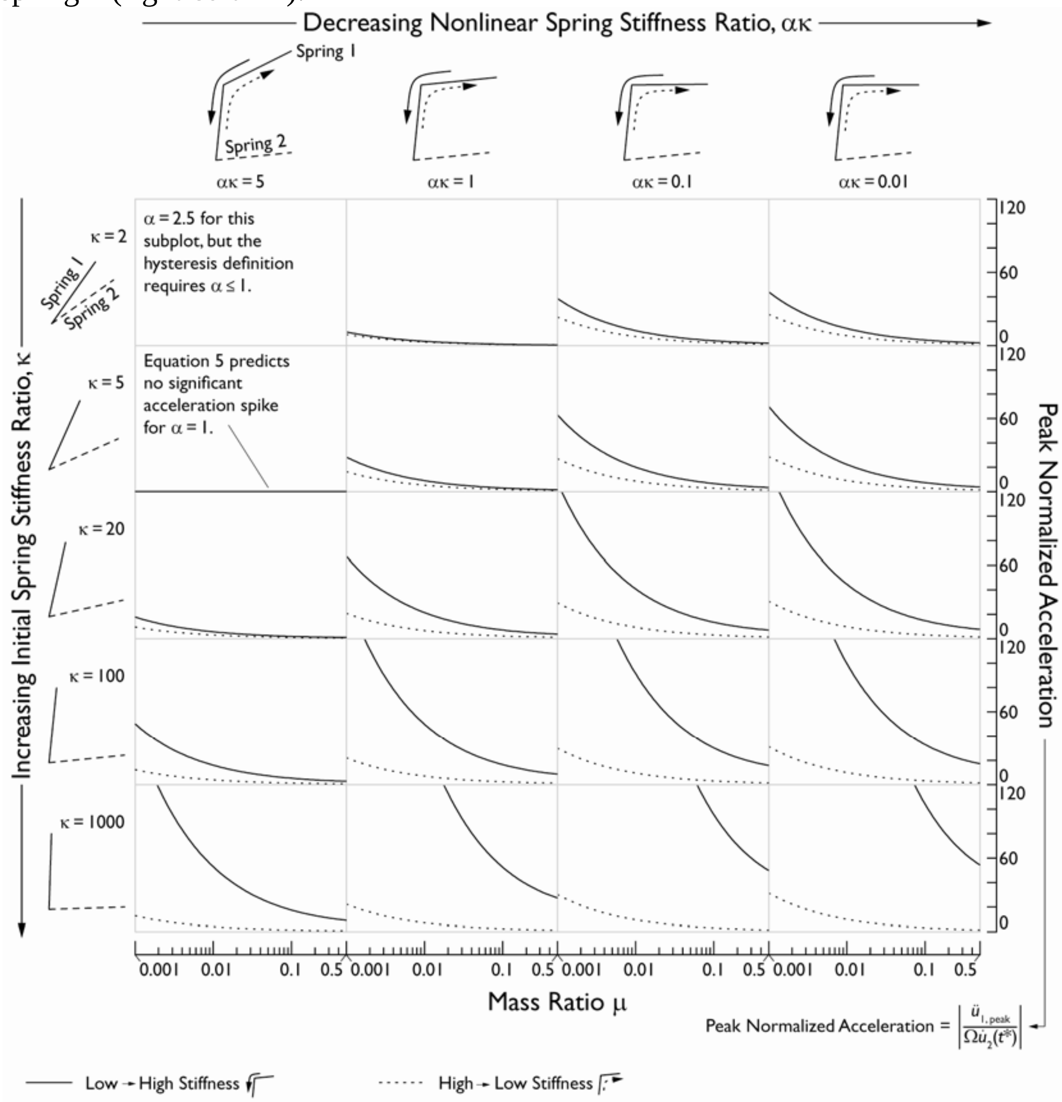

Figure 3. Effect of mass ratio μ, initial spring stiffness ratio κ, and nonlinear spring stiffness ratio ακ on peak normalized acceleration predicted by Eq. 5

This situation is typical of many base isolation systems and self-centering systems: prior to activation, these systems are usually close to rigid, but after activation, the nonlinear response of these systems often accounts for most of the global deformations. When modelling these systems, nonlinear elements, such as springs modeling the gap response of rocking systems, are often assigned an arbitrarily large initial stiffness that is intended to approximate rigid response. Fig. 3 shows that increasing this initial stiffness may greatly increase the calculated accelerations. However, the influence on calculated peak displacements is typically small (Wiebe and Christopoulos 2009b).

Acceleration Spikes in an MDOF System

In an earlier study of self-centering base-rocking systems with multiple rocking joints, Wiebe and Christopoulos (2009a) modelled the response of each rocking joint by using a rotational spring with a self-centering moment-rotation response. For a twenty-storey wall, up to ten rocking sections were considered over the height of the structure. These springs were intended to be nearly rigid in the elastic range, but to provide most of the system deformation in the nonlinear range. The peak computed displacements, drifts, and moments were similar regardless of the number of rocking joints, but the peak floor accelerations were very large, up to 17g, at the floor levels adjacent to self-centering joints.

Based on the insights gained from the 2DOF example that has been presented, the analyses that were performed by Wiebe and Christopoulos (2009a) were recalculated after reducing the stiffness of the rotational springs by factors of 10 and of 50. The results of these analyses are superimposed on the original results in Fig. 4. The displacement envelope (Fig. 4a) changes by no more than 4% with the reduced spring stiffnesses. The acceleration envelope (Fig. 4b), however, is reduced by up to 56% at the levels with the highest accelerations, although it also increases at some other levels. The effect on the hysteretic response of the base hinge (Fig. 4c) is very small.

A new hysteretic model has recently been developed that allows the corners of a flag-shaped hysteresis to be rounded (Wiebe and Christopoulos 2009b). A smooth transition between stiffnesses is generated using a degree of rounding that is a ratio of the anticipated maximum deformation, with a degree of rounding of 0.0 corresponding to a sharp-cornered hysteresis. Fig. 5 shows the response envelopes for the same system as Fig. 4, but with varying degrees of rounding of the hysteretic corners. The effect is similar: while the peak displacements are nearly unaffected by rounding the hysteretic corners, the peak accelerations are significantly reduced by even small amounts of rounding.

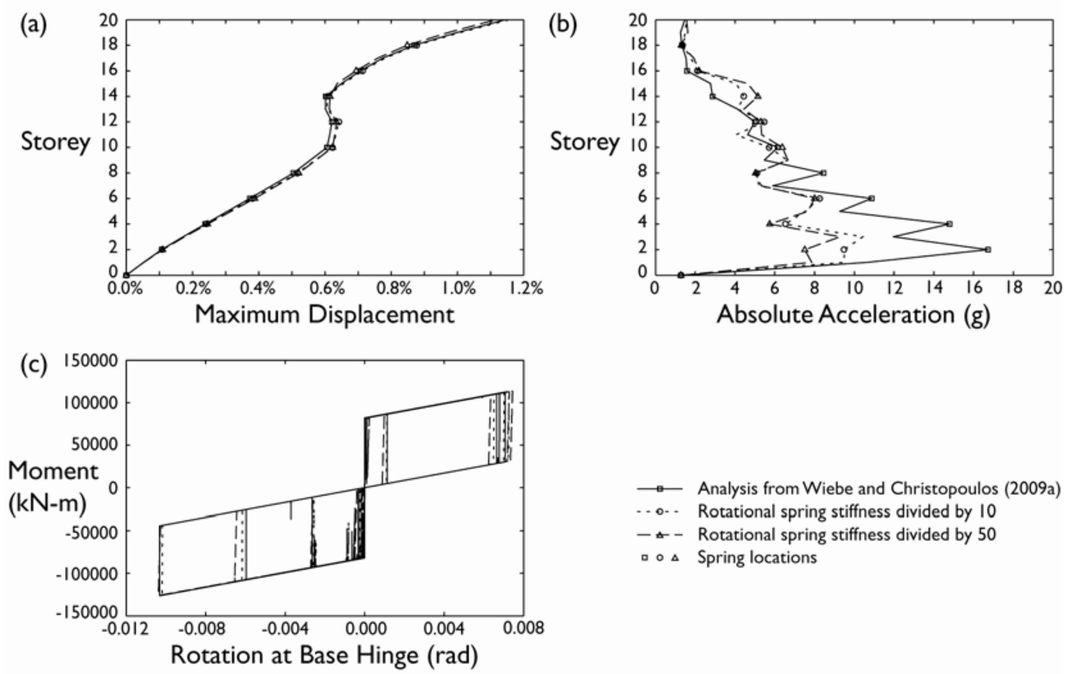

Figure 4. Response envelopes for systems with varying rotational spring stiffness: (a) displacement; (b) acceleration; (c) moment-rotation response of base hinge

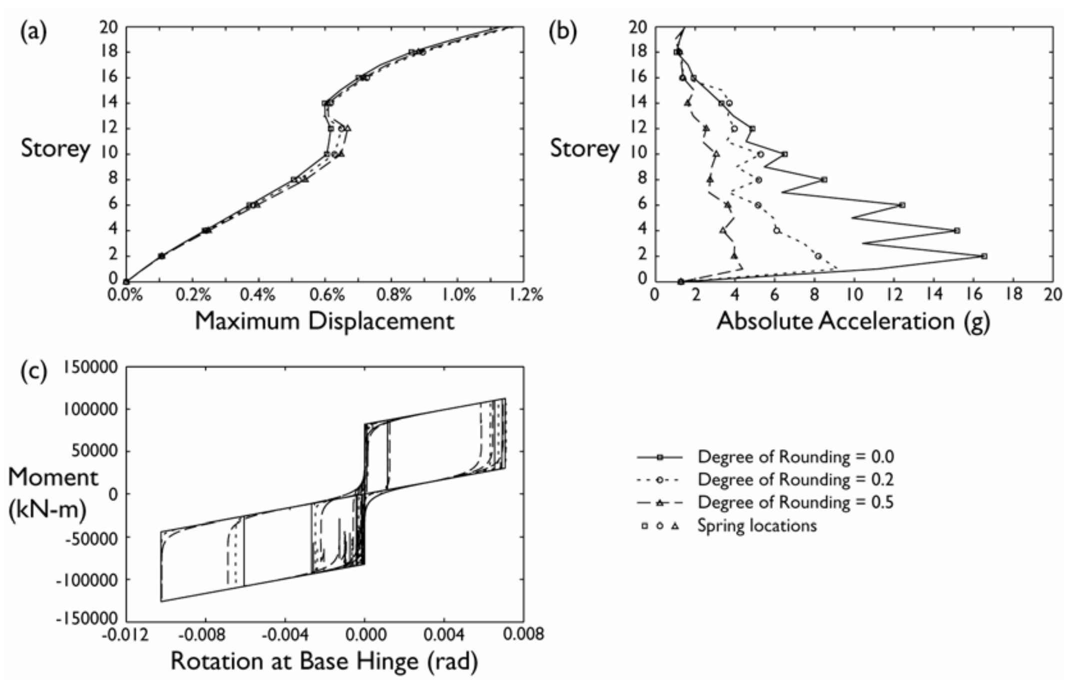

Figure 5. Response envelopes for systems with varying degrees of rounding: (a) displacement; (b) acceleration; (c) moment-rotation response of base hinge

Conclusions

Peak floor accelerations are an important aspect of the performance-based seismic design and response of structures, and some studies have suggested that abrupt stiffness changes may cause very large floor acceleration demands. By examining a two-degree-of-freedom representation of a nonlinear system, this paper has shown that an abrupt stiffness change in one element is likely to result in a force imbalance between the nonlinear member and the combined effect of all adjacent members. This force imbalance can cause a significant high-frequency acceleration spike at the location of the force imbalance.

Using a mathematical model of the behaviour of a two-degree-of-freedom system after a stiffness change, the magnitude of the acceleration spike was shown to increase with the velocity of the system before the stiffness change, the natural frequency of the system, and the ratio of the initial stiffness of the nonlinear spring to the stiffness of the rest of the structure. The acceleration spike decreases as the mass considered becomes larger and as the nonlinear stiffness of the spring becomes larger. Acceleration spikes are typically greatest after stiffness increases at high velocity, which occur in self-centering systems but not in traditional systems. The common modelling practice of assigning nonlinear elements an arbitrarily large initial stiffness in order to model near-rigid behaviour may result in very large accelerations being calculated.

The applicability of these findings was demonstrated with an example of a self-centering multi-degree-of-freedom system. Very large accelerations had been calculated for this system as it was originally modelled, but reducing the initial stiffness of the self-centering elements by a factor of 50 reduced the peak accelerations by more than 55%, while the effect on peak displacements was minimal. Similarly, implementing a new hysteretic model that rounds the corners of a flag-shaped hysteresis reduced the peak accelerations by nearly 80%, again with very little effect on peak displacements.

Based on these results, parametric studies regarding stiffness assumptions for nonlinear systems are recommended whenever accelerations are of particular concern for analytical models. The true rigidity of a nonlinear element that is initially nearly rigid is often not known precisely, but the accelerations near that element may be very sensitive to the rigidity that is assumed. Furthermore, idealizing the gradual stiffness change that occurs by using a piecewise linear hysteretic model may overestimate the peak accelerations, even while accurately calculating peak displacements. These factors should be considered in the analysis and design of nonlinear systems.

Acknowledgments

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