



METHODS, MODELS, AND SOFTWARE FOR SEISMIC RISK ASSESSMENT

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ABSTRACT

This paper describes the use of reliability methods, such as FORM, to carry out probabilistic performance and risk assessment. Conversely, it is common in the existing literature to employ total probability integration in conjunction with conditional probabilities. The reliability-based approach that is adopted in this paper employs probabilistic models that are capable of simulating all possible response scenarios. Each scenario corresponds to a unique realization of the intervening random variables. This paper presents a new software framework to carry out this type of analysis, with probabilistic models for hazards, structures, and consequences. The software has a graphical user interface and is freely available at www.inrisk.ubc.ca. The methodology and computer implementation is provided in this paper, together with a simple example. A companion paper at this conference presents a comprehensive example: a regional risk analysis of a university campus with more than 600 buildings. Ongoing work addresses a larger metropolitan area.

Introduction

This paper supports the trend towards “performance-based earthquake engineering” and the extended use of “seismic risk assessment.” Although the practical meaning of these terms may vary, they invariably include the probabilistic prediction of some uncertain future event(s). Examples of such predictions include the probability or rate of exceeding a monetary loss for a commercial building subjected to strong ground motion, including the downtime loss during repair. The prediction of such events involves several steps and significant uncertainty. The steps are often divided into hazard modelling, structural modelling, damage modelling, and consequence modelling. This is common in traditional infrastructure risk analysis (McGuire 2004). It is also adopted in more recent performance-based engineering approaches. For example, in the Pacific Earthquake Engineering Research Centre, the steps are identified as the prediction of intensity measures, engineering demand parameters, damage measures, and decision variables (Deierlein *et al.* 2003).

The obvious presence of uncertainty has motivated the use of probabilistic methods. One

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approach applies the theorem of total probability in conjunction with conditional probabilities. In contrast with total probability and/or conditional probabilities, this paper adopts the use of comprehensive and flexible probabilistic models in conjunction with reliability methods. Reliability methods include the first-order reliability method (FORM) and importance sampling. Such methods have been developed since the 1970's (Cornell 1967; Ditlevsen and Madsen 1996; Madsen *et al.* 2006; Melchers 1999).

The essence of the approach that is adopted in this paper, which was referred to as unified reliability analysis in (Haukaas 2008), is to describe all uncertainty by random variables and define a limit-state function that identifies the event of interest. For instance, the limit-state function may define the event that the total loss exceeds a specific value. To evaluate the probability associated with the limit-state function, referred to as the failure probability in the classical structural reliability literature, it is necessary to evaluate multiple probabilistic models for hazards, structures, and consequences. The description of a framework to carry out this analysis is the subject matter herein. This paper introduces the methodology, the framework of probabilistic models, and the software that is developed to carry out the analysis. Another paper at this conference applies the software to carry out regional risk assessment for the campus of the University of British Columbia, Vancouver (Mahsuli and Haukaas 2010). Ongoing work addresses the larger Metro Vancouver region.

Methodology

For the reader who is familiar with structural reliability methods, there is nothing new in the adopted analysis approach. The problem contains two ingredients; random variables and limit-state functions, and the objective is to compute the associated probability. For the reader who is unfamiliar with reliability methods, the following brief review is provided.

Formulation of Reliability Problems

To carry out a reliability analysis, all uncertainty is discretized in the form of random variables. These random variables are collected in the vector \mathbf{x} and characterize both irreducible (aleatory) and reducible (epistemic) uncertainty. Examples of random variables include the epicentre location, intensity model parameters, material properties, geometry properties, damage model parameters, and loss model parameters (Mahsuli and Haukaas 2010). These random variables enter into the probabilistic models that are described later.

Next, a limit-state function, $g(\mathbf{x})$, is defined. It is defined such that $g \leq 0$ denotes the event of interest. For example, the limit-state function $g(\mathbf{x}) = 10^6 - l(\mathbf{x})$, where l denotes the monetary loss, produces the probability that the loss exceeds one million. Two tasks are needed to compute this probability: definition of probabilistic models that calculate the loss, l , for given realizations of random variables \mathbf{x} , and execution of the reliability analysis.

Probabilistic Models

In approaches that are based on total probability integration and conditional probabilities, the objective of a model is to produce conditional probabilities of the form $P(B(\mathbf{x}) > b | A = a)$, where $P(\)$ denotes the probability, A is the input variable to the model, and B is the output

variable. Lowercase letters denote a specific outcome of a variable. One example is a damage model that according to this scheme, delivers the probability that the damage exceeds a damage state, b , given the site intensity, a . The calculation of such conditional probabilities requires some form of reliability analysis.

In the approach adopted in this paper, the phrase “probabilistic model” has a different meaning. Keeping the notation from above, a probabilistic model in this paper is defined as $b=B(\mathbf{x},a)$. In words, a realization of the output is computed for given values of the random variables and the value a from upstream models. Figure 1 provides a simple sketch of the definition of this type of probabilistic models. Each model takes realizations of random variables as well as input from upstream model(s). During the evaluation of the limit-state function, the model generates the output that is either used by downstream models or enters into the limit-state function.

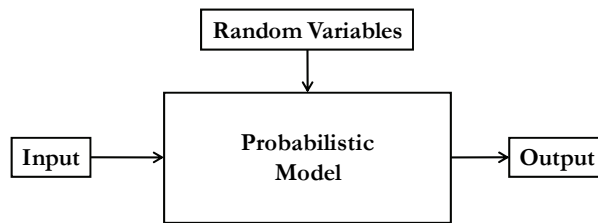


Figure 1. The general format of a probabilistic model.

It is noted that this type of probabilistic model does *not* produce a probability. Rather, it produces an outcome of one or more uncertain responses. In other words, it simulates a possible scenario. A list of properties of a probabilistic model is provided by Haukaas and Bohl (2009).

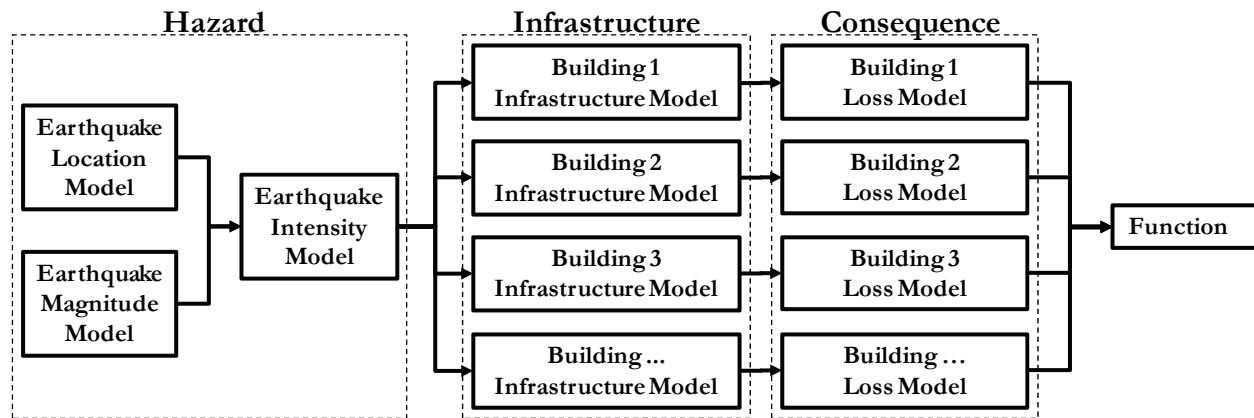


Figure 2. A schematic example of multiple models for calculating the total monetary loss due to earthquake damage to several buildings.

Although it is stated above that there is nothing new in the analysis compared with classical reliability methods, the multi-model aspect of the analysis is unique compared to contemporary approaches for performance-based earthquake engineering. Figure 2 shows a schematic overview of a multi-model reliability analysis. Hazard models provide output, *i.e.*, the

intensity at a site, which enters into infrastructure models, which in turn provide responses that enter into consequence models. All models contribute to the summarized loss in the “Function,” *i.e.*, the limit-state function. A simple example of this type of analysis is carried out in this paper to demonstrate the new software, while the companion paper at this conference presents a more comprehensive example (Mahsuli and Haukaas 2010).

Reliability Methods

Upon specifying the random variables, the models, and the limit-state function, the reliability analysis is carried out to determine the probability. In this paper, it is recommended to first carry out a histogram (relative frequency) analysis to get a sense of the probability distribution of the response(s) of interest. Subsequently, to determine the probability of specific events, there are several reliability methods available. FORM is recommended here, followed by importance sampling around the most probable point obtained by FORM. This strategy has several advantages. It provides a balance between accuracy and computational cost; it provides importance measures that identify the most important sources of uncertainty; it provides sensitivity results with respect to decision variables to be used in decision/optimization analysis; and it yields an error check in the form of importance sampling analysis. The disadvantage of the FORM/importance sampling approach is that the derivative of the limit-state function with respect to the random variables are needed; and that all model responses must be “smooth” with respect to variations in the random variable values, *i.e.*, the derivatives must be continuous. To this end, it is stressed that the new software provides efficiently and accurately computed derivatives by the direct differentiation method (DDM).

In the context of reliability methods, it is interesting to note the role of response surface and neural network approaches. These have gained popularity in the structural reliability community. Essentially, these approaches develop a metamodel for the limit-state function before carrying out the reliability analysis by, *e.g.*, FORM or sampling. By approximating $g(\mathbf{x})$ by, *e.g.*, a polynomial expression, the subsequent reliability analysis becomes computationally inexpensive because of the ease of evaluation of the limit-state function. In the approach adopted in this paper, response surface methods are not used for the reliability analysis. In other words, the limit-state function is *not* approximated by a metamodel. Rather, any of the individual hazard, infrastructure, and/or consequence models may be synthesized into a metamodel. One example in the ongoing work is the use of a comprehensive and computationally expensive model that forecasts the transportation demand in the Metro Vancouver region. Instead of entering this entire model into the probabilistic model framework, which has been explored at a steep computational cost, a metamodel has been developed that approximates the heavy numerical analyses. This modelling is carried out by a Bayesian approach, which is outside the scope of this paper. This is how the metamodel approaches fit into the proposed framework.

Software

An object-oriented software for multi-model reliability analysis is developed and presented in this paper. It is created with the C++ programming language (Stroustrup 2000). The need for a graphical user interface motivated an extensive use of the Qt application framework (Blanchette and Summerfield 2006). The use of Qt also facilitates the deployment of the software on several operating systems, including Microsoft Windows[®] and Mac OS X[®]. The

software is named Rt, which acknowledges Qt and abbreviates the words “Reliability tools.” Acknowledgements are also given to the GNU Scientific Library (Galassi *et al.* 2006) for processing of mathematics and matrices, the muParser Library (Berg 2009) for function evaluation, the Qwt Library (Rathmann and Wilgen 2009) for plotting, and Google Maps® Application Programming Interface (Google 2009) for visualization of locations. Rt is made available online at the Software page of www.inrisk.ubc.ca. This site also contains video tutorials for new users of the software.

User Interface

Figure 3 shows a screenshot of the graphical user interface of Rt. The main window has four sub-windows or “panes.” The Objects Pane is on the left and contains a tree view of the objects that the user creates. These objects are classified into Domain and Analysis objects. The Domain objects contain the data, which is classified into Parameters, Functions, and Models. The Analysis contains the algorithms to carry out the reliability analysis. The user creates and deletes objects through the Objects Pane.

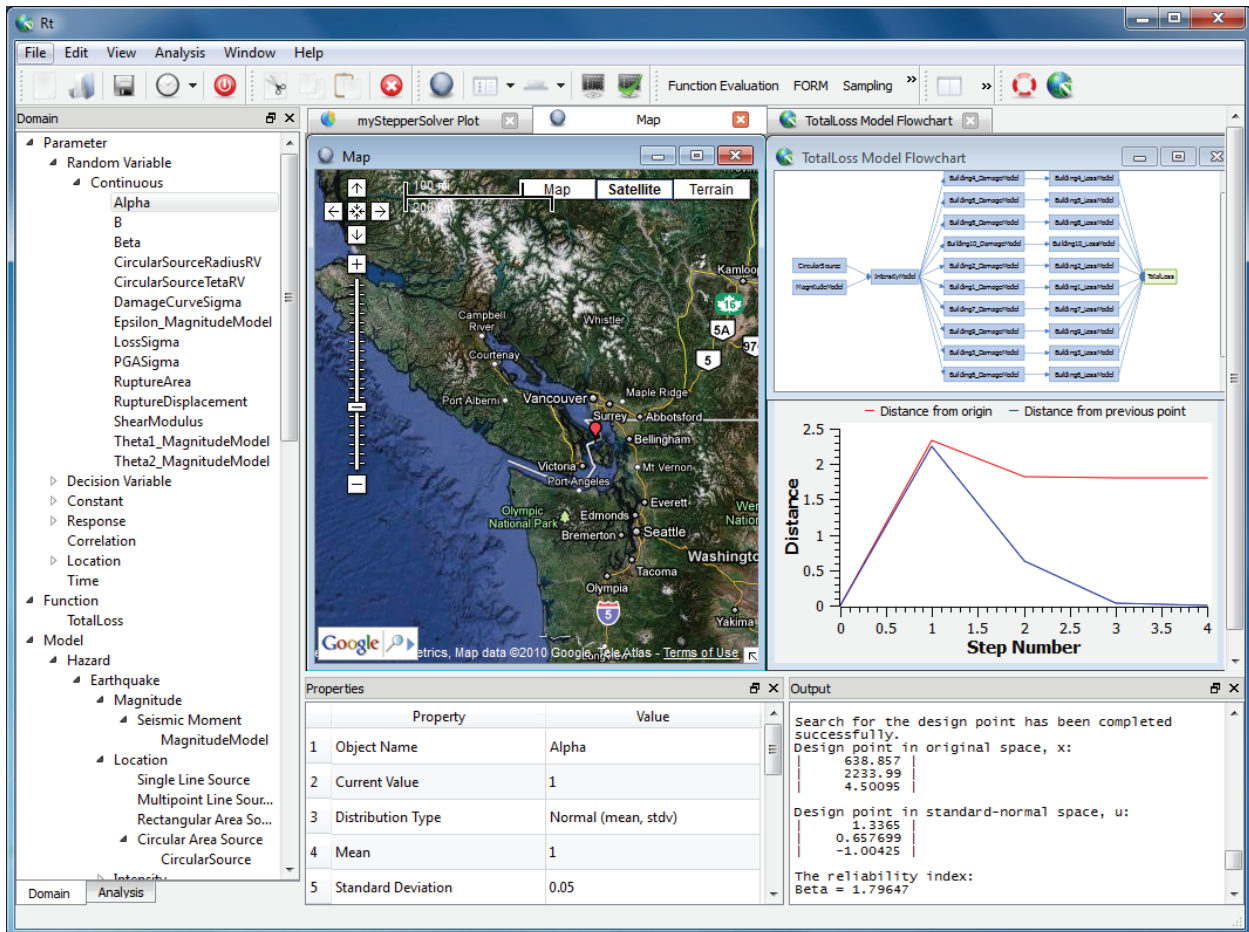


Figure 3. Screenshot of the graphical user interface of Rt.

The Properties Pane at the bottom left of Figure 3 displays an interactive view of the properties of each object. Upon selecting an object in the Objects Pane, its properties are

displayed in the Properties Pane. It is from this pane that the user can change, *e.g.*, the mean and standard deviation of a random variable. The Output Pane at the lower right displays the output of the analysis. This output includes messages related to the progress of the analysis, error reports, and analysis results, such as event probabilities. Finally, the Visualization Pane displays results, diagrams, flowcharts, and regional maps. For example, the integration with Google Maps[®] renders the visualization of locations, such as the location of buildings or the epicentre of an earthquake.

Parameterization

Contrary to many engineering analysis software, Rt is fully “parameterized.” In the object-oriented programming terminology, this means that each parameter, such as a modulus of elasticity, is instantiated at run-time as an object from a parameter class. Conversely, the common approach is that parameters are declared as integer or double precision data members of, say, a material class. In this traditional approach, the value of parameters is protected by the object-oriented principle of “encapsulation.” This is safe from a data protection viewpoint, but cumbersome in reliability and optimization analysis. This is because such analyses require the modification of parameter values during the analysis. An example of how to break encapsulation to facilitate parameter value updating during the analysis is found in OpenSees (McKenna *et al.* 2003), where designated member functions are called to modify the value of the data members. This is sometimes referred to as a “top-down” approach, because the new parameter values are sent from the top of the object hierarchy into the object that needs it, *e.g.*, a material object.

The alternative “bottom-up” approach is adopted in Rt. In this approach, individual objects exist for all parameters and these objects are being passed as “pointers” to be kept by models. One parameter object can be sent to multiple models. At run time, the models look up the value of each parameter instead of having the value as a data member. As an example, consider a finite element model in which the concrete strength is uncertain. In the parameterized bottom-up approach, a random variable object will be instantiated and passed to the finite-element structural model. In fact, this is what makes the finite element model a probabilistic model. At run-time, the structural model looks up the current realization of that random variable and runs the finite element analysis. Subsequently, the value (current realization) of the random variable is updated by the orchestrating reliability analysis, which also has access to the pointer to this parameter.

Two comments are made about the bottom-up parameterization: First, it is noted that the object-oriented principle of encapsulation is broken. In reliability and optimization analysis, this is a necessity. This implies that care must be exercised by the programmer to avoid unintended modification of data during runtime. On the downside, it is also noted that the bottom-up approach is more computationally demanding since parameter value look-ups are required every time the model is evaluated. Second, it is noted that the selected parameterization approach provides an exceptionally powerful strategy for efficient multi-model reliability and optimization analysis. Responses are carried from one model to become input to other models. Furthermore, the input to a model can be given as any parameter; either a response from an upstream model or a random variable, decision variable, or constant. The model itself need not know where the input is coming from. It is the authors’ experience that the opportunities offered by the bottom-

up parameterization approach far outweighs the downsides in reliability and optimization analysis applications.

Model Framework

The central theme in this paper is reliability analysis with multiple probabilistic models. To carry out this type of analysis the models must be evaluated sequentially and must communicate information from “upstream” models to “downstream” ones. For example, a set of earthquake hazard models produce the realization of magnitude and location of the epicentre of an earthquake. This information is communicated into an intensity model, which produces location-specific intensity or ground motion record(s). In turn, this information enters into downstream building, damage, and loss models.

The models in Rt are of two types. They are either built-in, in which case the code to evaluate the model exists in Rt, or they are interfaces to external software. One example of external software linked with Rt is OpenSees (McKenna *et al.* 2003). The user may create the structural model and structural analysis in OpenSees and carry out the reliability analysis with Rt. In this case, Rt sends the current realization of parameters to OpenSees and obtains the structural response (and response sensitivities if available) after the OpenSees analysis is done. Other external software models are available, and more is under development.

The companion paper provides a detailed description of several hazard, infrastructure, and consequence models that are available in Rt. In the present paper, two generic models in Rt are briefly described. One is called “Algebraic Expression” and one is called “Random Variable with Random Parameters.” The former model allows the user to create any mathematical expression to be used as a model. Any parameter, such as random variables and responses from other models, can be used in the expression, which makes this a flexible model. The latter model allows the user to characterize the distribution parameters of random variables as random variables without a need for a nested reliability analysis. As seen in the Model Manual that is posted at www.inrisk.ubc.ca, this model takes three inputs. The first input is a standard normal random variable. This random variable is transformed inside this model to represent the random variable with the distribution type specified in the next item. The second is the distribution type for the random variable. The third is a list of parameters, typically random variables, which represents the uncertain distribution parameters. For example, the Normal distribution needs a list of two parameters as the third input: the mean and the standard deviation.

Analysis Options

Currently, the analysis tools in Rt includes FORM and sampling reliability analysis. The derivatives needed in FORM are computed by finite difference or direct differentiation. Although direct differentiation is preferred, the finite difference approach is quite efficient because only the models affected by the perturbed parameter are re-analyzed. Variable and model inference options are also available to use data to develop model with Bayesian inference.

Example

To demonstrate the use of Rt to carry out a multi-model reliability analysis, a simple

example is presented here. The objective is to carry out a reliability analysis for a few buildings subjected to earthquake ground motion. The epicentre location is randomly distributed along a linear line source and a seismic moment computation is used to determine the uncertain magnitude. In turn, the magnitude and distance between the hypocentre and the building locations are input into a generic attenuation relationship to determine the intensity at the building sites. The intensity is given to building models and damage models to compute the final loss. Uncertainty is present in every model in the form of random variables that conceivably represent inherent aleatory uncertainty and reducible epistemic model uncertainty. Details are not provided here; the reader is encouraged to visit www.inrisk.ubc.ca for video tutorials that explain this and other examples.

Figure 4 shows the overview of the models in the form of a flowchart that is obtained in Rt by the “Evaluate Function” analysis option.

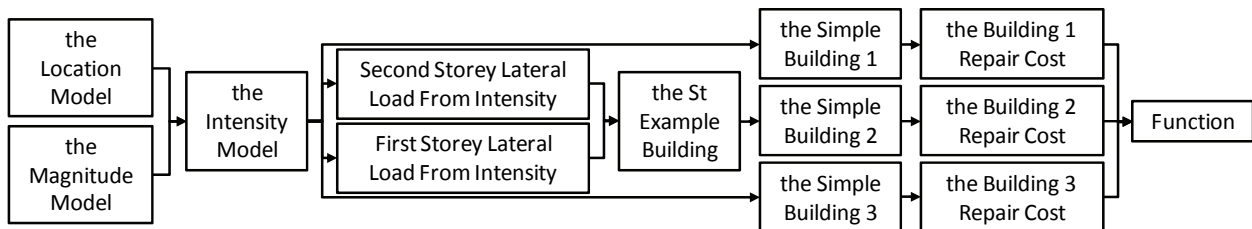


Figure 4. The flowchart of the analysis model stream visualized in Rt.

Next, Figure 5 demonstrates the histogram that is produced in Rt by generating 10,000 sample outcomes of the random variables. This loss curve indicates that losses up to almost \$160 million have been observed. However, most of the probability density is in the range of much smaller losses.

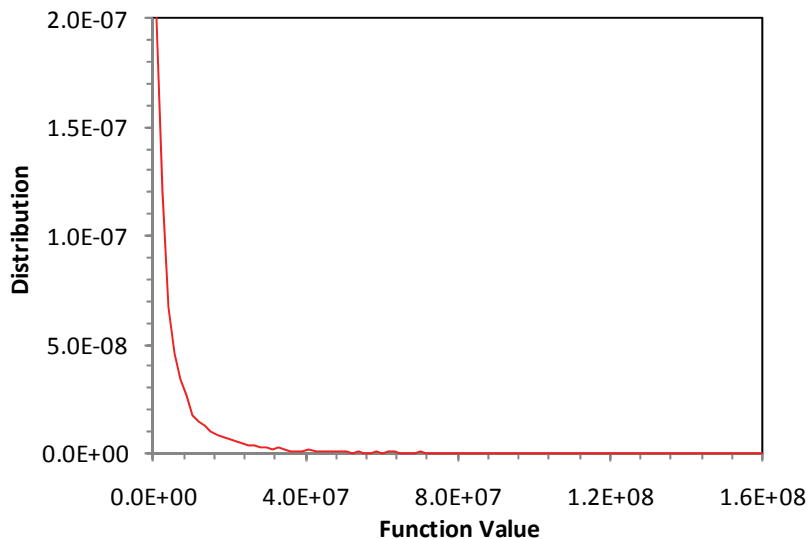


Figure 5. The histogram of the monetary loss produced by Rt.

The question is now asked; what is the probability that the loss exceeds \$50 million. For

this purpose, reliability analysis by FORM is carried out with the limit-state function $g=50,000,000 - l$, where l is the total loss summed over the buildings. By using the improved HLRF search algorithm (Zhang and Der Kiureghian 1995) with the Armijo rule to search for the design point, the probability is 0.01518, which has an associated reliability index equal to 2.17. Figure 6 illustrates the progression of the search towards the design point. The horizontal axis in this figure counts the number of trial points in the space of random variables. After six steps, the upper line, which shows the distance from the origin in the standard normal space of random variables to the trial point, shows a convergence to the reliability index 2.17. The lower line shows the distance from the previous trial point to the current trial point.

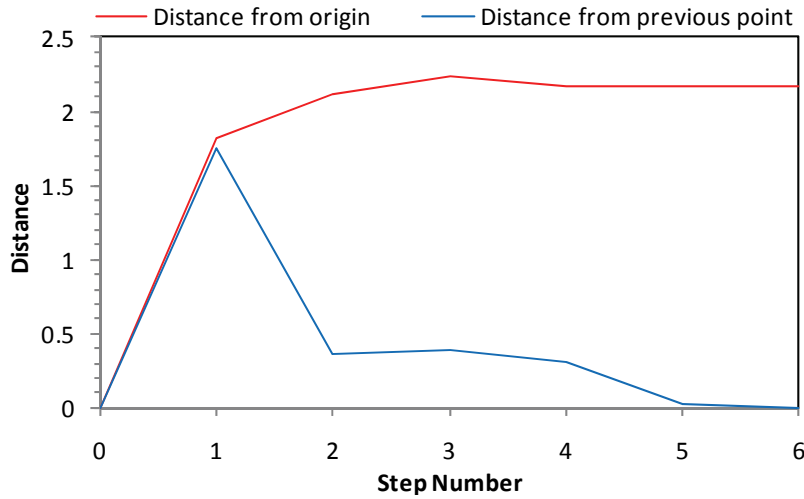


Figure 6. The diagram of the progression of the search towards the design point in FORM produced by Rt.

Finally, importance sampling around the design point from FORM is carried out. After 26,501 samples, the coefficient of variation of the sampling result was 2%, corresponding to a failure probability of 0.02017. This is a typical result, which shows that the limit-state function is only slightly nonlinear. It is noted that the importance sampling analysis would not have been possible without FORM since sampling around the mean is excessively time-consuming and computationally expensive. The importance sampling result is used as the final probability value.

Conclusions

This paper puts forward a new software for risk analysis and performance-based engineering. Probabilities for response events, *e.g.*, monetary loss events, are computed by reliability analysis and probabilistic models.

Acknowledgements

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