Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 1312

AUTOMATIC DAMAGE LOCALIZATION OF BUILDINGS BASED ON DOMINANT FREQUENCY SHIFTS THROUGH INCOMPLETE MEASUREMENTS

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ABSTRACT

This paper presents an automatic damage localization technique for buildings based on changes of dominant frequencies identified from incomplete measurements. Natural frequency is global measure of structural characteristics. The changes in structural condition cause different combinations of the change in natural frequencies, the shifts of multiple natural frequencies can provide the information on the location of damaged stories. The mode shapes are assumed to remain unchanged in a slight or moderate damage. A database with the pattern of modal frequency changes under different damage scenarios is established. The location of damage is then identified through comparison of the pattern of the damaged building and the pattern in the database. Moreover, a fuzzy inference system (FIS) was applied to develop the pattern recognition system. Finally, a multi-story shear building was considered to examine the accuracy and applicability of the proposed damage localization technique via experimental data. The acceptance of the assumption was also verified.

Introduction

In recent years, with the advanced systems of data acquisition and signal processing, there has been an increasing interest in the structural health monitoring (SHM) in methodologies that are capable of detecting and quantifying structural damage in areas of a structure, at which some non-destructive tests (NDT) for detail damage evaluation are implemented. Calculating the change of modal frequency to detect damage is wildly used in structural health monitoring (SHM) systems because damage is always accompanied by a reduction of stiffness as well as modal frequency. Damage in different locations and components actually leads to different frequency changes in various modes. Nevertheless, it remains difficult to determine the damage location just by observing the changes of modal frequencies.

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Among the modal parameters of a structure system, the mode shape is obviously the only location related parameter. Therefore, many researchers have attempted to establish mode-shape-based indices, such as modal curvature index (Pandey et al. 1991; Farrar and Jauregui 1998), index MAC (Modal Assurance Criterion) (Allemang and Brown 1982) and index COMAC (Coordinate Modal Assurance Criterion) (Iieven and Ewins 1988), to identify damage and its locations. All above indices have simple expressions and have been applied in identifying the location of damage. However, it has been shown that they have low sensitivity to damage in some cases (Ndambi et al. 2002; Brasiliano et al. 2004). Considering both modal frequencies and mode shapes to detect the occurrence and location of damage may be a more reliable way than relying on either one of them. The modal flexibility damage index (MFDI) (Pandey and Biswas 1994) may be the most well-known one. The principle of this method is on the basis of the comparison of the flexibility matrices obtained from two sets of mode shapes. Moreover, this method involves the normalization of mode shape since the mode shape values are not fixed. The advantage for the mode-shape-based technique is containing spatially related information, thus damage location is available. While the disadvantage of this technique is that large number of measurement locations are required to accurately characterize mode shapes.

Many researchers developed other accurate damage indices for various types of structures. Brasiliano et al. 2004 evaluated the residual error method in the movement equation to verify its efficiency when applied to continuous beams and frame structures. Kim and Chun 2004 derived an index to apply to buildings. Kim et al. 2003 employed frequency-based and mode-shape-based damage detection methods for locating and quantifying damage in pre-stressed concrete beams. Bernal 2002 and Bernal and Gunes 2002 proposed a technique to localize damage in structures using damage locating vectors (DVLs) that have the property of inducing stress fields whose magnitude is zero in the damaged elements. The DLVs are associated with sensor coordinates and are computed systematically as the null space of the change in measured flexibility. Wang et al. 2007 developed a story damage index (SDI) and expressed as a simple formula based on modal frequency and mode shape obtained from real earthquake records. Morita et al. 2005 proposed a damage detection technique that requires only the change in dominant frequencies. The technique has also been verified by the shaking table test. For all the aforementioned methods, the incomplete mode shape is the primary problem because sensors are usually partially distributed in a real instrumented case.

In structural engineering, it is not practical to have measurements at all degrees. In most applications, an incomplete set of recorded time histories is available and this impairs the applicability of the SHM methods. In the case of an incomplete set of measurements, only limited information about the system parameters can be retrieved (Lus et al. 2003). In addition, dealing with a limited set of input-output measurements raises issues related to the uniqueness of the identified solution (Franco et al.).

Based on the discussions mentioned above, studies on damage localization technique for structures with incomplete measurements are quite important. This paper presents an automatic damage localization technique for buildings based on changes of dominant frequencies identified from incomplete measurements. First, the modal property of the undamaged building should be known by system identification (Lin et al. 2005; Morita et al. 2005) or finite element model. Then, a database with the pattern of modal frequency changes under different damage scenarios is established. The location of damage is then identified through comparison of the pattern of the damaged building and the pattern in the database. Moreover, a fuzzy inference system (FIS) is applied to develop the pattern recognition system. Following the theoretical derivation, the proposed damage localization technique is demonstrated by the shaking table test data of a three-story benchmark building and some conclusions are drawn.

Damaged Localization using Multiple Natural Frequency Shifts

Dynamic and characteristic equations

Considering an N–story shear building frame with mass m_l at the l-th floor and with stiffness k_l and damping c_l at the l-th story, the equation of motion of the linear building frame under ground acceleration $\ddot{u}_g(t)$ can be written as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{1}$$

where **M**, and **K** are the $N \times N$ mass and stiffness matrices respectively:

$$\mathbf{M} = diag\begin{bmatrix} m_1 & m_2 & \cdots & m_l & \cdots & m_{N-1} & m_N \end{bmatrix} \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & \cdots & & & & \cdots & 0 \\ -k_2 & k_2+k_3 & -k_3 & & & & & \vdots \\ & & \ddots & & & & & \\ & & -k_l & k_l+k_{l+1} & -k_{l+1} & & & \\ & & & & \ddots & & & \\ \vdots & & & & -k_{N-1} & k_{N-1}+k_N & -k_N \\ 0 & \cdots & & & \cdots & -k_N & k_N \end{bmatrix} \tag{3}$$

In Eq. 1, **C** is the $N \times N$ damping matrix. $\mathbf{u}(t)$ represents the $N \times 1$ vector of the floor displacements relative to the ground at a time t; and **r** indicates the $N \times 1$ influence vector. In Eq. 2, *diag* represents the diagonal matrix. The characteristic equation of the system can be represented as

$$\mathbf{K\Phi} = \boldsymbol{\omega}^2\mathbf{M\Phi} \tag{4}$$

where

$$\boldsymbol{\omega}^2 = diag[\omega_1^2 \quad \omega_2^2 \quad \cdots \quad \omega_i^2 \quad \cdots \quad \omega_{N-1}^2 \quad \omega_N^2],\ (\omega_1^2 \le \omega_2^2 \le \cdots \le \omega_i^2 \le \cdots \le \omega_{N-1}^2 \le \omega_N^2) \tag{5a}$$

$$\mathbf{\Phi}_{N\times N} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_i & \cdots & \boldsymbol{\varphi}_{N-1} & \boldsymbol{\varphi}_N \end{bmatrix} \tag{5b}$$

In Eq. 5, ω_i and $\boldsymbol{\varphi}_i$ denote the i-th natural frequency and mode shape vector, respectively.

Damage localization using multiple natural frequency shifts

The squared natural frequency of Eq. 4 can be rewritten as

$$\omega_i^2 = \frac{\boldsymbol{\varphi}_i^T\mathbf{K}\boldsymbol{\varphi}_i}{\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_i} \tag{6}$$

The change in squared natural frequency due to damage of the building can be represented as

$$\Delta\omega_i^2 = \omega_{0i}^2 - \omega_{Di}^2 = \frac{\boldsymbol{\varphi}_{0i}^T\mathbf{K}_0\boldsymbol{\varphi}_{0i}}{\boldsymbol{\varphi}_{0i}^T\mathbf{M}\boldsymbol{\varphi}_{0i}} - \frac{\boldsymbol{\varphi}_{Di}^T\mathbf{K}_D\boldsymbol{\varphi}_{Di}}{\boldsymbol{\varphi}_{Di}^T\mathbf{M}\boldsymbol{\varphi}_{Di}} \tag{7}$$

where subscripts 0 and D represent undamaged and damaged states, respectively. For "slight" and "moderate" damage, assume the mode shapes remain unchanged.

$$\mathbf{\Phi}_D \approx \mathbf{\Phi}_0 \tag{8}$$

Eq. 7 reduce to

$$\Delta\omega_i^2 = \frac{\boldsymbol{\varphi}_{0i}^T(\mathbf{K}_0 - \mathbf{K}_D)\boldsymbol{\varphi}_{0i}}{\boldsymbol{\varphi}_{0i}^T\mathbf{M}\boldsymbol{\varphi}_{0i}} = \frac{\boldsymbol{\varphi}_{0i}^T\Delta\mathbf{K}\boldsymbol{\varphi}_{0i}}{\boldsymbol{\varphi}_{0i}^T\mathbf{M}\boldsymbol{\varphi}_{0i}} \tag{9}$$

The change in stiffness matrix $\Delta\mathbf{K}$ can be decomposed into the changes in story stiffness of each story as follows

$$\Delta\omega_i^2 = \frac{\sum_{l=1}^{N}\Delta k_l\left(\Delta\phi_{il}\right)^2}{\boldsymbol{\varphi}_{0i}^T\mathbf{M}\boldsymbol{\varphi}_{0i}} \tag{10}$$

where

$$\Delta k_l = k_{l0} - k_{lD} = k_{l0}\cdot\alpha_l;\quad \Delta\phi_{il} = \begin{cases}\phi_{il} - \phi_{i(l-1)}, & l = 2\sim N\\ \phi_{il}, & l = 1\end{cases} \tag{11}$$

In Eq. 11, Δk_l is the change in story stiffness of the l-th story while α_l is the story stiffness reduction ratio (SSSR) at the l-th story. Dividing both sides of Eq. 10 by ω_{i0}^2 gives

$$\Omega_i = \frac{\Delta\omega_i^2}{\omega_{i0}^2} = \sum_{l=1}^{N}\frac{k_{0l}\left(\Delta\phi_{il}\right)^2}{\boldsymbol{\varphi}_{0i}^T\mathbf{K}_0\boldsymbol{\varphi}_{0i}}\alpha_l = \sum_{l=1}^{N}\frac{S_{0il}}{S_{0i}}\alpha_l \tag{12}$$

where Ω_i is the squared natural frequency change ratio (SFCR) in the i-th mode. S_{0i} is the i-th modal strain energy of the system while S_{0il} is the i-th modal strain energy stored in the l-th story. The i-th modal strain energy ratio (MSER) of the l-th story is defined as

$$\lambda_{il} = \frac{k_{0l}\left(\Delta\phi_{il}\right)^2}{\boldsymbol{\varphi}_{0i}^T\mathbf{K}_0\boldsymbol{\varphi}_{0i}} = \frac{S_{0il}}{S_{0i}} \tag{13}$$

From Eq. 13, Eq. 12 can be written as

$$\Omega_i = \sum_{l=1}^{N}\lambda_{il}\cdot\alpha_l \tag{14}$$

If P ($P \le N$) modes are considered, Eq. 14 can be written as

$$\boldsymbol{\Omega}_{P\times 1} = \boldsymbol{\Lambda}_{P\times N}\cdot\boldsymbol{\alpha}_{N\times 1} \tag{15}$$

Eq.15 can be expressed as a matrix form in follows

$$\begin{bmatrix}\Omega_1\\ \Omega_2\\ \vdots\\ \Omega_i\\ \vdots\\ \Omega_{P-1}\\ \Omega_P\end{bmatrix}_{P\times 1} = \begin{bmatrix}\lambda_{11} & \lambda_{12} & \cdots & \lambda_{1l} & \cdots & \lambda_{1(N-1)} & \lambda_{1N}\\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2l} & \cdots & \lambda_{2(N-1)} & \lambda_{2N}\\ \vdots & \vdots & \ddots & & & \vdots & \vdots\\ \lambda_{i1} & \lambda_{i2} & & \lambda_{il} & & \lambda_{i(N-1)} & \lambda_{iN}\\ \vdots & \vdots & & & \ddots & \vdots & \vdots\\ \lambda_{(P-1)1} & \lambda_{(P-1)2} & \cdots & \lambda_{(P-1)l} & \cdots & \lambda_{(P-1)(N-1)} & \lambda_{(P-1)N}\\ \lambda_{P1} & \lambda_{P2} & \cdots & \lambda_{Pl} & \cdots & \lambda_{P(N-1)} & \lambda_{PN}\end{bmatrix}_{P\times N}\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_l\\ \vdots\\ \alpha_{N-1}\\ \alpha_N\end{bmatrix}_{N\times 1} \tag{16}$$

In Eq. 15, $\boldsymbol{\Lambda}_{P\times N}$ (MSER) represents the relationship matrix between the SSSR vector $\boldsymbol{\alpha}_{N\times 1}$ and the SFCR vector $\boldsymbol{\Omega}_{P\times 1}$.

Damage at single floor

If the damage only occurs in the r-th story, vector α_r will be much lager than other α_l in the vector $\boldsymbol{\alpha}_{N\times1}$. In other words, if the damage concentrates on the r-th story, stiffness reduction does not occur at other stories than the r-th story, Eq. 15 reduces to

$$\boldsymbol{\Omega}_{P\times1} = \boldsymbol{\lambda}_r \cdot \alpha_r \tag{17}$$

where $\boldsymbol{\Omega} \in (P\times1)$ represents the SFCR vector while $\boldsymbol{\lambda}_r \in (P\times1)$ represents the multiple modal strain energy ratios at r-th story. In Eq. 17, it can be observed that α_r is a constant, vector $\boldsymbol{\Omega}$ is proportional to vector $\boldsymbol{\lambda}_r$. When the elements of the $\boldsymbol{\Omega}$ and $\boldsymbol{\lambda}_r$ are normalized and drawn in bar charts, the damaged story can be located by comparing the patterns of $\boldsymbol{\Omega}$ and $\boldsymbol{\lambda}_r$. This means that a damaged story can be located by comparing the patterns of SFCR and MSER. Figure 1 shows the relationship between $\boldsymbol{\Omega}$ and $\boldsymbol{\lambda}_r$ (damage only at r-th story).

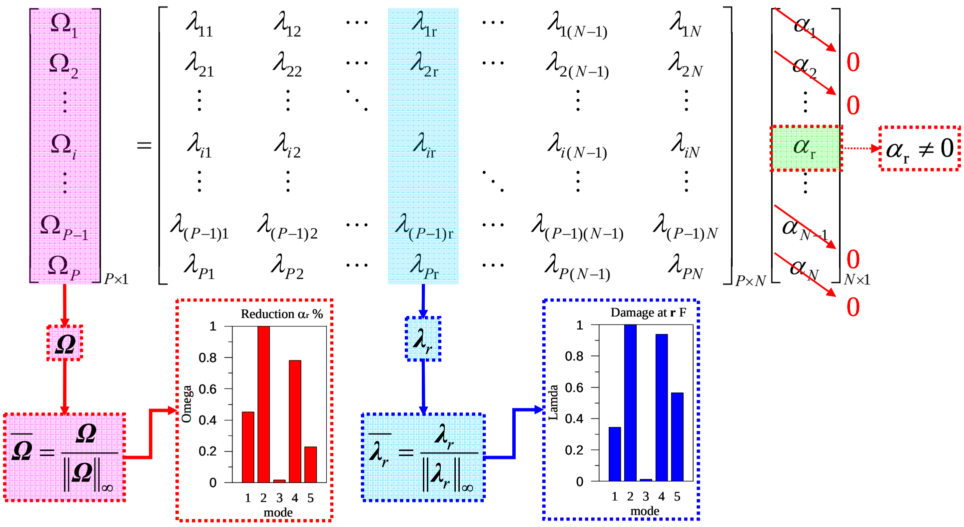


Figure 1. Relationship between $\boldsymbol{\Omega}$ and $\boldsymbol{\lambda}_r$ (damage only at r-th story).

The story stiffness reduction ratio α_r can be determined by

$$\alpha_r = \frac{\Omega_i}{\lambda_{ir}} \tag{18}$$

Damage at multiple floors

If Eq. 15, when damage at multiple floors, the stiffness reduction ratio vector can be determined by

$$\boldsymbol{\alpha}_{N\times1} = \boldsymbol{\Lambda}^{\oplus}_{P\times N} \cdot \boldsymbol{\Omega}_{N\times1} \tag{19}$$

Note that in Eq. 15, when P ($P \le N$) modes are considered, more unknown than equations, although Eq. 19 gives a minimum-norm solution, which may lose the accuracy in estimating the story stiffness reduction ratio $\boldsymbol{\alpha}_{N\times1}$ vector. If $P = N$, from Eq. 15, $\boldsymbol{\alpha}_{N\times1}$ can directly computed by

$$\boldsymbol{\alpha}_{N\times1} = \boldsymbol{\Lambda}^{-1}_{N\times N} \cdot \boldsymbol{\Omega}_{N\times1} \tag{20}$$

Although Eq. 20 has unique solution for $\boldsymbol{\alpha}_{N\times 1}$, it is very sensitive to the error of mode shapes, more studies should be conducted in this area.

Pattern Recognition using Fuzzy Inference System

As mentioned above, the damaged story can be located by comparing the patterns of SFCR and MSER. In this section, fuzzy inference system (FIS) is applied to develop an automatic damage localization system. The fuzzy set theory (Zadeh 1965) was first proposed by Zadeh in 1965. Mamdani applied Zadeh's theories of linguistic approach and fuzzy inference on the automatic operating control of a steam generator (Mamdani 1974). The advantage of the FIS is its ability to handle nonlinearities and uncertainties in the system. Nevertheless, for a more realistic implementation, system nonlinearities can be overcome easily by introducing human expertise into fuzzy IF-THEN rules (Choi et al. 2004). Figure 2 shows the flowchart of pattern recognition using FIS. The FIS consists of three basic parts; fuzzification where continuous input variables are transformed into linguistic variables, fuzzy rule inference that handles rule inference consisting of fuzzy IF–THEN rules, and defuzzification that ensures output variable. The design of the FIS includes the definition of input (SFCR pattern) and output (damaged floor) variables, the selection of data manipulation method, the membership function design and the rule base design. Using fuzzy rules (MSER patterns) and membership functions, FIS converts linguistic variables into numerical values.

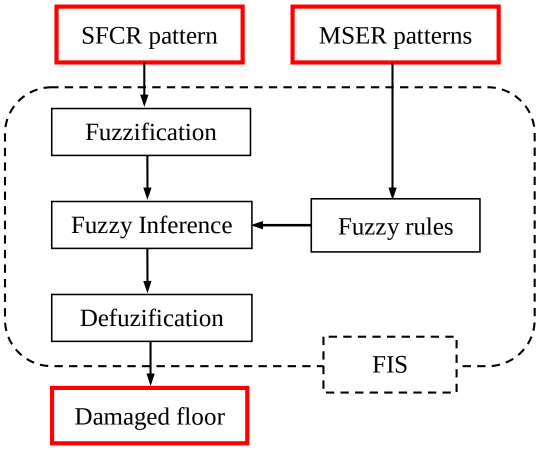


Figure 2: Flowchart of pattern recognition using fuzzy inference system (FIS).

Verification via Experimental Data

Description of test frame and experiment

The experimental structure used in this study was a full-scale three-story steel frame mounted on the shake table at the National Center for Research on Earthquake Engineering (NCREE) as shown in Figure 3. This structure is regarded as a benchmark building designed for the demonstration of research on structural control and health monitoring in Taiwan (Table 1). It is a uniform building with 18 tons in weight and 9 m in height. The dimension of the rectangular floor is 3 m × 2 m. The weight of each floor comes from the composite frame-plate structure and additional lead blocks atop. It is supported by four columns with H-shape (H150×150×7×10) section.

In order to verify the damage localization technique, three types of frame are used to represent the “undamaged” and “damaged” structure, which is shown in Figure 4. The description of these two types of frame is as follows. (2) Undamaged: the three-story benchmark steel structure. (2) Damaged: two weak elements (Figure 4) at the bottom of 1F. Figure 4 also shows the locations of the installed accelerometers. The undamaged structure applied complete measurement to identify the modal parameters, which are shown in Table 2. On the other hand, the damaged structure only measure 1^{st} floor acceleration. Table 3 represents the identified modal parameters of the damaged structure. The System Realization using Information Matrix (SRIM) technique (Juang 1997, Lin et al. 2005) is applied to compare the identified building parameters

Figure 3. Photo of the three-story steel frame.

Table 1. Dimensions of the steel frame.

Floor Dimension:	3mx2m
Floor Height:	3m
No. of floor:	3 stories
Floor mass:	6 ton/floor
Beam:	H150x150x7x10
Column:	H150x150x7x10
Floor plate:	25mm

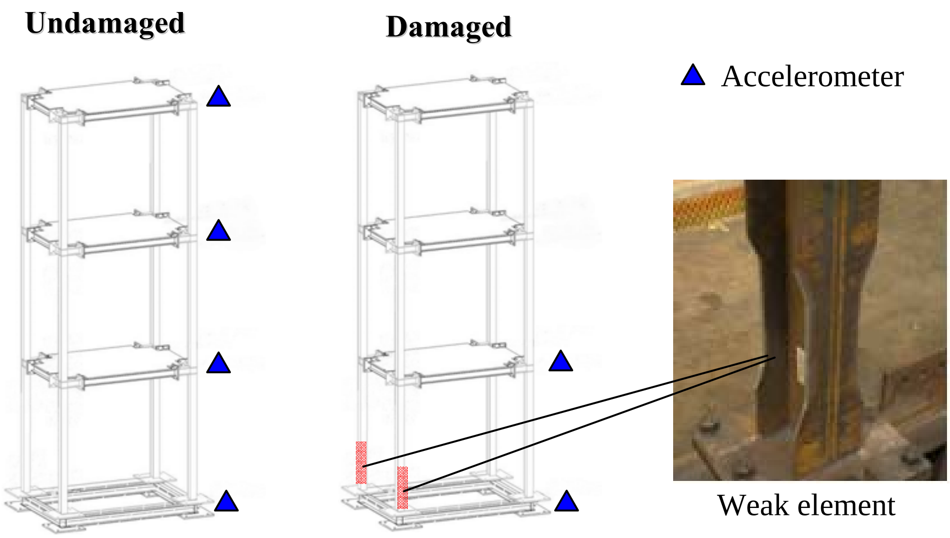


Figure 4. Undamaged and damaged test frames.

Table 2. Identified modal parameters of the undamaged structure.

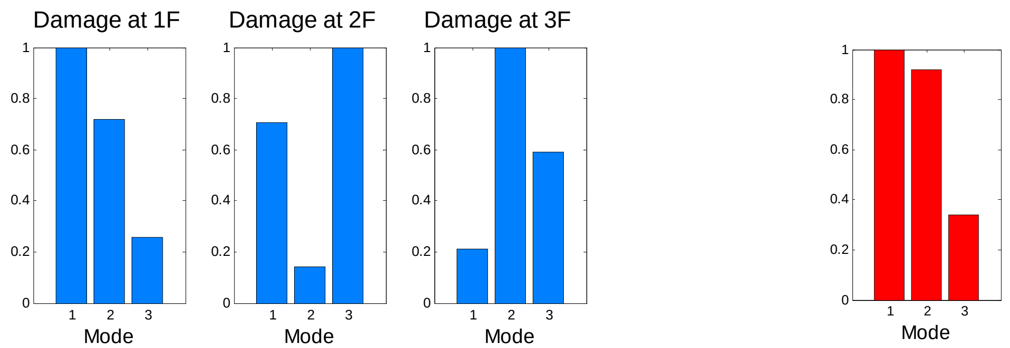
System Modal Parameter – (undamaged)				
Mode		1	2	3
Frequency (Hz)		1.07	3.26	5.13
Damping ratio (%)		1.97	0.21	0.18
Mode shapes	1	0.41	-1.00	-0.85
	2	0.79	-0.50	1.00
	3	1.00	0.84	-0.46

Table 3. Identified modal parameters of the damaged structure.

System Modal Parameter – (damaged)				
Mode		1	2	3
Frequency (Hz)		1.03	3.16	5.08
Damping ratio (%)		0.97	0.32	0.20
Mode shapes	1	N/A	N/A	N/A
	2	N/A	N/A	N/A
	3	N/A	N/A	N/A

Comparison of the SFCR and MSER patterns

From Tables 2 and 3, the normalized SFCR and MSER patterns can be established. The "normalized" means the biggest component of the SFCR and MSER is scaled to 1. Figure 5 shows the normalized MSER patterns and the normalized SFCR pattern. As mentioned above, fuzzy inference system (FIS) is applied to develop the pattern recognition system. In the selection of membership functions, four membership functions shown in Figure 6(a) are used for the input variables (SFCR) in the FIS. These four membership functions are labeled by S=small, M=middle, L=large, and XL=very large. Moreover, as shown in Figure 6(b), the output variable uses three membership functions labeled from 1 to 3, which represent the damaged floor. The fuzzy rules (based on the MSER patterns) have shown in Table 4. The defuzzificaion process used in this study is the center of gravity. The output of the FIS is 1.04, which indicates the damaged floor is 1F. On the other hand, α_1 is 0.15, which is computed by Eq. 18.



(a) MSER patterns (b) SFCR pattern

Figure 5. Patterns of MSER and SFCR.

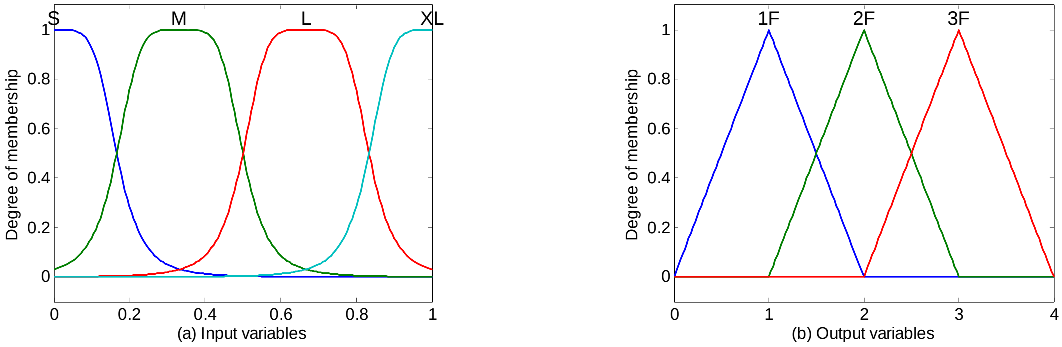


Figure 6. Membership functions of input and output variables.

Table 4. Fuzzy rules of the fuzzy inference system (FIS).

Input (SFCR)	Mode 1	XL	L	M
	Mode 2	L	S	XL
	Mode 3	M	XL	L
Output (damaged story)		1F	2F	3F

Assumption verification

The derivation of this study is based on the assumption of Eq.8. In order to check the acceptance of the assumption, a modal assurance criterion (*MA*C) is defined as

$$MAC(\varphi_{0i}, \varphi_{Di}) = \frac{\left|\varphi_{0i}^T \cdot \varphi_{Di}\right|^2}{\left|\varphi_{0i}^T \cdot \varphi_{0i}\right|\left|\varphi_{Di}^T \cdot \varphi_{Di}\right|} \tag{21}$$

Full measurements and the SRIM technique (Juang 1997) are applied to determine the mode shapes of the undamaged and damaged cases. The *MAC* of the 1st to the 3rd modes are 0.9997, 0.9998 and 0.9997. In this case (α_1=0.15), the assumption of Eq.8 is quite reliable.

Conclusions

This paper presents an automatic damage localization technique for buildings based on changes of dominant frequencies identified from incomplete measurements. For slight or moderate damage, assume the mode shapes remain unchanged. A database with the pattern of modal frequency changes under different damage scenarios is established. The location of damage is then identified through comparison of the pattern of the damaged building and the pattern in the database. A fuzzy inference system (FIS) is applied to develop the pattern recognition system. Finally, a multi-story shear building is considered to examine the proposed damage localization technique via shaking table test. The dominant frequencies of the damaged structure are identified from incomplete measurements. From the experimental data of the benchmark model (damage at single floor), it is shown that the localization technique can localize damage floor correctly. The modal assurance criterion (*MA*C) showed that the assumption is reliable.

Acknowledgments

This work was supported by the National Science Council of the Republic of China under Grants NSC 93-2625-Z-005-009 and NSC 93-2811-Z-005-002. These supports are greatly appreciated.

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