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DESIGN OF CAT-IN-A-BOX PARAMETRIC EARTHQUAKE CAT BOND TRIGGERS

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ABSTRACT

A catastrophe –cat– bond is an instrument used by insurance and reinsurance companies, by governments or by pools of nations to cede catastrophic risk to the financial markets. Cat-in-a-box parametric triggers whose outcomes depend only on the physical parameters of an earthquake published by respected agencies can be used to determine whether the bond principal is paid. Since the outcome of the trigger is not influenced by the parties involved, the associated moral hazard is practically inexistent. These instruments are especially appealing in developing countries where the absence of ground motion recording stations precludes the use of more complex indices. Sensitivity analyses to different design assumptions show that these transactions can be affected by a large negative basis risk, namely the risk that the bond will not trigger for events within the risk level transferred, unless a sufficiently small geographic resolution is selected to define the trigger zones. This paper proposes a methodology for designing cat-in-a-box earthquake cat bonds aimed at minimizing basis risk. A hypothetical cat bond is designed for Costa Rica as an illustration of the methodology.

Introduction

Cat bonds are a type of insurance-linked security (ILS), a class of financial instruments that allow insurers, reinsurers, governments and catastrophe pools to cede risks of losses due to natural hazards to the capital markets. Cat bonds offer an enormous supply for reinsurance surpassing the capacity of traditional providers and they are therefore well suited to provide cover for potentially very large losses. Cat bonds are also fully collateralized, meaning that the full amount invested is held by a dedicated entity, typically known as a special purpose vehicle (SPV), which makes the settlement of losses a quick and swift process.

In order to provide transparency to investors and sponsors and eliminate moral hazard (the possibility that the parties involved manipulate the outcome of the payment mechanism to their advantage) cat bonds often use parametric triggers. These triggers consist of a set of conditions on several easily obtainable physical characteristics of the earthquake, so that a payment occurs in case that the event fulfils some established criteria (Cummins, 2007; Croson and Kunreuther, 1999).

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This paper treats trigger mechanisms known as "first generation" parametric triggers. For earthquake risk transactions these triggers typically use the event magnitude, the location of the epicenter, and the depth of the hypocenter as event parameters. Since they often involve the description of some zones in which the events must be located in order to trigger the bond, these cat bonds are also known as "cat-in-a-box" bonds. "Second generation" parametric triggers, not treated in this paper, involve geographically distributed intensity information to define the payment mechanism, usually through a mathematical index that acts as a proxy of expected loss. The application of this second group of triggers requires the existence of a network of recording stations on the field that measure the desired parameter and publish it for broad usage. Since the absence of a geographically distributed reporting network of intensity values makes it impossible to develop a second generation trigger, first generation triggers are very appealing in most locations.

The ability to use parameters published by the USGS to construct parametric trigger mechanisms in any country in the world, has made these structures very interesting, not only to hedge losses from insurance or reinsurance companies but to transfer losses experienced by governments to the capital markets. An example of these transactions is Cat-Mex, a first generation parametric structure designed to cover the government of Mexico from emergency losses ensuing from an earthquake event (Cardenas et al., 2007; AIR, 2005).

The main objective in the construction of a parametric trigger is to faithfully represent the risk being transferred through a set of trigger conditions. The discrepancies between the actual risk and the risk represented by the trigger constitute the basis risk. This paper illustrates a straightforward methodology to design first generation or cat-in-a-box catastrophe bonds for earthquake peril geared towards minimizing basis risk. The paper draws from previous sensitivity analyses carried out on the influence of the cat bond parameters on basis risk (Franco, 2009). The methodology is illustrated through the design of a hypothetical cat bond for Costa Rica.

The paper is structured in three main parts that follow this introduction: In the first, the general methodology proposed is presented along with some common terms used throughout the paper. The main fundamental assumptions as well as several basic quantities involved in the trigger design are also introduced. The second section deals with the selection of a geographic resolution for the cat bond zone definition. The discretization of the geographic domain affects the basis risk and constitutes one of the most important design drivers. The third section treats the problem of selecting a number of appropriate zones. This decision is affected by the accuracy desired in the cat bond as well as by the simplicity requirements of the zone definitions. This trade-off, determined primarily by the wish to produce transactions of market appeal, is illustrated through the computation of several optimal design options.

Design Methodology and Definitions

The methodology suggested in this paper consists of a seven-step process, illustrated in figure 1. The first step consists of choosing a loss level against which financial protection is desired. The sponsor of the cat bond will typically specify this loss as a requirement for the design of the trigger mechanism. This loss is referred to here as the loss threshold L, which can also be expressed in terms of its return period.

The second step requires the existence of a probabilistic risk model. It will be assumed that an earthquake risk model is available and that the output of the model can be obtained in the form of a catalog of N stochastic events that constitutes a representative sample of the seismicity of the region of interest. For the events in this catalog, the following characteristics must be specified: latitude of the epicenter y_i, longitude of the epicenter x_i, moment magnitude m_i, depth d_i, and the estimated monetary loss to the portfolio of interest L_i. The parameters associated to an event can be obtained from any of the earthquake risk models available in the market or in the literature. HAZUS, for instance, can be used to produce the parameters needed to apply the methodology described here. Loss results require a loss estimation process, also incorporated into HAZUS and into many other earthquake risk tools. Furthermore, the methodology presented here uses monetary loss to establish a target for the trigger mechanism but other metrics may also be used for different purposes, such as expected fatalities or injuries, for example.

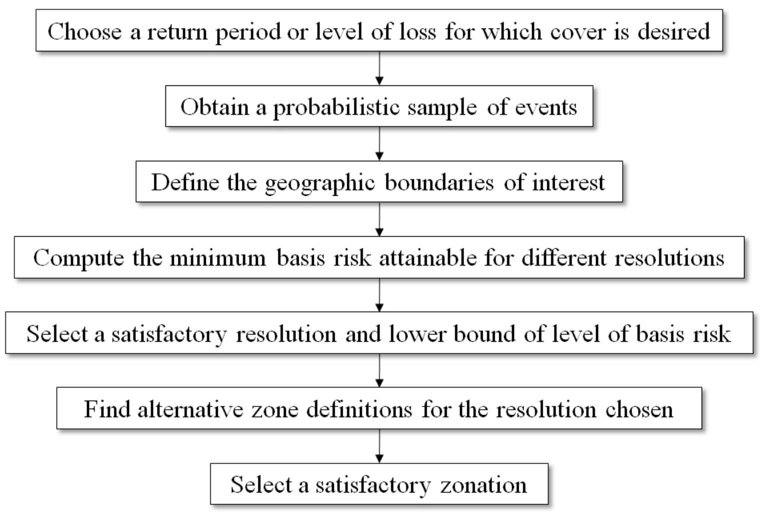

Figure 1. Cat-in-a-box design methodology.

In this analysis and without loss of generality, the earthquake model developed by AIR, a catastrophe risk modeling company, is used in order to construct a hypothetical cat bond for Costa Rica to cover losses to a nominal portfolio somewhat representative of the exposure value throughout the country. The stochastic catalog associated with this probabilistic model represents 10,000 possible years of seismicity for the geographic domain of interest. It is beyond the scope of this paper to describe the generation of this particular seismic catalog and it is neither critical nor particular to the cat bond design methodology discussed here. Suffice it to say, however, that it is constructed through a sampling process involving regional and fault-specific magnitude-rate distributions defined based on historical data as well as data obtained from GPS measurements and paleo-seismic studies covering the region under study.

The third step in the design methodology involves the selection of the geographic domain. In this paper the domain is defined by the area located within parallels 6.5°N and 13.5°N and within meridians 89°W and 79°W, a wide area that includes Costa Rica and all the seismogenic sources that may produce earthquakes affecting its territory.

The fourth step in the methodology, namely the choice of the resolution of the geographic discretization, requires a substantive analytical process. Before tackling this problem, several necessary definitions are introduced here. As mentioned above, the fundamental design parameter of the trigger is the loss threshold L. This value is a loss over which the sponsor desires protection and is typically associated with large return periods. For each event i, $i=1,2,\ldots,N$, it is possible to assign a binary outcome B_i that describes whether the event loss L_i obtained from the model is greater than the threshold L:

$$B_i = \begin{Bmatrix} 0, \text{if } \mathrm{L}_i < L \\ 1, \text{if } \mathrm{L}_i \geq L \end{Bmatrix} \tag{1}$$

The objective in the design of the parametric trigger can be restated as the construction of an operator B' such that it produces the same outcomes B_i without knowledge of the loss L_i and with the sole information of the event's depth, magnitude, and location. The operator B' is defined with the help of geographic areas within which certain thresholds on magnitude and depth are specified. Although these geographic areas can be in general defined arbitrarily, in this work it will be considered that the geographic domain can be split up in $k=1,2,\ldots,K$ boxes that together contain all earthquakes in the given catalog. The geographic boxes are defined as a grid, specifying the coordinates of the lower left corner (x_0,y_0), the number of boxes in the x direction, N_x, the number of boxes in the y direction, N_y, and the length of the side of the square boxes, d.

A given event then pertains to one and only one of these boxes (the particular case in which an epicenter strictly falls on a box boundary can be easily overcome establishing that all boundaries belong to one of the adjacent boxes). In addition, a set of "zones" are defined for $z=1,2,\ldots,Z$ with associated thresholds of magnitude and depth, M_z and D_z. Each box k corresponds to a zone type that will determine what magnitude and depth thresholds are applicable in that geographic area:

$$(M_k, D_k) \in \{(M_1, D_1), (M_2, D_2), \ldots, (M_Z, D_Z)\} \tag{2}$$

The value of the operator B'_i for an event i that belongs to box k is defined as:

$$B'_i = \begin{Bmatrix} 0, \text{if } \mathrm{m}_i < M_k \text{ or if } \mathrm{d}_i > D_k \\ 1, \text{if } \mathrm{m}_i \geq M_k \text{ and if } \mathrm{d}_i \leq D_{k\,i} \end{Bmatrix} \tag{3}$$

It will be assumed that the difference between the desired event outcome B_i and the outcome dictated by the cat bond trigger B'_i represents the basis risk of the structure for event i. Cummins (2007) defines basis risk as "the risk that the loss payout of the bond will be greater or less than the sponsoring firm's actual losses." In order to interpret this definition within the framework of the parametric binary trigger under study, "the sponsoring firm's actual losses" are defined here as either zero or equal to the loss threshold L. An event that produces a loss equal or higher than L should trigger the cat bond and pay the amount L to the sponsor. In contrast, an event that produces a loss that is lower than the threshold L should not trigger the bond and would therefore pay 0 to the sponsor. If the trigger does not reflect these situations correctly, basis risk appears.

The design of the cat bond relies on a stochastic simulation. Therefore, the "actual" loss of an event i is approximated with the "modeled" loss L_i, introduced above. The basis risk treated in this work is exclusively introduced by the cat bond trigger mechanism. There are other sources of basis risk, most importantly that introduced by the model itself. However, the study of the basis risk introduced by the model, sometimes referred to as "model risk," needs a detailed assessment of the hypotheses of the model, something which is beyond the scope of this work.

Two sources of basis risk are considered: Positive basis risk is defined as the basis risk that ensues when an event that produces a loss lower than the threshold triggers the cat bond. Positive risk favors the sponsor and disfavors the investor. Negative basis risk is defined as the basis risk that ensues when an event that produces a loss equal or higher than the threshold loss does not trigger the cat bond. Negative risk favors the investor and disfavors the sponsor. Note that both negative and positive basis risk are represented by positive integers in this study and their sum constitutes the total basis risk of the cat bond. In order to produce a cat bond structure of market appeal, both the total basis risk and its bias towards negative or positive risk should be minimal.

It is then possible to define the basis risk for each box k as the sum of the positive and negative basis risk contributions within that box:

$$E_k = E_k^+ + E_k^- \tag{4}$$

The positive, E_k^+, and negative, E_k^-, basis risk contributions are defined based on the number of events that do not trigger as desired:

$$E_k^+ = \sum\nolimits_{\forall i \text{ in Box } k} \begin{Bmatrix} 1 \text{ if } B'_i > B_i \\ 0 \text{ if } B'_i = B_i \end{Bmatrix} \quad \text{and} \quad E_k^- = \sum\nolimits_{\forall i \text{ in Box } k} \begin{Bmatrix} 1 \text{ if } B'_i < B_i \\ 0 \text{ if } B'_i = B_i \end{Bmatrix} \tag{5}$$

If there are no events in a particular box, the associated basis risk values are zero. The total risk of the structure will simply be the sum of the basis risk across all boxes:

$$E = \sum_{k=1}^{K} E_k; \quad E^+ = \sum_{k=1}^{K} E_k^+; \quad E^- = \sum_{k=1}^{K} E_k^- \tag{6}$$

Consider that the cat bond trigger will fail to produce the desired outcome in a number of events equivalent to the basis risk of equation (6) in the sample of 10,000 years represented by the catalog. The average annual probability of failure of the cat bond trigger is therefore $E/10{,}000$. For convenience, basis risk will be defined in terms of events and not probability.

Selecting a Satisfactory Resolution

Depending on the parameters that define the cat bond trigger mechanism such as the number of zones or the size of the boxes, basis risk will vary. A lower bound of basis risk for a given box side length d can be obtained for a chosen level of loss. The main assumption in the calculation of the lower bound is that there are as many zone definitions as there are boxes, or *Z*=*K*, which means that a magnitude and a depth thresholds can be tailored to each of the particular boxes. Although this type of solution would find little market acceptance due to its complexity, it has two main interesting uses: First, since this solution is not restricted by the number of zones, the basis risk associated with it constitutes a lower bound of basis risk achievable with the given geographic resolution and the given loss threshold. Secondly, the lower bound can be used as a reference target once the design constrains the number of zones *Z*<*K*. Since the *Z*=*K* solution assumes a specific pair of thresholds tailored to each of the boxes, it is relatively straightforward to find the optimal values through an exhaustive search of all possible threshold pairs within each box *k*. Figure 2 shows a sensitivity analysis of the lower bound solutions obtained for loss thresholds corresponding to the 20, 50, 100, 250, and 500-year return periods, as well as for resolutions of 1°, 0.5°, 0.25°, 0.1°, and 0.05°. Row A of figure 2 shows that basis risk is typically dominated by the negative basis risk contribution in the optimal lower bound solutions. Since the loss threshold is a requirement imposed at the beginning of the cat bond design (step 1), these results are useful to decide on a target resolution *d* that satisfies the basis risk accuracy requirements. Observe also the increasing balance between positive and negative basis risk as the resolution increases (*d* decreases). For instance at the 20-year return period threshold loss it is in theory possible to almost balance negative and positive basis risk by using small boxes of 0.05° of side length (about 5km) while decreasing total basis risk, thus making the structure appealing to sponsor and investor.

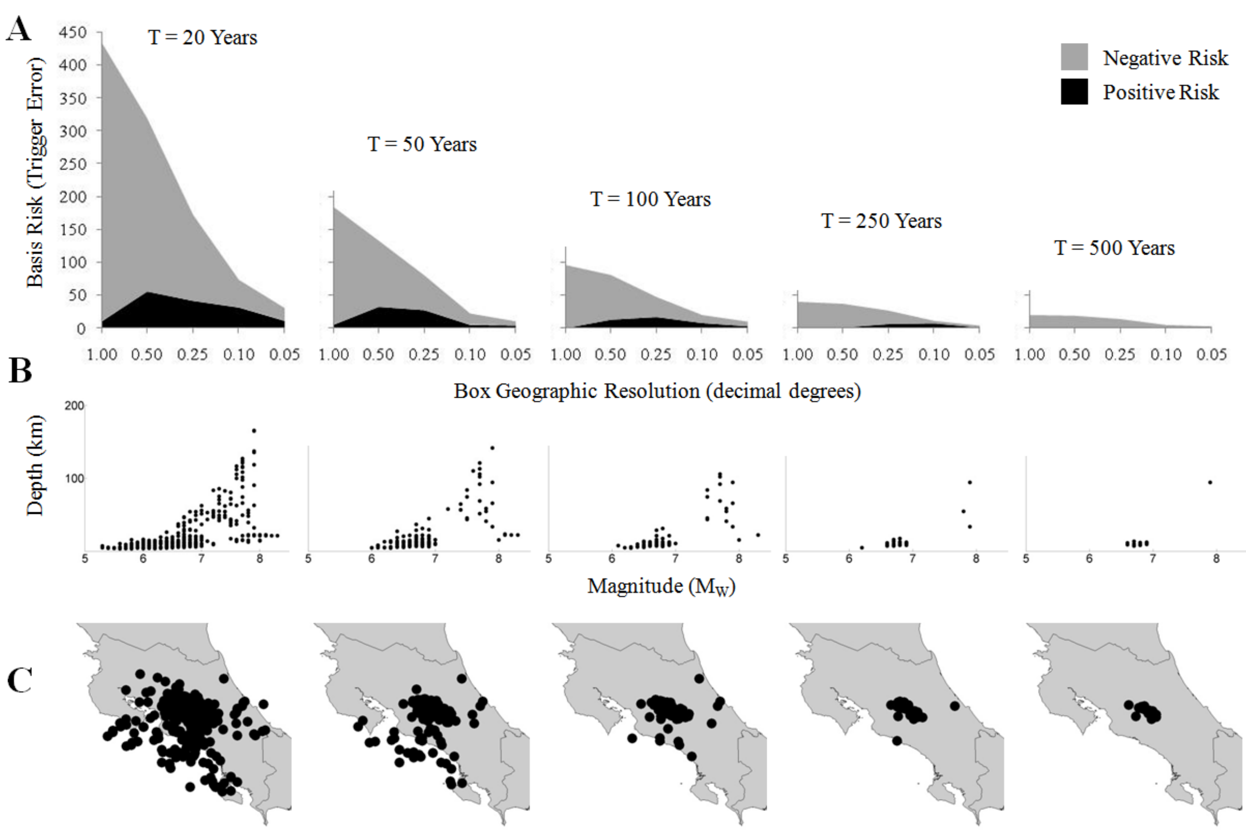

Figure 2. Lower bound basis risk and distribution of triggering events for different cases of loss threshold and geographic resolution (Franco, 2009).

Observe in rows C and B of figure 2 that the events that trigger the cat bond are clustered in more narrowly defined regions geographically as well as in their magnitude-depth distribution as the loss threshold increases. This is due to the fact that smaller losses can ensue due to a greater number of combinations of geographic location, magnitude, and depth, while very large losses only happen in locations of very high exposure (near the San Jose area in the case of Costa Rica) combined with relatively large magnitude events of shallow depths. This is in general favorable for this type of trigger structure and it is indeed the reason why they are useful in risk transactions. Since the losses for which the sponsor typically seeks cover are relatively large, it is possible to confine the triggering events to small windows of magnitudes, depths, and locations.

Usually, at high levels of loss threshold, the number of triggering events will be much smaller than the number of events that do not trigger the cat bond. Therefore, given a window of magnitudes, depths, and coordinates containing the triggering events, it is relatively easy to trap many non-triggering events alongside them (and increase the positive basis risk). An algorithm designed to minimize total basis risk will have a preference towards not trapping the many non-triggering events at the cost of not catching the few triggering events. Thus, solutions obtained with the requirement of minimizing total basis risk will tend to miss some triggering events, increasing the negative basis risk bias as seen in figure 2.

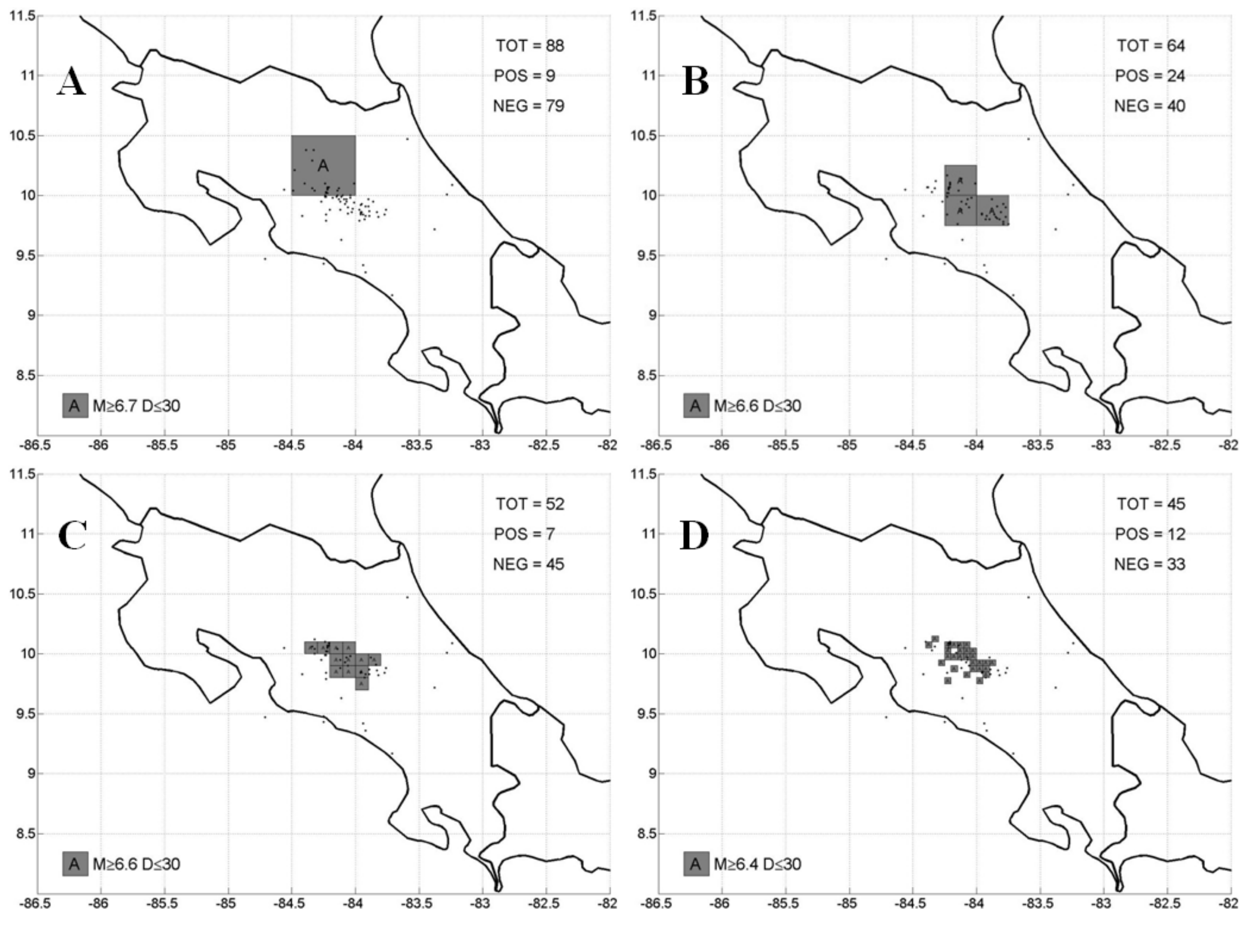

Figure 3. Optimal trigger conditions obtained for Z=2 and increasing resolution.

Reducing the size of the boxes (increasing the geographic resolution) helps to reduce total basis risk in general, and negative basis risk in particular. Caution needs to be exercised, however, not to forget that as the resolution increases, the model accuracy becomes the relevant driver in the overall basis risk of the transaction. Naturally, if the boxes were designed to be so small as to include only one event and the zones were sufficient, the basis risk could practically be reduced to zero. A perfect optimization of the trigger mechanism, however, will emphasize the basis risk introduced by the model and the inherent uncertainty in the occurrence of earthquakes. This problem can be controlled by using larger stochastic samples, by introducing random variability in the events present in the catalog, and by using the proposed methodology with moderation with the understanding that minimizing the basis risk introduced by the trigger might occur at the cost of eliminating the flexibility of the trigger to account for uncertainty in the events.

Selecting a Satisfactory Zonation

Cat bonds actually found in the market typically use a small number of zones where the threshold magnitude and threshold depth are defined. It can be shown that this requirement can be satisfied through an optimization process that searches for the best zone thresholds associated with the minimum basis risk across the domain (Franco, 2009). *Z* is then arbitrary and typically *Z*<<*K* in contrast to the previously discussed lower bounds where *Z*=*K*.

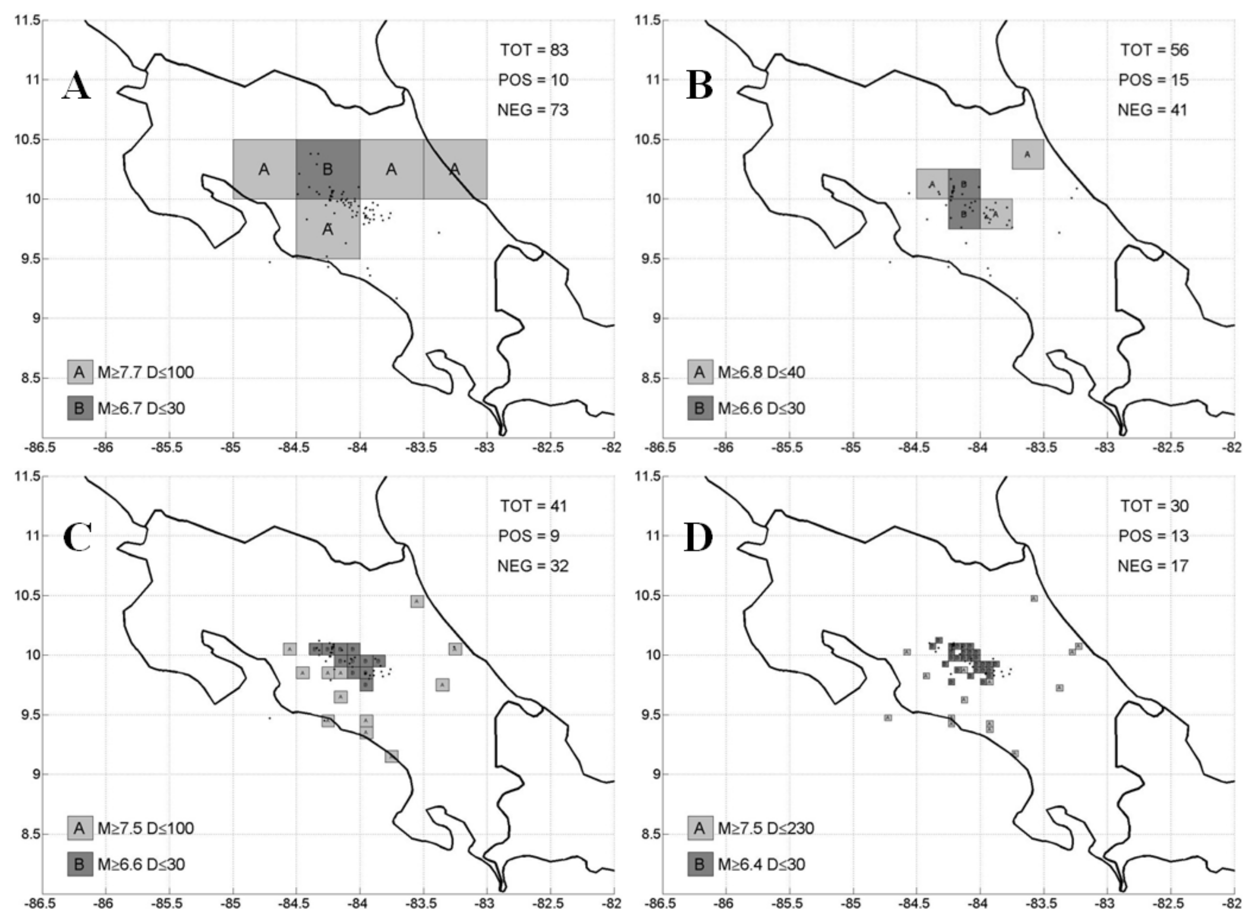

Figure 4. Optimal trigger conditions obtained for *Z*=3 and increasing resolution.

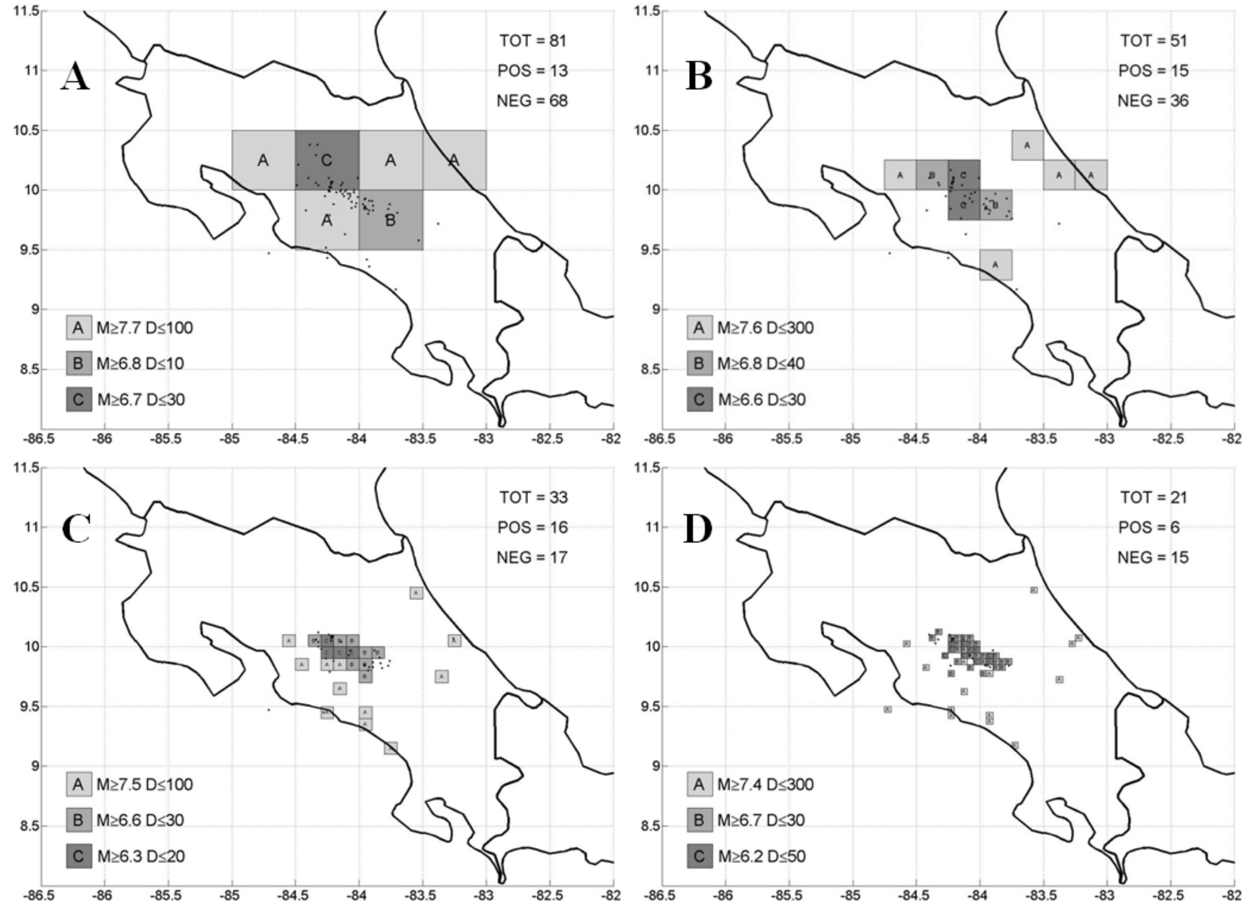

Figure 5. Optimal trigger conditions obtained for *Z*=4 and increasing resolution.

The sixth step in the methodology consists of calculating alternative scenarios considering different number of zones *Z*. The optimization process yields a set of threshold values for the selected zones in order to minimize basis risk in as far as it is possible with the established constraints. Solutions obtained through this exercise are presented here assuming a loss threshold corresponding to the 100 year return period, for *Z*=2, 3, and 4 and for *d*=0.5° (A), 0.25° (B), 0.1° (C), and 0.05° (D) in figures 3, 4, and 5. Note that a minimum of two zones are typically necessary since one of the zones corresponds to the area where it is more favorable from a basis risk minimization perspective to ignore all events due to their low losses regardless of their depth and magnitude. The optimal trigger conditions are summarized in figures 3, 4, and 5 for the different respective zonations and increasing resolutions. The dots in these figures depict the events that were incorrectly triggered. It can be seen that as the resolution and the number of zones increase, the number of dots (the basis risk) decreases.

There is a negligible computational cost in performing a more intricate analysis with a high resolution and a high number of zones, therefore the optimal combination of these factors are actually determined by the requirements of the sponsor of the cat bond and the appetite of the market as well as by the considerations cited earlier about model risk and large resolutions. The seventh and last step of the methodology consists of selecting the best option for the cat bond out of all those available. Complex solutions that offer very low basis risk will not necessarily be successful, since the cat bond might not be appealing to investors. It also might have little tolerance for uncertainty in the parameters of the event and modeling inaccuracies.

Figure 3 shows the application of only one trigger zone and one non-triggering zone (*Z*=2) at the four different resolutions considered. Observe that as the resolution increases, the trigger zone is defined more smoothly around the San Jose area in order to include and discard the appropriate events. At the highest resolution considered of 0.05° in figure 3(D), the triggering zone becomes disjointed, since there are gaps without triggering events in the vicinity of San Jose. It is at this point that the increase of accuracy needs to be judged from a common sense perspective. The numerical algorithm identifies these zones as non-trigger zones given the data sample of 10,000 years but it is not unlikely that triggering events could occur in those gaps. A reasonable choice needs to be made in these cases. For instance, a sponsor might prefer to increase the basis risk to 52 (instead of 45) but provide a more smooth trigger zone that would accommodate some degree of uncertainty in the events. Analogous observations can be made for figures 4 and 5.

Conclusions

This paper has illustrated a straightforward methodology to construct first generation or cat-in-a-box catastrophe bonds with an application to Costa Rica. The process is geared towards minimizing the basis risk introduced by the trigger mechanism in an automated and transparent fashion. Once an event catalog is available, the computational design cost of different cat bond trigger mechanisms is minimal, thus reducing the overall costs associated with these tools. The methodology can yield a variety of potential solutions with varying loss threshold, geographic resolution, basis risk, and number of zones that provide potential cat bond sponsors with a wide array of choices of accuracy, complexity, and market appeal. This design process leads to a "menu" of cat bond options whose quality can be quantified numerically but also compared visually to gauge their acceptance in the risk transfer market.

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