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DYNAMIC BEHAVIOUR OF CONCRETE LIQUID TANKS UNDER HORIZONTAL AND VERTICAL GROUND MOTIONS USING FINITE ELEMENT METHOD

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ABSTRACT

In this study, the finite element method is used to investigate the seismic behavior of concrete liquid tanks. This method is capable of considering both impulsive and convective responses of liquid-tank system. Two different finite element models corresponding with rectangular and cylindrical tank configurations are studied under the effects of both horizontal and vertical ground motions using the scaled earthquake components of the 1940 El-Centro earthquake record. The containers are assumed fixed to the rigid ground. Fluid-structure interaction effects on the dynamic response of fluid containers are taken into account incorporating wall flexibility. The results show that the effect of vertical acceleration on the dynamic response of liquid tanks is found to be less significant when horizontal and vertical ground motions are considered together.

Introduction

Liquid storage tanks are critical lifeline structures which their use has become widespread during the recent decades. These structures are extensively used in water supply facilities, oil and gas industries and nuclear plants for storage of a variety of liquid or liquid-like-materials such as oil, liquefied natural gas (LNG), chemical fluids and wastes of different forms. These tanks can be exposed to a wide range of seismic hazards and interaction with other sectors of built environment. Heavy damages have been reported due to strong earthquakes such as Niigata in 1964, Alaska in 1964, Parkfield in 1966, Imperial County in 1979, Coalinga in 1983, Northridge in 1994 and Kocaeli in 1999.

Problems associated with liquid tanks involve many fundamental parameters. In fact, the dynamic behavior of liquid tanks is governed by the interaction between fluid and structure as well as soil and structure along their boundaries. Housner (1963) developed the most commonly used analytical model in which hydrodynamic pressure induced by seismic excitations is separated into impulsive and convective components using lumped mass approximation. The fluid was assumed incompressible, inviscid and the structure was assumed to be rigid. This model has been adopted with some modifications in most of the current codes and standards.

Yang (1976) studied the effects of wall flexibility on the pressure distribution in liquid and corresponding forces in the tank structure through an analytical method. Also, Veletsos and

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Yang (1977) developed flexible anchored tank linear models and found that the pressure distribution for the impulsive mode of rigid and flexible tanks were similar, but also discovered that the magnitude of the pressure was highly dependent on the wall flexibility.

Haroun (1984) presented a very detailed analytical method in the typical system of loading in rectangular tanks. Also, Haroun and Tayel (1985) used the finite element method (FEM) for analyzing dynamic response of liquid tanks subjected to vertical seismic ground motions.

Veletsos and Tang (1986) analyzed liquid storage tanks subjected to vertical ground motion on both rigid and flexible supporting media.

Kim et al. (1996) further developed analytical solution methods and presented the response of filled flexible rectangular tanks under vertical excitation. Park et al. (1992) performed research studies on dynamic response of the rectangular tanks. They used the boundary element method (BEM) to obtain hydrodynamic pressure distribution and finite element method (FEM) to analyze the solid wall.

Dogangun et al. (1997) investigated the seismic response of liquid-filled rectangular storage tanks using analytical methods, and the finite element method implemented in the general purpose structural analysis computer code SAPIV.

Kianoush and Chen (2006) investigated the dynamic behavior of rectangular tanks subjected to vertical seismic vibrations in a two-dimensional space. The importance of vertical component of earthquake on the overall response of tank-fluid system was discussed. In addition, Kianoush et al. (2006) introduced a new method for seismic analysis of rectangular containers in two-dimensional space in which the effects of both impulsive and convective components were accounted for in time domain.

Livaoglu (2008) evaluated the dynamic behavior of fluid–rectangular tank–foundation system with a simple seismic analysis procedure. Ghaemmaghami and Kianoush (2009) used a finite element method to investigate the dynamic behavior of liquid tanks in two-dimensional space.

In this study, a comprehensive investigation of dynamic behavior of concrete rectangular tanks was carried out using the finite element method (FEM) in which the coupled fluid-structure equations were solved using direct integral method. Effects of wall flexibility, damping properties of liquid and sloshing motion were taken into account. Also, both horizontal and vertical components of an earthquake were applied in the procedure to investigate the effect of vertical ground acceleration on the dynamic responses.

Numerical procedure

In liquid domain, the hydrodynamic pressure distribution is governed by the pressure wave equation. Because of the small volume of containers, the velocity of pressure wave assumed to be infinity. Assuming that water is incompressible and neglecting its viscosity, the small-amplitude irrotational motion of water is governed by the two-dimensional wave equation:

$$\nabla^2 P(x, y, z, t) = 0 \tag{1}$$

Where $P(x, y, t)$ is the hydrodynamic pressure in excess of hydrostatic pressure.

The hydrodynamic pressure in equation (1) is due to the horizontal and vertical seismic excitations of the walls and bottom of the container. The motion of these boundaries is related to hydrodynamic pressure through boundary conditions. For earthquake excitation, the appropriate boundary condition at the interface of liquid and tank is governed by:

$$\frac{\partial P(x,y,z,t)}{\partial n} = -\rho a_n(x,y,z,t) \tag{2}$$

Where ρ is the density of liquid and a_n is the component of acceleration on the boundary along the direction outward normal n. No wave absorption is considered in the interface boundary condition.

Accounting for the small-amplitude gravity waves on the free surface of the liquid, the resulting boundary condition is given as:

$$\frac{1}{g}\frac{\partial^2 P}{\partial t^2} + \frac{\partial P}{\partial z} = 0 \tag{3}$$

In which y is the vertical direction and g is the gravitational acceleration.

Applying the small-amplitude wave boundary condition will lead to an evaluation of convective pressure distribution in the liquid domain which is of great importance in liquid containers. However, due to the large amplitude of sloshing under the strong seismic excitations and turbulence effects in liquid tanks, more complicated boundary conditions on the surface of liquid are needed to accurately model the convective motions such as works done by Chen et al. (1996). Neglecting the gravity wave effects leads to the free surface boundary condition which is appropriate for impulsive motion of liquid. The related governing equation is given as:

$$P(x,y,H_l,t) = 0 \tag{4}$$

Where H_l is the height of liquid in the container.

Using finite element discretization and discretized formulation of equation (1), the wave equation can be written as the following matrix form:

$$[G]\{\ddot{P}\} + [H]\{P\} = \{F\} \tag{5}$$

In which $G_{i,j} = \sum G_{i,j}^e$, $H_{i,j} = \sum H_{i,j}^e$ and $F_i = \sum F_i^e$. The coefficients $G_{i,j}^e$, $H_{i,j}^e$ and F_i^e for an individual element are determined using the following expressions:

$$\begin{aligned}
G_{i,j}^e &= \frac{1}{g}\int_{A_e} N_i N_j \, dA \\
H_{i,j}^e &= \int_{V_e} \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right) dV \\
\{F\} &= \{F_i\} - \rho [Q]^T (\{\ddot{U}\} + \{\ddot{U}_g\}) \\
F_i^e &= \int_{A_e} N_i \frac{\partial P}{\partial n} dA
\end{aligned} \tag{6}$$

Where N_i is the shape function of the *i*th node in the liquid element, $\{\ddot{U}\}$ is the acceleration vector of nodes in the structure domain, $\{\ddot{U}_g\}$ is the ground acceleration vector applied to the system and $[Q]$ is the coupling matrix. A_e and V_e are the integration over side and area of the element, respectively.

In the above formulation, matrices $[H]$ and $[G]$ are constants during the analysis while the force vector $\{F\}$, pressure vector $\{P\}$ and its derivatives are the variable quantities. In the coupling system of liquid–structure the pressures are applied to the structure surface as the loads on the container walls. The general equation of fluid–structure can be written in the following form:

$$
\begin{aligned}
&[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{f_1\}-[M]\{\ddot{U}_g\}+[Q]\{P\}=\{F_1\}+[Q]\{P\} \\
&[G]\{\ddot{P}\}+[C']\{\dot{P}\}+[H]\{P\}=\{F\}-\rho[Q]^T(\{\ddot{U}\}+\{\ddot{U}_g\})=\{F_2\}-\rho[Q]^T\{\ddot{U}\}
\end{aligned} \tag{7}
$$

Where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices of structure. The term $[C']$ is the matrix representing the damping of liquid which is dependent on the viscosity of liquid and wave absorption in liquid domain and boundaries and is rigorously determined. As previously discussed, the matrix $[Q]$ transfers the liquid pressure to the structure as well as structural acceleration to the liquid domain.

The direct integration scheme is used to find the displacement and hydrodynamic pressure at the end of time increment *i+1* given the displacement and hydrodynamic pressure at *i*. The Newmark-β method is used for discretization of both equations (implicit-implicit method). In this method $\{\dot{U}\}_{i+1}, \{U\}_{i+1}, \{\dot{\mathrm{P}}\}_{i+1}$ and $\{P\}_{i+1}$ can be written as follows:

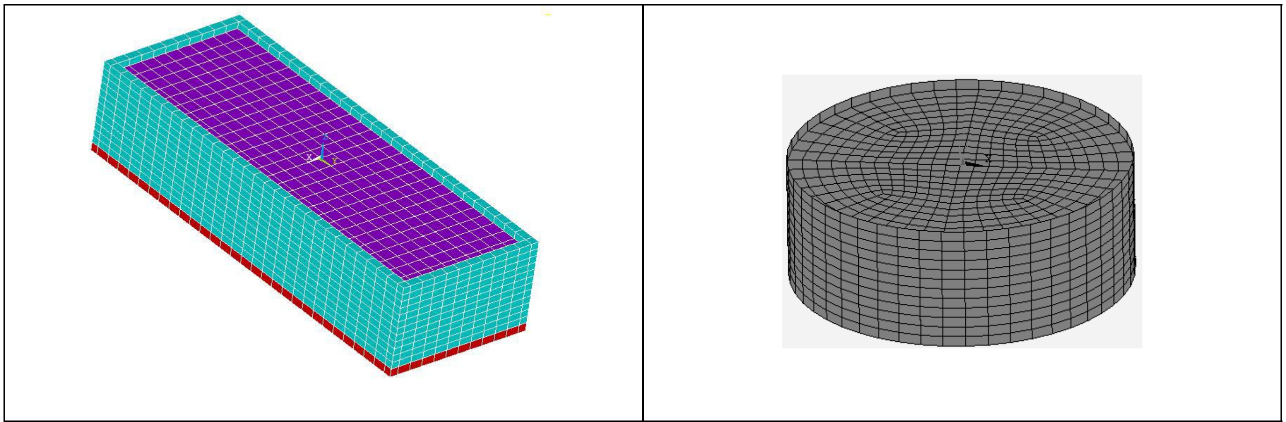
$$
\begin{aligned}
\{\dot{U}\}_{i+1} &= \{\dot{U}\}_{i+1}^{p} + \gamma\Delta t\{\ddot{U}\}_{i+1} \\
\{\dot{U}\}_{i+1}^{p} &= \{\dot{U}\}_{i} + (1-\gamma)\Delta t\{\ddot{U}\}_{i} \\
\{U\}_{i+1} &= \{U\}_{i+1}^{p} + \beta\Delta t^2\{\ddot{U}\}_{i+1} \\
\{U\}_{i+1}^{p} &= \{U\}_{i} + \Delta t\{\dot{U}\}_{i} + (0.5-\beta)\Delta t^2\{\ddot{U}\}_{i} \\
\{\dot{P}\}_{i+1} &= \{\dot{P}\}_{i+1}^{p} + \gamma\Delta t\{\ddot{P}\}_{i+1} \\
\{\dot{P}\}_{i+1}^{p} &= \{\dot{P}\}_{i} + (1-\gamma)\Delta t\{\ddot{P}\}_{i} \\
\{P\}_{i+1} &= \{P\}_{i+1}^{p} + \beta\Delta t^2\{\ddot{P}\}_{i+1} \\
\{P\}_{i+1}^{p} &= \{P\}_{i} + \Delta t\{\dot{P}\}_{i} + (0.5-\beta)\Delta t^2\{\ddot{P}\}_{i}
\end{aligned} \tag{8}
$$

Where γ and β are integration parameters. Further descriptions regarding the direct integration method can be found in the studies done by Mirzabozorg et al. (2003).

In the proposed FE procedure, Rayleigh damping is used in the direct step-by-step integration method. The stiffness proportional damping equivalent to 5% of critical damping corresponding to first mode of vibration is assumed as structural damping for concrete material. For sloshing and impulsive behaviors of water 0.5 percent and 5 percent of critical damping are applied, respectively. These values are chosen as conservative damping ratios based on studies done by Veletsos and Tang (1986) and Veletsos and Shivakumar (1997).

Finite element implementation

In this study, a finite element model is developed to investigate the dynamic behavior of liquid tanks. An eight node isoparametric element with three translational degrees of freedom in each node is used in the finite element procedure to model the tank walls and the base slab. The liquid domain is modeled using 8-node isoparametric fluid elements with pressure degree of freedom in each node. Two different model configurations associated with rectangular and cylindrical tanks are investigated in three-dimensional space. The finite element (FE) model configurations for both tanks are shown in Figure 1.



(a)	(b)

Figure 1: Finite element model of liquid tank: (a) Rectangular tank (b) Cylindrical tank

The dimensions and properties of rectangular and cylindrical tanks are as follows:
Rectangular tank:

$\rho_w = 2300 kg/m^3 \quad \rho_l = 1000 kg/m^3 \quad E_c = 26.44 GPa \quad \nu = 0.17$

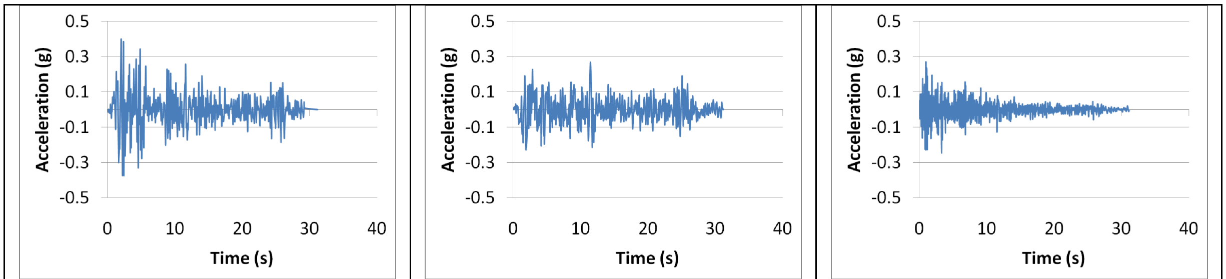
$L_x = 15m \quad L_y = 30m \quad H_w = 6.0m \quad H_l = 5.5m \quad t_w = 0.6m$

Circular tank:

$\rho_w = 2300 kg/m^3 \quad \rho_l = 1000 kg/m^3 \quad E_c = 20.77 GPa \quad \nu = 0.17$

$D = 47.9m \quad H_w = 6m \quad H_l = 5.5m \quad t_w = 0.5m$

Where L_x and L_y are half of the length of rectangular tank in x and y directions and D is the diameter of the circular tank. t_w, H_w and H_l are thickness of the wall, height of the wall and liquid depth, respectively.
It is also assumed that the tank is anchored at its base and the effects of uplift pressure are not considered. The horizontal and vertical components recorded for 1940 El-Centro earthquake are used as excitations of the tank–liquid system. The components are scaled in such a way that the peak ground acceleration in the horizontal direction is 0.4g, as shown in Figure 2.



(a) (b) (c)

Figure 2: Scaled Components of the 1940 El-Centro earthquake: (a) longitudinal component (b) transversal component (c) vertical component

Results of analysis

Two concrete liquid container models given in Figure 1 are used basically for the example analyses in time-domain. It should be noted that the longitudinal component of earthquake is applied to the tanks.

The transient and maximum values of shear, moment, and hoop force along the height of the tank wall acting per unit width of the wall due to horizontal and vertical excitations were determined using the FEM. As an illustration, variations of base moment and base shear with time are shown for rectangular and cylindrical models in Figures 3 and 4, respectively.

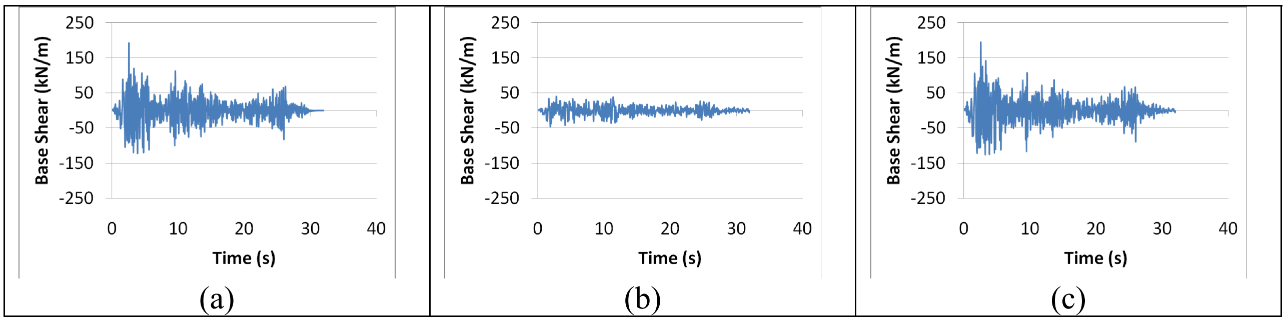


Figure 3: Time history of base moment response due to impulsive behaviour of rectangular tank model: (a) Horizontal excitation (b) Vertical excitation (c) Horizontal and vertical excitation

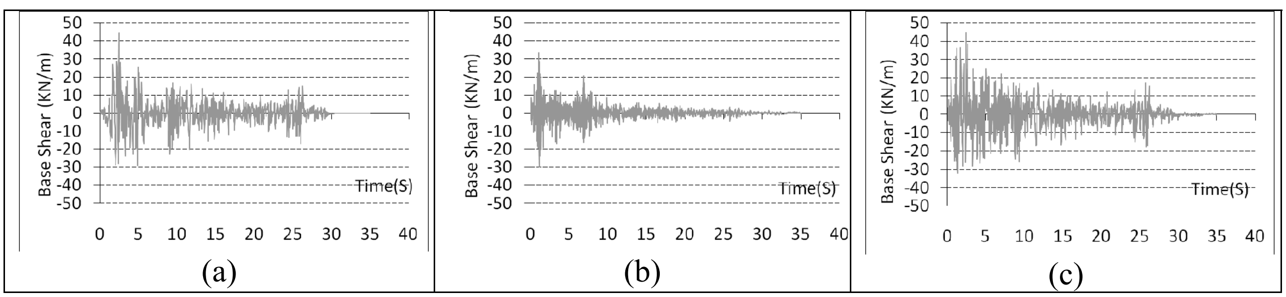


Figure 4: Time history of base shear response due to impulsive behaviour of cylindrical tank model: (a) Horizontal excitation (b) Vertical excitation (c) Horizontal and vertical excitation

A detailed comparison among peak impulsive and convective pressure distributions over the height of the rectangular tank wall measured at the middle section of longer wall is presented in Figure 5. Due to single vertical component, a linear impulsive pressure distribution is obtained which its peak value at the tank bottom is almost equal to peak vertical acceleration multiplied by the liquid height and water density. Also, the dynamic pressure distribution along the side walls under combined horizontal and vertical ground motions indicates that the effect of vertical ground motion is negligible on the seismic behavior of fluid-structure system.

In addition, the effect of vertical acceleration on convective pressure distribution is found insignificant.

For circular tank model, hydrodynamic pressure distribution corresponding to horizontal and vertical excitations over the height of the tank wall are shown in Figure 6.

In cylindrical tank model, the impulsive pressure distribution is obtained due to single vertical component which its peak value at the tank bottom is almost 45 percent higher than the value obtained by product of maximum vertical acceleration, height of the tank and water density.

Also, the dynamic pressure distribution over the tank wall under combined horizontal and vertical ground motions indicates that the effect of vertical ground motion is negligible on the seismic behavior of fluid-structure system as shown previously in rectangular tank.

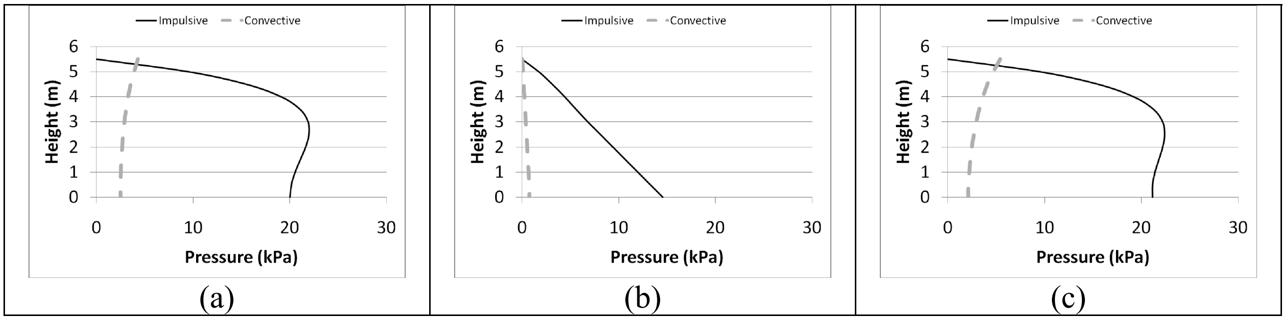


Figure 5: Pressure distribution along height of rectangular tank model measured at the middle section of longer wall: (a) Horizontal excitation (b) Vertical excitation (c) Horizontal and vertical excitation

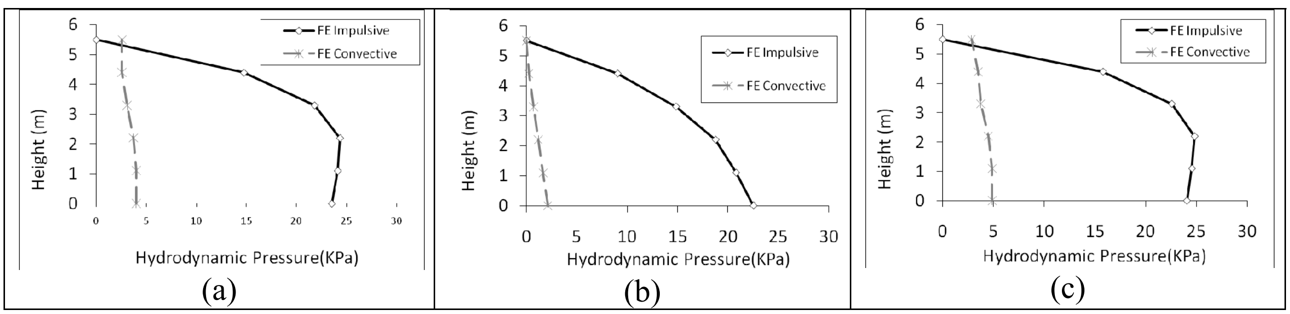


Figure 6: Pressure distribution along height of cylindrical tank model: (a) Horizontal excitation (b) Vertical excitation (c) Horizontal and vertical excitation

The maximum hydrodynamic impulsive pressure under horizontal excitation occurs at $\theta = 0$ corresponding to direction of horizontal ground motion. For cylindrical tank at the water depth of 4.4m according to FE this maximum value is 27.2 kPa, however ACI 350.3-06 predicts the maximum pressure of 45.2 kPa at this location. Under vertical excitation, the uniform hydrodynamic pressure having the maximum magnitude of 23.6 kPa was obtained from FE. The code predicts slightly higher value of 28.9 kPa at this depth.

The calculated results show that the effect of vertical ground motion could be as high as that of the horizontal excitation when considered separately, however this effect is of little significance when horizontal and vertical earthquake components are considered to be applied simultaneously to the structure.

The vertical acceleration is found to have a negligible effect on the convective response. This has also been concluded by Haroun who stated that little or no sloshing occurs due to transient vertical motion. Therefore, in analyzing the seismic response of cylindrical liquid-storage tanks to vertical random excitations, the effect of convective fluid motion can be neglected.

Conclusions

In this study, a finite element method is introduced that can be used for the dynamic analysis of partially filled fluid container under horizontal and vertical ground excitations in three-dimensional space. The liquid sloshing is modeled using an appropriate boundary condition and the damping effects due to impulsive and convective components of the stored liquid are modeled using the Rayleigh method. Two different configurations including cylindrical and rectangular tank models are considered to investigate the effect of geometry on the response of liquid-structure system in time-domain. Effect of wall flexibility on the dynamic response of system is taken into account.

The impulsive and convective pressure distribution along the wall height is measured for all cases when the sloshing height reaches its maximum value.

Although the FE convective responses are in satisfactory agreement with corresponding responses obtained by current design code, a discrepancy is seen between corresponding impulsive values. On the other hand, higher result values are obtained using current design code.

Applying the vertical excitation combined with horizontal excitations does not affect the dynamic behavior significantly. This point is valid for both rectangular and cylindrical tanks.

It is clear that the dynamic behavior of liquid concrete tanks depends on a wide range of parameters such as seismic properties of earthquake, tank dimensions and fluid-structure interaction which should be considered in future research studies. This study shows that the proposed FE method can be used for time history analysis of liquid tanks. One of the major advantages of this method is in accounting for three-dimensional geometry effects, damping properties of liquid domain and calculating impulsive and convective terms, separately.

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