



ENERGY-BASED DAMAGE INDEX AND CYCLIC DRIFT CAPACITY FOR STEEL STRUCTURES

E. Bojórquez¹, A. Reyes-Salazar¹, A. Terán-Gilmore² and S. E. Ruiz³

ABSTRACT

Although several studies have been devoted to calibrate damage indices for steel and reinforced concrete members with the purpose of overcoming some of the shortcomings of the parameters currently used during seismic design; there is a challenge to study and calibrate the use of such indices for the practical structural evaluation of complex structures. This paper introduces an energy-based damage model for multi-degree-of-freedom (MDOF) steel framed structures that accounts explicitly for the effects of cumulative plastic deformation demands. The model has been developed by complementing the results obtained from experimental testing of steel members and those derived from analytical studies regarding the height distribution of plastic demands on several steel frames designed according to Mexico City Building Code (MCBC). Through the development of hazard curves, the limitations of the maximum inter-story drift demand as a performance parameter to achieve adequate damage control are discussed. The concept of cyclic drift capacity, which incorporates information of the influence of cumulative plastic deformation demands, is introduced as an alternative for seismic design of structures subjected to long duration ground motions.

Introduction

Currently, the maximum inter-story drift and ductility demands are targeted as response parameters to achieve adequate structural performance of earthquake-resistant structures. Nevertheless, the use of these parameters is not completely justified for buildings subjected to long duration ground motions. In fact, ample evidence suggests that the structural performance of buildings subjected to long duration ground motions is not adequately characterized through maximum deformation demands (Hancock and Boomer 2006). Therefore, in some cases, the effect of cumulative plastic deformation demands must be explicitly considered.

¹Facultad de Ingeniería, Universidad Autónoma de Sinaloa.

²Departamento de Materiales, Universidad Autónoma Metropolitana.

³Instituto de Ingeniería, Universidad Nacional Autónoma de México.

Although different energy-based methodologies that aim at providing earthquake-resistant structures with adequate energy dissipating capacity have been proposed (Akiyama 1985, Akbas et al. 2001, Choi and Kim 2006, Bojórquez et al. 2008a); currently, the most popular response parameter worldwide for seismic design of buildings is the maximum inter-story drift. Although the use of energy concepts during the practical earthquake-resistant design requires further developments, the influence of cumulative demands should be incorporated into seismic design of structures subjected to long duration motions. A viable manner to account explicitly for cumulative plastic deformation demands is the use of a cyclic (reduced) drift capacity, which in concept is similar to the target ductility concept formulated by Fajfar (1992). The aim of this paper is to firstly, introduce an energy-based damage index which explicitly accounts for the effects of cumulative plastic deformation demands in steel frames. Secondly, to compare demand hazard curves obtained for moment-resisting steel frames in terms of the energy-based damage index and the maximum inter-story drift. And finally, to provide for the steel frames drift capacity thresholds (denoted cyclic drift capacity) that account for cumulative damage, and that yield adequate levels of reliability.

Energy-based damage index for steel framed structures

Energy-based methodologies are focused at providing structures with energy dissipating capacities that are larger or equal than their expected energy demands (Akiyama 1985, Uang and Bertero 1990). The design requirement of an earthquake-resistant structure can be formulated in these terms as:

$$\textit{Energy Capacity} \geq \textit{Energy Demand} \quad (1)$$

Among all the energies absorbed and dissipated by a structure, the plastic dissipated hysteretic energy E_H is clearly related to structural damage. E_H can physically be interpreted by considering that it is equal to the total area under all the hysteresis loops that a structure undergoes during a ground motion. Therefore, it is convenient to express Eq. 1 in terms of plastic dissipated hysteretic energy:

$$E_{HC} \geq E_{HD} \quad (2)$$

where E_{HC} is the plastic hysteretic energy capacity and E_{HD} its corresponding energy demand. Eq. 2 can be reformulated as an energy-based damage index:

$$I_{DE} = \frac{E_{HD}}{E_{HC}} \leq 1 \quad (3)$$

Within the context of Eq. 3, the performance level or condition that implies that the energy demand on the system is equal to its corresponding capacity will be considered as the failure of that system. Hence, while I_{DE} equal to one corresponds to failure of the structural system; a value of zero implies no structural damage (elastic behavior implies no structural damage). From a physical point of view, this equation represents a balance between the structural capacity and

demand in terms of energy. In this sense, this formulation follows the direction initially established by Housner in 1956 for an energy-based design. According to Eq. 3, structural damage depends on the balance between the plastic hysteretic energy capacity and demand on the structure. While the plastic hysteretic energy demand can be obtained through dynamic analysis, a challenge exists to define the plastic hysteretic energy capacity of a structure. Nevertheless, flexural plastic behavior is usually concentrated at the ends of the structural members that make up a frame; and in the particular case of W steel shapes, in the flanges. The plastic hysteretic energy capacity of a steel member that forms part of a structural frame can be estimated as follows (Akbas et al. 2001):

$$E_{HCm} = 2 Z_f f_y \theta_{pa} \quad (4)$$

where Z_f is the section modulus of the flanges; f_y , the yield stress; and θ_{pa} , its cumulative plastic rotation capacity. Note that the above equation considers that plastic energy is dissipated exclusively through plastic behavior at both ends of a steel member. Eq. 4 can be used together with Eq. 3 to evaluate the level of structural damage in steel members. However, it is convenient to normalize E_H for damage evaluation purposes (Krawinkler and Nassar 1992, Terán-Gilmore and Simon 2006):

$$E_N = \frac{E_H}{F_y \delta_y} \quad (5)$$

where F_y and δ_y are the strength and displacement at first yield, respectively. Eq. 3 can be expressed in terms of E_N as follows:

$$I_{DEN} = \frac{E_{ND}}{E_{NC}} \leq 1 \quad (6)$$

where the parameters involved in Eq. 6 have similar meanings as those used in Eq. 3. The advantage of formulating the problem in terms of E_N is that this is a more stable parameter, and thus, can be used in quantitative terms for practical purposes. The energy-based damage index proposed herein corresponds to the ratio between the normalized hysteretic energy demand and normalized hysteretic energy capacity, and the condition of failure is assumed to be I_{DEN} equal to one. In the case of MDOF steel structures, the principal challenge for the practical use of Eq. 6 is the definition of the energy capacity of the structure in terms of that of its structural members. Through the consideration that in regular steel frames the energy is dissipated exclusively by the beams (which is a reasonable assumption for strong column-weak beam structural systems), their energy capacity can be estimated through a modified version of Eq. 4 (Bojórquez et al. 2008a):

$$E_{NC} = \frac{\sum_{i=1}^{N_s} (2 N_B Z_f F_y \theta_{pa} F_{EHi})}{C_y D_y W} \quad (7)$$

where N_s and N_B are the number of stories and bays in the building, respectively; F_{EHi} , an energy participation factor that accounts for the different contribution of each story to the energy dissipation capacity of a frame; W , the total weight of the structure; and finally, C_y and D_y , the

seismic coefficient and displacement at first yield, which are obtained through pushover analysis.

From extensive statistical studies, F_{EH} can be estimated as (Bojórquez et al. 2008a):

$$F_{EH} = \min(F_{EH}^*, 1) \quad (8)$$

where:

$$F_{EH}^* = \frac{1}{(-0.0675\mu + 2.82)h/H} \exp\left\{-\frac{1}{2}\left[\frac{(\ln(h/H) - \ln(0.031\mu + 0.3461))}{0.06\mu + 0.39}\right]^2\right\}$$

Eq. 7 shows the role of the cumulative plastic rotation capacity of the structural members in the total energy dissipation capacity of a frame. Based on the results of several experimental tests of steel members collected by Akbas (1997), Bojórquez et al. (2008b) found that the cumulative plastic rotation capacity of steel members is well represented by a lognormal probability density function with a median value equal to 0.23.

Maximum interstory drift index

The demand hazard curves in terms of maximum inter-story drift and the energy-based damage index introduced herein cannot be compared directly unless the maximum inter-story drift is normalized by its respective structural capacity. In this manner, a normalized damage measure in terms of inter-story drift (denoted inter-story drift damage index) needs to be formulated:

$$I_{D\gamma} = \frac{\gamma_D}{\gamma_u} \quad (9)$$

where $I_{D\gamma}$ characterizes damage in terms of maximum inter-story drift; and γ_D and γ_u represent the demand and capacity of the structure in these terms, respectively. From pushover analyses of the steel frames under consideration, γ_u was found to be close to 0.05 (Bojórquez et al. 2009).

Steel moment resisting frames and ground motion records

Structural models

Six moment resisting steel frames having 4, 6, 8, 10, 14 and 18 stories were considered for the studies reported herein. The frames are denoted F4, F6, F8, F10, F14 and F18, respectively. As shown in Fig. 1, the frames (designed according to the MCBC) have three eight meter bays and inter-story heights of 3.5 meters. Each frame was provided with ductile detailing and its lateral strength was established according to the MCBC. A36 steel was used for the beams and columns of the frames. An elasto-plastic model with 3% strain-hardening was used to model the cyclic behavior of the steel elements. As discussed by Bojórquez and Rivera (2008), this model provides a good approximation to the actual hysteretic behavior of steel members.

The columns in the first story were modeled as clamped at their bases. Second order effects were explicitly considered, and 3% of critical damping was used for the two first modes of the frames during the nonlinear dynamic analyses. Relevant characteristics for each frame are summarized in Table 1.

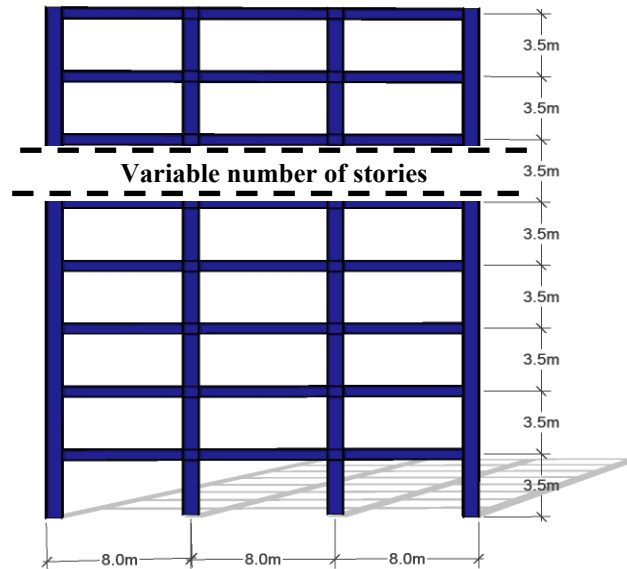


Figure 1. Geometrical characteristic of the MDOF steel frames.

Table 1. Relevant characteristics of the moment resisting steel frames.

Frame	Number of Stories	T_1 (s)	C_y	D_y (m)
F4	4	0.90	0.45	0.136
F6	6	1.07	0.42	0.174
F8	8	1.20	0.38	0.192
F10	10	1.37	0.36	0.226
F14	14	1.91	0.25	0.30
F18	18	2.53	0.185	0.41

Seismic records

A set of 23 narrow-band ground motions recorded at Lake Zone sites of Mexico City was considered. Particularly, all motions were recorded at sites having soil periods of two seconds during seismic events with magnitudes of seven or larger, and having epicenters located at distances of 300 km or more from Mexico City. Some important characteristics of the records are summarized in Table 2. While PGA and PGV denote the peak ground acceleration and velocity, respectively; the duration was estimated according to Trifunac and Brady (1975). The seismic records under consideration were established by rotating both horizontal components of motion recorded at a given station so that their Arias intensity (Arias 1970) was maximized. The narrow-band records exhibit similar values of parameter N_p , which is an indicator of the characteristics of their spectral shape (Bojórquez and Iervolino 2009). As a result, there is a

strong similarity between their spectral shapes. This is illustrated in the log-log plot included in Fig. 2, which shows response spectra of all records scaled to the same spectral acceleration for a period of 1.2 sec (fundamental period of vibration of frame F8). The similitude exhibited by all spectra indicates that the spectral acceleration is a good indicator of the damage potential of the ground motions, and emphasizes the good correspondence that exists between parameter N_p and the spectral shape.

Table 2. Seismic records.

Record	Date	Magnitude	Station	PGA (cm/s ²)	PGV (cm/s)	Duration (s)
1	25/04/1989	6.9	Alameda	45.0	15.6	45.34
2	25/04/1989	6.9	Garibaldi	68.0	21.5	73
3	25/04/1989	6.9	SCT	44.9	12.8	65.73
4	25/04/1989	6.9	Sector Popular	45.1	15.3	79.13
5	25/04/1989	6.9	Tlatelolco TL08	52.9	17.3	56.55
6	25/04/1989	6.9	Tlatelolco TL55	49.5	17.3	49.91
7	14/09/1995	7.3	Alameda	39.3	12.2	53.6
8	14/09/1995	7.3	Garibaldi	39.1	10.6	86.8
9	14/09/1995	7.3	Liconsa	30.1	9.62	50.94
10	14/09/1995	7.3	Plutarco Elías Calles	33.5	9.37	77.57
11	14/09/1995	7.3	Sector Popular	34.3	12.5	100.76
12	14/09/1995	7.3	Tlatelolco TL08	27.5	7.8	85.76
13	09/10/1995	7.5	Cibeles	14.4	4.6	83.06
14	09/10/1995	7.5	Córdoba	24.9	8.6	94.10
15	09/10/1995	7.5	Liverpool	17.6	6.3	104.95
16	09/10/1995	7.5	Plutarco Elías Calles	19.2	7.9	104.44
17	11/01/1997	6.9	CU Juárez	16.2	5.9	62.09
18	11/01/1997	6.9	Centro urbano Presidente Juárez	16.3	5.5	60.71
19	11/01/1997	6.9	García Campillo	18.7	6.9	84.89
20	11/01/1997	6.9	Plutarco Elías Calles	22.2	8.6	56.34
21	11/01/1997	6.9	Est. # 10 Roma A	21.0	7.76	76.09
22	11/01/1997	6.9	Est. # 11 Roma B	20.4	7.1	74.06
23	11/01/1997	6.9	Tlatelolco TL55	13.4	6.5	55.37

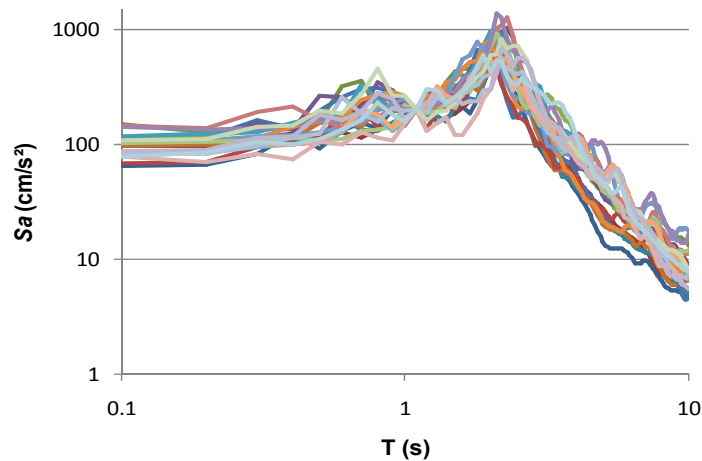


Figure 2. Elastic response spectra for records scaled up to the same value of $Sa(T_1)$ for T_1 of 1.2s (3% of critical damping).

Seismic vulnerability assessment

One of the main objectives of Earthquake Engineering is to quantify, through the consideration of all possible earthquake ground motion intensities at a site, the seismic reliability implicit in structures. Probabilistic seismic demand analysis (PSDA) is used as a tool for estimating the reliability of structures through the evaluation of the mean annual frequency of exceeding a specified value of an earthquake demand parameter EDP (e.g. inter-story drift, normalized plastic hysteretic energy, etc). Based on past studies (Esteva 1967, Cornell 1968) and considering the total probability theorem, a probabilistic seismic demand analysis can be carried out through the consideration of the mean annual rate of exceeding a given value of EDP :

$$\lambda_{EDP}(x) = \sum v_i \int \int \int P[EDP > x | IM, M, R] f(IM | M, R) f(M, R) dr dm d(im) \quad (10)$$

where $\lambda_{EDP}(x)$ is the mean annual frequency of EDP exceeding the value x , v_i is the rate of earthquakes for source i , $f(IM | M, R)$ is the conditional distribution function of the intensity measure (IM) given values of magnitude (M) and distance (R), $f(M, R)$ is the joint probability density function of M and R , and finally, $P[EDP > x | IM, M, R]$ is the probability of EDP exceeding x given IM , M and R (if x corresponds to the capacity of the structure, this term represent the fragility curves of the system). If $P[EDP > x | IM, M, R] = P[EDP > x | IM]$, then the IM is said to be *sufficient* (Bazzurro 1998, Shome 1999) since its ability to predict the structural response is independent of M and R , given IM . It has been shown that the spectral acceleration at first mode of vibration $Sa(T_1)$ is *sufficient* with respect to magnitude and distance (Shome 1999). As suggested before, the records used herein allow the use of a scaling criteria based on $Sa(T_1)$: a) First, due to sufficiency of $Sa(T_1)$ with respect to M and R ; b) Second, due to the similar spectral shape of the records; and c) Third, because no bias in nonlinear structural response is observed for different scaling levels of the records under consideration (Bojórquez et al. 2009). Within this context, Eq. 10 can be expressed as:

$$\lambda_{EDP}(x) = \int_{Sa(T_1)} P[EDP > x | Sa(T_1) = sa] d\lambda_{Sa(T_1)}(sa) \quad (11)$$

where $d\lambda_{Sa(T_1)}(sa) = \lambda_{Sa(T_1)}(sa) - \lambda_{Sa(T_1)}(sa + dsa)$ is the differential of the ground motion hazard curve expressed in terms of $Sa(T_1)$. Eq. 11 was used to evaluate the structural reliability of the steel frames in terms of two $EDPs$: inter-story drift index and energy damage index. A lognormal distribution is considered to evaluate $P[EDP > x | Sa]$.

Cyclic drift capacity for steel moment resisting frames

In this study, structural reliabilities were established through the use of Eq. 11 and the ground motion seismic hazard curves corresponding to the *Secretaria de Comunicaciones y Transportes* (SCT) site in Mexico City and established by Alamilla (2001). The soil at the site

exhibits a period of two seconds, which is considered representative of the sites where the ground motions under consideration were recorded. A $\theta_{pa} = 0.23$ was used to characterize the normalized plastic hysteretic energy capacity of the beams. Fig. 3 summarizes the demand hazard curves in terms of $I_{D\gamma}$ and I_{DE_N} for frame F4 (values larger than one were plotted for illustrative purposes). Three zones can be appreciated in the figures. The first one corresponds to small values of I_D (blue box), which would commonly be associated to the serviceability limit state. In this range of I_D , the mean annual rate of exceedance is larger for $I_{D\gamma}$ than for I_{DE_N} . This seems logical since the level of displacement control required by serviceability implies minimum or no plastic demands in the structural elements. A second zone, corresponding to intermediate values of I_D (close to 0.5), can be noticed in Fig. 3 (orange box). In this zone, both demand hazard curves exhibit similar ordinates, implying that design for intermediate levels of damage is not sensible to the measure of damage used to guarantee an adequate performance of the frame. Finally, a third zone (red box) can be appreciated for values of I_D close to one. Because this zone relates to failure, it is usually deemed as the most important in terms of practical seismic design. The results suggest that under some circumstances; an unsatisfactory design can be obtained if measures of the ground motion duration or of cumulative demands are not taken into account explicitly. This discussion is valid for all the frames under consideration.

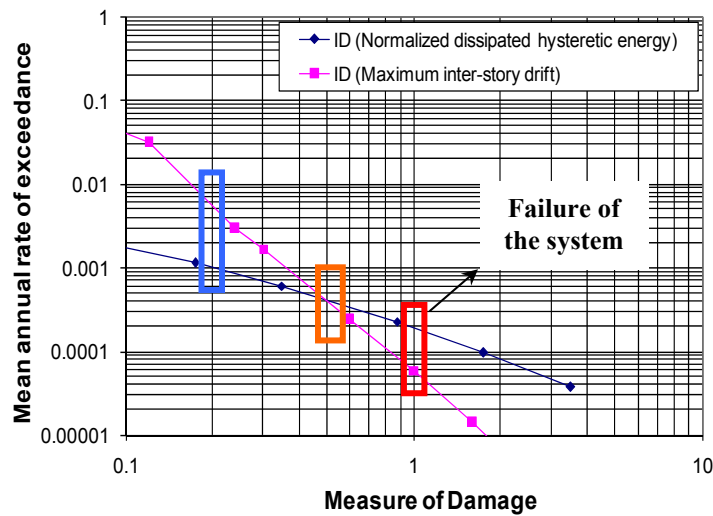


Figure 3. Hazard curves in terms of I_{DE_N} and $I_{D\gamma}$ for all frames under consideration.

Because in most seismic design codes the principal parameter to promote adequate structural performance is the maximum inter-story drift index, it is important to offer maximum inter-story drift thresholds that consider the effect of cyclic cumulative deformation (denoted cyclic drift capacity, γ_{CC}). These thresholds should be established in such manner that the structural reliabilities associated to failure of the frames under consideration are similar in terms of both damage indices used herein. Table 3 summarizes the reduced inter-story drift thresholds that account for energy demands. Note that the maximum reduction in terms of drift threshold occurs for the frames whose fundamental period of vibration is close to two seconds (period of the soil at the site). Appendix A of the Technical Requirements for Seismic Design of the MCBC

considers a threshold of 0.03 for seismic design of ductile steel frames. This threshold is conservative if compared to the 0.05 value estimated for the frames under consideration. Nevertheless, the cyclic inter-story drift ratio thresholds included in Table 3 indicate that the 0.03 threshold may not be conservative enough for structures located in the Lake Zone, particularly for those whose period of vibration is similar to that of the soil. In the latter case, a threshold or cyclic drift capacity of 0.02 would seem more appropriate.

Table 3. Cyclic drift capacity.

Frame	$T_I(s)$	γ_{CC}
F4	0.90	0.032
F6	1.07	0.029
F8	1.20	0.026
F10	1.37	0.023
F14	1.91	0.018
F18	2.53	0.023

Conclusions

An energy-based damage model for multi-degree-of-freedom steel framed structures that accounts explicitly for the effects of cumulative plastic deformation was proposed. The energy-based damage index was used to propose new thresholds for the maximum inter-story drift. The evaluation of the new drift limit, denoted cyclic drift capacity, was based in the use of the energy damage index introduced herein; and was targeted to achieve similar levels of structural reliability at failure of the frames in terms of inter-story drift and energy.

For the serviceability limit state, seismic design is controlled by maximum inter-story drift demands. Nevertheless, in terms of failure, the results presented herein suggest that under some circumstances; an unsatisfactory design can be obtained if measures of the ground motion duration or of cumulative demands are not taken into account explicitly. This is particularly important for structures located in very soft soils and having a fundamental period of vibration close to the dominant period of the soil.

Inter-story drift ratio thresholds currently used to promote adequate structural performance during severe ground motions usually yield conservative seismic design. Nevertheless, these thresholds need to be carefully assessed for the seismic design of structures located at sites capable of generating long duration motions. Particularly, a cyclic drift capacity of 0.02 seems a better value to achieve adequate structural performance of ductile steel structures located in the Lake Zone of Mexico City, than the current value of 0.03 formulated in current Mexican building codes.

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