Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 1424

AN EVALUATION OF INELASTIC RESPONSES OF HYSTERETIC SYSTEMS UNDER BIDIRECTIONAL SEISMIC EXCITATIONS

C. S. Lee[1] and H.P. Hong[2]

ABSTRACT

An assessment of the statistics of peak displacement ductility demand aimed at developing simple equations for estimating ductility demand of structures under bidirectional seismic excitations is carried out for idealized inelastic 2-degree-of-freedom systems. The hysteretic behavior of the structural system is represented by the Bouc-Wen model with biaxial interaction. Based on the dynamic analysis results, it was concluded that in general the ductility demand under bidirectional seismic excitations are much higher than that under unidirectional excitations. Therefore, underestimation of the damage and seismic risk is likely to occur if the effect of bidirectional excitations is ignored. Simple approximate equations are proposed for estimating the ductility demand and normalized dissipated hysteretic energy. These approximations used the results of unidirectional excitations for single-degree-of-freedom systems. Nonlinear dynamic responses obtained for the considered records suggest that the proposed approximations are adequate. The results also shown that the ductility demand under bidirectional excitations can be modeled by a lognormal or Frechet variate, depending on the vibration period, and that the coefficient of variation of ductility demand under bidirectional excitations is comparable to that obtained under unidirectional excitations.

Introduction

Statistics of the maximum inelastic displacement of a structure under seismic excitations are needed for an efficient quantitative seismic risk assessment of structures. The statistics used for such an assessment (ATC 2005, FEMA/NIBS 2003, Goda and Hong 2008) are often based on an equivalent single-degree-of-freedom (SDOF) system subjected to uni-directional excitation; while structures are subjected to multidirectional excitations are likely to be affect at least by the two orthogonal horizontal ground excitations. The impact of the bidirectional seismic excitations on the inelastic responses was considered by Yeh and Wen (1990), and by De Stefano and Faella (1996). The later was focused on bilinear hysteretic 2-degree-of-freedom (2DOF) system, representing a single column or an idealized symmetric one-story building with a rigid deck supported by identical columns. However, statistics of ductility demand with strength/stiffness

[1] PhD candidate, [2] Professor, Dept. of Civil and Environmental Engineering, University of Western Ontario, London, ON, N6A 5B9, Canada

degrading behavior and simple empirical equations for estimating the ductility demand are lacking.

The main objectives of the present study are to investigate statistics of the peak ductility demand, and to develop empirical equations to estimate ductility demand of structures under bidirectional seismic excitations. The structure is idealized as inelastic 2DOF systems with hysteretic behavior represented by the Bouc-Wen model with biaxial interaction (Park et al. 1986, Yeh and Wen 1990, Wang and Chang 2007). For the analyses, 381 California records from 31 seismic events selected from the Next Generation Attenuation Database (PEER Center, 2006) was employed. Details on the formulation, analyses and results are given in the following sections.

Structural Model

To evaluate the structural response under bi-directional excitations, a 2DOF Bouc-Wen hysteretic model is considered and shown in Figure 1. Similar to De Stefano and Faella 1996, it is viewed that the model can be used to represent a single column or an idealized symmetric one-story building with a rigid deck supported by identical columns under bidirectional excitations. The governing equations of motion along the *X* and *Y* axes for the considered model are (Park et al. 1986),

$$m\ddot{u}_x + c_x\dot{u}_x + \alpha k_x u_x + (1-\alpha)k_x z_x = -m\ddot{u}_{gx} \tag{1a}$$

and,

$$m\ddot{u}_y + c_y\dot{u}_y + \alpha k_y u_y + (1-\alpha)k_y z_y = -m\ddot{u}_{gy} \tag{1b}$$

where k, c, z, and $\ddot{u}_g$ are the stiffness, damping coefficient, hysteretic displacement and ground acceleration, respectively, the subscript x and y are used to indicate that the quantities are associated with *X*- and *Y*-axis respectively.

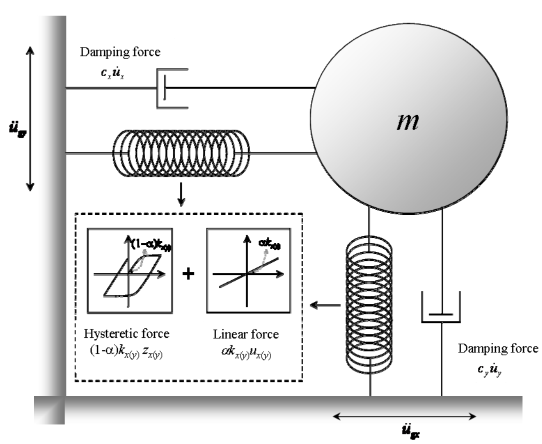

Figure 1. Two-degree-of-freedom hysteretic system subject to bidirectional seismic excitation.

For orthotropic system, k_x and k_y can differ and/or the yield displacements Δ_x and Δ_y, may not be equal. In such a case, the system can be transformed into an equivalent isotropic system

(Park et al. 1986), with the displacements denoted by u_x, z_x, u_{y1} and z_{y1}, where,

$$u_{y1} = \Delta \cdot u_y,\ z_{y1} = \Delta \cdot z_y, \text{and } q_{y1} = (Q_x / Q_y) q_y \tag{2}$$

in which $\Delta = \Delta_x / \Delta_y$, $q_y = \alpha k_y u_y + (1-\alpha) k_y z_y$; the yield strength along the Y-axis $Q_y = k_y \Delta_y$.

The transformed displacement u_{y1} and hysteretic displacement z_{y1}, and u_x and z_x are governed by the following coupled equations (Park et al. 1986, Wang and Wen 2000),

$$\dot{z}_x = \frac{1}{\eta}\left[A\dot{u}_x - \nu z_x I\right] \tag{3a}$$

and,

$$\dot{z}_{y1} = \frac{1}{\eta}\left[A\dot{u}_{y1} - \nu z_{y1} I\right] \tag{3b}$$

where,

$$I = |\dot{u}_x||z_x|^{n-1}\left[\beta + \gamma\,\mathrm{sgn}(\dot{u}_x z_x)\right] + |\dot{u}_{y1}||z_{y1}|^{n-1}\left[\beta + \gamma\,\mathrm{sgn}(\dot{u}_{y1} z_{y1})\right] \tag{3c}$$

in which $\eta = 1 + \delta_\eta E_{n,b}$ and the parameter δ_η controls the stiffness degradation; $\nu = 1 + \delta_\nu E_{n,b}$, and the parameter δ_ν controls the strength degradation; $E_{n,b}$ represents the normalized dissipated hysteretic energy for biaxial responses, which will be discussed in the following.

The definition of the dissipated hysteretic energy for biaxial responses was given by Park et al. (1986), Yeh and Wen (1990) and Wang and Chang (2007). These definitions are focused on $n = 2$. To define $E_{n,b}$ for different n values, let u and z denote the displacement and hysteretic displacement, respectively, for a displacement path along the Θ-axis, defined by a line passing through the origin in u_x and u_{y1} plane with a counterclockwise rotation angle θ from u_x-axis. It can be shown,

$$u_x = u\cos(\theta),\ \text{and}\ u_{y1} = u\sin(\theta) \tag{4a}$$

and,

$$z_x = z\cos(\theta),\ \text{and}\ z_{y1} = z\sin(\theta) \tag{4b}$$

By substituting Eq. (4) into Eqs. (3a) and (3b), it can be shown that each of these equations reduces to,

$$\dot{z} = \frac{1}{\eta}\left[A\dot{u} - \nu\left(\beta + \gamma\,\mathrm{sgn}(\dot{u}z)\right)z|\dot{u}||z|^{n-1}\left(\left|\cos^n\theta\right| + \left|\sin^n\theta\right|\right)\right] \tag{5}$$

and represents uniaxial Bouc-Wen hysteretic model. If A=1 and $\delta_\nu = 0$, it can be shown that the yield displacement along the Θ-axis (in the u_x and u_{y1} plane), Δ_θ, is given by,

$$\Delta_\theta = \Delta_x \left(\left| \cos^n \theta \right| + \left| \sin^n \theta \right| \right)^{-1/n} \tag{6}$$

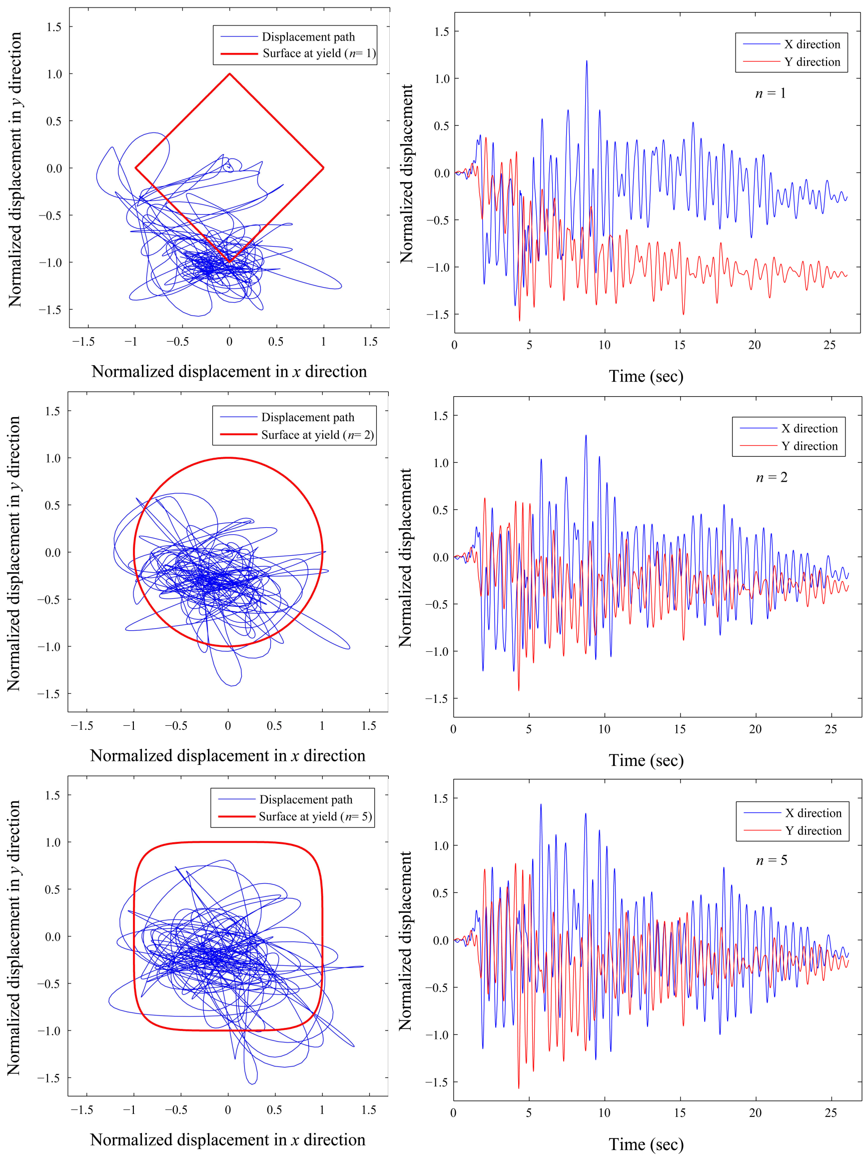

Figure 2. Surface at yield defined by isotropic biaxial Bouc-Wen model and response time-history for a selected record.

This equation shows the yield displacement along different directions could differ. If n equals 1, the surface at yield becomes a rhombus, implying a very significant interaction for the biaxial yield responses. If n equals 2, the surface at yield is a circle, representing equal yield capacity along any direction. The interaction may not be significant and could be ignored, if n is greater than about 5 since the surface at yield approach to a square. An illustration of the surface at yield as well as corresponding response time-history for a record is shown in Figure 2 for n equal to 1, 2 and 5. The surfaces at yield represented by n between 1 and 2 are commonly employed in structural analyses for a column (Takizawa and Aoyama 1976, MacGregor 1998, De Stefano and Faella 1996).

Since the normalized dissipated hysteretic energy for the uniaxial response is defined as a quantity that is proportional to the integration of the product of the normalized hysteretic displacement and the normalized velocity, by extending this definition for biaxial responses $E_{n,b}$, can be written as,

$$E_{n,b} = (1-\alpha)\int_0^t \frac{\vec{z}}{\Delta_\theta} \bullet \frac{\dot{\vec{u}}}{\Delta_\theta}\, dt \tag{7a}$$

where $\vec{z}$ represents the vector of hysteretic displacement that equals $\left(z_x,\ z_{y1}\right)$, and $\dot{\vec{u}}$ represents the velocity vector $\left(\dot{u}_x,\ \dot{u}_{y1}\right)$. Substituting Eq. (6) into (7a) results in,

$$E_{n,b} = \frac{1-\alpha}{\Delta_x^2}\int_0^t \left(z_x\dot{u}_x + z_{y1}\dot{u}_{y1}\right)\left(\left|\cos^n\theta\right| + \left|\sin^n\theta\right|\right)^{2/n} dt \tag{7b}$$

where z_x, z_{y1}, $\dot{u}_x$, $\dot{u}_{y1}$ and $\theta = \tan^{-1}(u_{y1}/u_x)$ are time-varying variables.

For n equal to 2. Eq. (7b) reduces to, except a constant, those used by Park et al. (1986), Yeh and Wen (1990), and Wang and Wen (2000).

Ductility Demand

Since the trajectory of the displacement of the mass of the system in terms of u_x and u_{y1} can be expressed in terms of $(\theta, u_\theta(t))$, where

$$\left(\theta, u_\theta(t)\right) = \left(\tan^{-1}\left(u_{y1}/u_x\right), \sqrt{u_x^2 + u_{y1}^2}\right) \tag{8}$$

one could introduce a normalized peak displacement under bidirectional excitations, $\mu_{b,\max}$ defined by,

$$\mu_{b,\max} = \max\left(\left|u_\theta(t)\right|/\Delta_\theta\right). \tag{9}$$

This definition and Eq. (6) leads to,

$$\mu_{b,\max} = \max\left(\left|u_x/\Delta_x\right|^n + \left|u_{y1}/\Delta_x\right|^n\right)^{1/n} = \max\left(\left|u_x/\Delta_x\right|^n + \left|u_y/\Delta_y\right|^n\right)^{1/n} \tag{10}$$

Note that $\mu_{b,\max} > 1$ represents the peak ductility demand for the system under bidirectional excitations, while $\mu_{b,\max} < 1$ simply indicates that the maximum displacement is less than the yield displacement. Eq. (10) suggests that $\mu_{b,\max}$ may be approximated by,

$$\mu_{b,\max} \approx \max\left|\left(\mu^{n}_{bx,\max} + \mu^{n}_{by,\max}\right)^{1/n}\right| \tag{11}$$

where $\mu_{xb,\max} = \max(|u_x / \Delta_x|)$ represents the normalized peak displacement along the X-axis, and $\mu_{by,\max} = \max(|u_y / \Delta_y|)$ represents the normalized peak displacement along the Y-axis. Alternately, $\mu_{b,\max}$ may be approximated by $\mu_{b,ap}$,

$$\mu_{b,ap} = \max\left|\left(\mu^{n}_{x,\max} + \mu^{n}_{y,\max}\right)^{1/n}\right| \tag{12}$$

where $\mu_{x,\max}$ denotes the normalized peak displacement along the X-axis if the excitations and responses along the Y-axis are ignored, and $\mu_{y,\max}$ denotes the normalized peak displacement along the Y-axis if the excitations and responses along the X-axis are ignored. Use of Eq. (13) is justified since $\mu_{x,\max}$ and $\mu_{y,\max}$ rather than $\mu_{xb,\max}$ and $\mu_{yb,\max}$ are likely to be available. The adequacy of these approximations is to be assessed in the following.

To facilitate the parametric studies of the system described previously, it is noteworthy that for a given ground motion record with its recording axes coincide with the structural axes, one could evaluate the peak linear elastic displacement responses d_{0x} and d_{0y}, and relate the inelastic displacements Δ_x and Δ_y of the system in terms of the normalized yield strength ϕ_x and ϕ_y,

$$\phi_x = \Delta_x / d_{0x}, \text{ and } \Delta_x = \phi_x d_{0x} \tag{13a}$$

and,

$$\phi_y = \Delta_y / d_{0y}, \text{and } \Delta_y = \phi_y d_{0y} \tag{13b}$$

such that the evaluated displacements are related to the normalized yield strength ϕ_x and ϕ_y. d_{0x} and d_{0y} can be calculated using Eq. (6) with α set equal to 1.0.

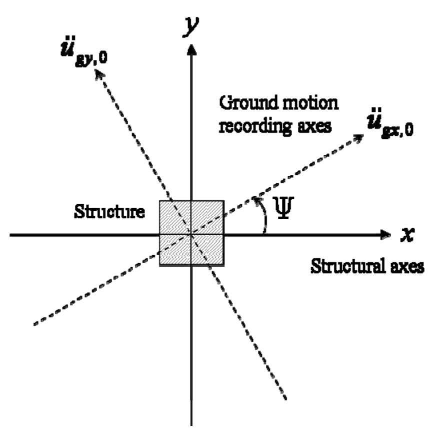

Figure 3. Orientation of the strong ground motion recording axes verses structural axes.

The orientation of the recording sensors may not coincide with structural axes, as shown in Figure 3. To investigate this orientation effect on ductility demand, the excitations along the structural axes, denoted by $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$, can be expressed as

$$\ddot{u}_{gx} = \ddot{u}_{gx,0} \cos\Psi + \ddot{u}_{gy,0} \sin\Psi \tag{14a}$$

and

$$\ddot{u}_{gy,\Psi} = -\ddot{u}_{gx,0} \sin\Psi + \ddot{u}_{gy,0} \cos\Psi \tag{14b}$$

where Ψ is defined in Figure 3, and $\ddot{u}_{gx,0}$ and $\ddot{u}_{gy,0}$ denotes the recorded ground motions.

Numerical evaluation of ductility demand

To assess $\mu_{b,\max}$ (see Eq. (10)), a set of 381 California records from 31 seismic events (Goda et al. 2009) is considered. The criteria used for selecting the records are detailed in Hong and Goda (2007) and Goda et al. (2009). To illustrate the effect of the responses with and without the effect of biaxial interaction on the estimated responses, the normalized displacements $\mu_{x,\max}$, $\mu_{y,\max}$, $\mu_{xb,\max}$, $\mu_{yb,\max}$, and $\mu_{b,\max}$ of a reference case (or Case 1) whose parameters are summarized in Table 1 are obtained by solving the governing equations, and the ratios $r_x = \mu_{xb,\max}/\mu_{x,\max}$, $r_y = \mu_{yb,\max}/\mu_{y,\max}$, $r_{ap} = \mu_{b,\max}$ /to $\mu_{b,ap}$ and $r_{b\text{-}u} = \mu_{b,\max}/\max(\mu_{x,\max}, \mu_{y,\max})$, are calculated using the response time-history for each record.

Table 1. Structural parameters for the considered cases, and mean and coefficient of variation values of $\mu_{x,\max}$, $\mu_{y,\max}$, $\mu_{xb,\max}$, $\mu_{yb,\max}$, $\mu_{b,\max}$, r_{ap}, $r_{b\text{-}u}$ (the first entry represent the mean, and the second entry represent the coefficient of variation).

Case[1]	Parameters varied	*Values*	$\mu_{x,\max}$	$\mu_{y,\max}$	$\mu_{xb,\max}$	$\mu_{yb,\max}$	$\mu_{b,\max}$	r_{ap}	$r_{b\text{-}u}$
1	–	–	2.02 0.32	2.01 0.32	2.01 0.37	2.03 0.36	2.54 0.35	0.87 0.15	1.10 0.18
2	T_{nx} & T_{ny}	0.1, 0.1	12.09 1.14	12.13 1.09	15.74 1.20	15.86 1.10	21.51 1.11	1.11 0.28	1.32 0.30
3	T_{ny}	0.25	2.02 0.32	7.42 0.96	2.53 0.50	7.47 0.95	7.70 0.92	0.97 0.13	1.07 0.16
4	ϕ_x & ϕ_y	0.25, 0.25	5.67 0.58	5.63 0.60	5.75 0.57	5.72 0.62	7.58 0.55	0.91 0.20	1.10 0.24
5	n	1	2.07 0.36	2.05 0.37	2.25 0.43	2.26 0.47	3.81 0.44	0.91 0.25	1.59 0.27

Note: For Case 1, T_{nx}=0.5, T_{ny}=0.5, ϕ_x=0.5, ϕ_y=0.5, n=2, δ_v=0, and δ_η=0.

By assuming that $\mu_{x,\max}$, $\mu_{y,\max}$, $\mu_{xb,\max}$, $\mu_{yb,\max}$, $\mu_{b,\max}$, r_{ap}, $r_{b\text{-}u}$ are independent of M, D, V_{s30} and PSA, the estimated means and coefficient of variation (cov) of these variables for the considered records are shown in Table 2. The statistics shown in Table 2 for $\mu_{x,\max}$, $\mu_{y,\max}$, $\mu_{xb,\max}$, $\mu_{yb,\max}$, and $\mu_{b,\max}$ are calculated from those samples whose values are greater than 1.0, which represent the ductility demand. Note that the percentage of number of records leading to the normalized responses of interest greater than 1.0, p_r, is greater than 99%. From the table, it can be observed that the means of $\mu_{xb,\max}$ and $\mu_{yb,\max}$ are similar to those of $\mu_{x,\max}$ and $\mu_{y,\max}$, respectively; but the cov values of $\mu_{xb,\max}$ and $\mu_{yb,\max}$ are slightly greater than those of $\mu_{x,\max}$ and $\mu_{y,\max}$,. The former implies that the consideration of biaxial response does not increase significantly the responses along the *X* or *Y* axes, although significant variability of the ratios of $\mu_{xb,\max}$ to $\mu_{x,\max}$, and of $\mu_{yb,\max}$ to $\mu_{y,\max}$ can be observed from the table. It indicates that the biaxial interaction could increase or reduce the response along a particular direction. The table also shows that the mean of $\mu_{b,\max}$ is greater than the means of $\mu_{x,\max}$ and $\mu_{y,\max}$ by about 30%, which indicates that the ductility demand for this considered structure under bidirectional seismic excitations is about 30% greater than that obtained by considering the unidirectional seismic excitations. The cov value of the ductility demand for the considered structure under bidirectional seismic excitations is similar to those under unidirectional seismic excitations. Since the mean of r_{ap} is smaller but near 1.0 and its cov value, which equals 0.15, is relatively small, it shows that $\mu_{b,\text{ap}}$ approximates $\mu_{b,\max}$ well.

In order to assign a probabilistic model to $\mu_{b,\max}$, samples of $\mu_{b,\max}$ are fitted to several well-known probability distribution types such as normal, lognormal, gamma, Weibull, Frechet distributions. Based on the Kolmogorov-Smirnov and Chi-square tests, it was concluded that the lognormal and Frechet distributions, among the considered distribution types, provides the best fit to the data. This fitted distribution is illustrated in Figure 4a. Similar analysis was carried out for r_{ap} and r_{b-u} and the best fit lognormal distributions for r_{ap} is depicted in Figures 4b.

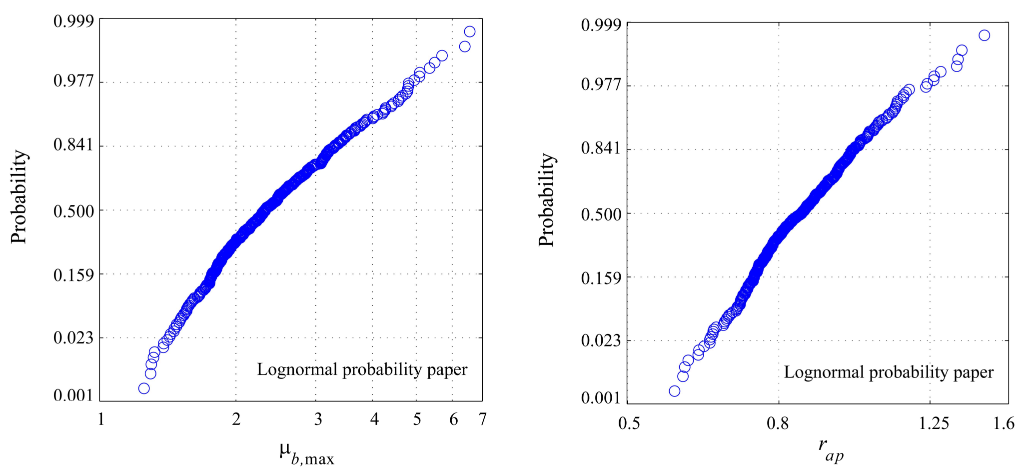

Figure 4. Fitted lognormal distributions for $\mu_{b,\max}$ and r_{ap} for Case 1

To investigate the impact of some of the structural parameters on the normalized displacement, the analyses carried out for Case 1 are repeated but varying one or two structural parameters as shown in Table 1 (i.e., Cases 2 to 5). The obtained results are shown in Table 1 as well. More extensive parametric studies are to be reported in a near future. The general observations that can be drawn from the table are:

1) Ductility demand under bidirectional seismic excitations could be much greater than that obtained under unidirectional excitation if the normalized yield strengths in both directions are similar and small. However, if the normalized yield strengths in both orthogonal directions differ significantly, the ductility demand under biaxial excitations is similar to that obtained along the axis with smaller normalized yield strength under unidirectional excitation.
2) In some cases, the mean of $\mu_{b,\max}$ can be about 55% greater than the means of $\mu_{x,\max}$ and

$\mu_{y,\max}$, which indicates that the ductility demand for structure under bidirectional seismic excitations is about 55% greater than that obtained by considering the unidirectional seismic excitations.

3) Since the mean of r_{ap} is near 1.0 and its cov value is relatively small as compared to that of $\mu_{b,\max}$, $\mu_{b,\mathrm{ap}}$ provides a good approximation to the ductility demand for structures under bidirectional excitations. For practical applications, therefore, this approximation is recommended.

4) The degree of uncertainty in the ductility demand (i.e., cov of ductility demand) for structures under bidirectional or unidirectional excitations are similar.

To evaluate the recording orientation effect on ductility demand, $\Psi = 30^{o}$, and the principal axes defined by Arias intensity coinciding with the structural axes are considered for Case 1. The obtained results are shown in Table 2, indicating that the orientation of ground excitations do not affect the statistics of ductility demand significantly, although it seems that the mean ductility demand is slightly increased if the principal axes defined by Arias intensity coinciding with the structural axes and without considering biaxial interactions. This needs to be verified further by considering different structural properties.

Table 2. Mean and coefficient of variation values of $\mu_{x,\max}$, $\mu_{y,\max}$, $\mu_{xb,\max}$, $\mu_{yb,\max}$, $\mu_{b,\max}$, r_{ap}, $r_{b\text{-}u}$ (the first entry represent the mean, and the second entry represent the coefficient of variation) Considering Case 1.

Angle	$\mu_{x,\max}$	$\mu_{y,\max}$	$\mu_{xb,\max}$	$\mu_{yb,\max}$	$\mu_{b,\max}$	r_{ap}	$r_{b\text{-}u}$
30^{o}	2.06 0.39	2.07 0.31	2.07 0.39	2.06 0.35	2.60 0.35	0.87 0.17	1.10 0.19
Principal axes coinciding with structural axes	2.09 0.35	2.00 0.33	2.03 0.37	2.00 0.35	2.51 0.37	0.85 0.16	1.07 0.18

Conclusions

Structures are subjected to at least two orthogonal horizontal seismic excitations; simple rules for estimating the inelastic responses or displacement ductility demands are lacking for such cases and considering possible biaxial interactions. Parametric studies to develop such simple rules are explored for idealized inelastic 2DOF systems (without considering torsional effects) whose hysteretic behavior is represented by the Bouc-Wen model with biaxial interaction. Conclusions that can be drawn from numerical results for a set of 381 California records presented in this study include:

1) Use of the ductility demand obtained under unidirectional excitations to represent the ductility demand under bidirectional excitations leads to, on average, an underestimation of about 30% to 55%. This underestimation is highly dependent on the shape of the considered surface at yield and the natural periods of the system. The underestimation of the damage and seismic risk is likely to occur if the effect of bidirectional excitations is ignored.

2) A simple approximate equation is developed in this study for estimating the ductility demand of a structure under bidirectional excitations. Statistical analysis results suggest that the developed approximation is adequate, the ductility demand under directional

excitations can be modeled by a lognormal or Frechet variate, and the coefficient of variation of ductility demand for structure under unidirectional or bidirectional excitations are similar.

Acknowledgments

The financial supports of the Natural Science and Engineering Research Council of Canada and the University of Western Ontario are gratefully acknowledged.

References

Applied Technology Council (ATC) (2005). *Improvement of nonlinear static seismic analysis procedures - FEMA-440*, Washington, DC

De Stefano, M. and Faella, G. (1996). "An evaluation of the inelastic response of systems under biaxial seismic excitations." *Engineering structures*, 18, 724-731.

Federal Emergency Management Agency (FEMA), and the National Institute of Building Sciences (NIBS) (2003). *HAZUS-Earthquake: technical manual*, Washington, D.C.

Foliente, G.C. (1995). "Hysteresis modeling of wood joints and structural systems." *J. Structural Eng*, **121**(6), 1013-1022.

Goda, K. and Hong, H.P. (2008). "Estimation of seismic loss for spatially distributed buildings." *Earthquake Spectra*, 24(4), 889-910.

Goda, K., Hong, H.P. and Lee, C.S. (2009). "Probabilistic Characteristics of seismic ductility demand of SDOF systems with Bouc-Wen Hysteretic Behavior." *Journal of Earthquake Engineering*, 13(5), 600-622.

Hong, H.P. and Goda, K. (2007). "Orientation-dependent ground motion measure for seismic hazard assessment." *Bulletin of the Seismological Society of America*, 97(5), 1525-1538.

Ma, F., Zhang, H., Bockstedte, A., Foliente, G.C. and Paevere, P. (2004). "Parameter analysis of the differential model of hysteresis," *Transactions of the ASME.*, 71(3), 342-349.

MacGregor, J. (1997). *Reinforced concrete: mechanics and design*, Upper Saddle River, N.J.: Prentice Hall.

Park, Y.J., Wen, Y. K. and Ang, A.H-S. (1986). "Random vibration of hysteretic systems under bi-directional ground motions." *Earthquake Eng. Struct. Dyn*, 14, 543–557.

Pacific Earthquake Engineering Research (PEER) Center (2006). *Next Generation Attenuation database*, http://peer.berkeley.edu/nga/index.html. (last accessed April 4th, 2006).

Takizawa, H. and Aoyama, H. (1976). "Biaxial effects in modeling earthquake response of R/C structures." *Earthquake Eng. Structural Dynamics*, 4, 523–552.

Taylor, R. (2008). *FEAP - A Finite Element Analysis Program User Manual*, University of California at Berkeley, Berkeley, California.

Wang, C.-H. and Chang, S.-Y. (2007). "Development and validation of a generalized biaxial hysteresis model." *J. Engrg. Mech.*, 133(2), 141–152.

Wen, Y.K. (1976). "Method for random vibration of hysteretic systems." *J. Engrg. Mech. Div.*, 102(2), 249–263.

Yeh, C.-H. and Wen, Y.K., (1990). "Modeling of nonstationary ground motion and analysis of inelastic structural response." *Struct. Safety*, 8, 281–298.