



A GROUND MOTION SELECTION AND MODIFICATION METHOD PRESERVING CHARACTERISTICS AND ALEATORY VARIABILITY OF SCENARIO EARTHQUAKES

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ABSTRACT

In performance-based seismic design of civil infrastructure, earthquake ground motion is one of the primary sources of uncertainty in assessing the seismic performance of the civil system. It is critical to develop systematic methods to select and modify from current ground-motion databases to provide a group of earthquake motions that can realistically represent important aspects of the design motion that control the nonlinear response of civil engineering facilities. The paper presents a new ground-motion selection and modification (GMSM) method that preserves the characteristics and aleatory variability of scenario earthquakes. The resulted ground motions sets realistically represent the statistical distribution (mean, standard deviations) and correlations of the response spectra, with other selection criterion to incorporate ground motion characteristics such as earthquake magnitude, distance and site conditions etc.

Numerical analyses of a 20-story RC frame structure were performed using generated record sets of different sizes. The proposed GMSM method has demonstrated excellent capacity to generate “scenario-compatible” ground-motion sets that can accurately predict the full distribution of the engineering demand parameters under the earthquake scenario. The proposed method shows great potential in performance-based earthquake design of nonlinear civil systems.

Introduction

In recent years, performance-based seismic design of civil infrastructure has become more and more important in preventing human losses and structural damages from earthquakes. Researchers and practitioners generally agree that earthquake ground motion is one of the primary sources of uncertainty in assessing the seismic performance of the civil system. Due to the lack of recorded data for the design-level earthquakes (which are usually rare events), it is critical to develop systematic methods and useful tools to select and modify from current ground-motion databases to provide a group of earthquake motions that can realistically represent important aspects of the design motion that control the nonlinear response of civil engineering facilities.

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Although many ground-motion selection and modification (GMSM) methods exist, there is no consensus as to the accuracy and performance of these methods. Since traditionally the seismic hazard at a site for design purposes has been represented by a design spectrum, most existing ground-motion selection and modification (GMSM) models are mainly focused on developing time history sets that, in aggregate, have response spectra that “resemble” a *single* target response spectrum. Sometimes, modifications to existing ground-motion time histories are necessary to achieve a desirable spectral shape, including “simple-scaling” approach that scales the amplitude of time histories to achieve an average fit to the spectrum (eg. Wang *et. al*, 2009). “Spectrum-matching” approaches adjust the ground-motion time history in frequency content so that the modified one is a very close match to the design spectrum, and the modification can be made either in the time domain (eg. Abrahamson 1992) or in the frequency domain (eg. Bolt and Gregor, 1993). Each approach has its proponents and appears to be a generally acceptable method. Besides, methods focusing on other response characteristics of the nonlinear system, such as a proxy response, or inelastic displacement, were also pursued by several researchers (eg. Watson-Lamprey and Abrahamson 2006; Shantz 2006).

Current GMSM efforts are mainly focused on predicting the median response of the engineering demand parameters (EDP) under a prescribed seismic demand. Preliminary results from COSMOS 2007 workshop concluded that for a first-mode dominated structure, such as tall buildings, time histories that closely match target spectrum conditioned on the period of the first mode of the structure can yield good estimate of the median response of EDPs (eg. maximum inter-story drift ratio) for that scenario (Haselton eds. 2009). There is no guidance on GMSM regarding nonlinear response analysis of geotechnical structures, such as liquefiable soil ground, earth slopes and earth dams. The seismic responses of these facilities are significantly different from those of buildings in that under strong shaking, the soil response is a broadband phenomenon that is not controlled by a couple of spectral periods as is seen in buildings. Dynamic soil response is nonlinear, and it is affected by ground-motion amplitude and frequency contents over a broad range of periods. A GMSM procedure that incorporates the characteristics and variability of ground motion holds the key to developing predictive models to evaluate the seismic performance of these systems.

To predict the full distribution of EDPs under a scenario earthquake, the aleatory variability of ground motions should be carefully incorporated in the ground-motion selection model to fully quantify the seismic demand. The importance of capturing the variability in seismic analysis is reflected in the recent ATC-58 guideline (Applied Technology Council, 2009), which recommended *randomly* gathering eleven ground motions from the chosen magnitude and distance bin and then scaling them to match the targeted spectrum value at the fundamental period of the structure. However, the randomness nature in the selection procedure makes it difficult to represent the true variability of ground motions.

In this paper, we present a new GMSM method that preserves the intrinsic characteristics and variability of the scenario earthquakes. The method will be useful to the study the full distribution of engineering demand parameters under a scenario earthquake, particularly, for the broadband nonlinear systems whose seismic response is controlled over a large range of periods, such as liquefiable ground, the deformation in earth structures and slope-retaining wall system *etc.*

Aleatory Variability of Ground Motions

Statistical analysis shows that the probability distribution of ground-motion spectral acceleration at individual periods can be well approximated by lognormal distributions, given a certain magnitude and distance *etc.* Ground-motion attenuation models (eg. Chiou and Youngs, 2008, Campbell and Bozorgnia, 2008) usually provide the mean value of log spectral acceleration $\mu_{\ln S_a}$ and the standard deviation of log spectral acceleration $\sigma_{\ln S_a}$ of a scenario earthquake based on regression analysis of a large ground-motion dataset.

The correlation between spectral values at different periods is an intrinsic property of ground motions (Baker and Cornell 2006). Based on regression analysis of the PEER-NGA strong motion database, the theoretical correlation coefficients were given by Baker and Jayaram (2008). The formulation is valid over a period of range (0.01 – 10 sec), and more importantly, the resulted covariance matrix is a symmetric positive semi-definite matrix that allows the random sample generation, as a sample covariance matrix should be always at least positive semi-definite. The spectral correlation is one of the most important properties in quantifying the variability of ground motions, since it describes the correlation of the seismic demands over frequency content.

The correlation coefficient can be calculated from a set of response spectra using the following formulation:

$$\rho_{\ln S_a(T_1), \ln S_a(T_2)} = \frac{\sum_{i=1}^n (\ln S_a^{(i)}(T_1) - \overline{\ln S_a(T_1)}) (\ln S_a^{(i)}(T_2) - \overline{\ln S_a(T_2)})}{\sqrt{\sum_{i=1}^n (\ln S_a^{(i)}(T_1) - \overline{\ln S_a(T_1)})^2 \sum_{i=1}^n (\ln S_a^{(i)}(T_2) - \overline{\ln S_a(T_2)})^2}} \quad (1)$$

where $\ln S_a^{(i)}(T_1)$ is the logarithm of the spectral acceleration of the *i*-th record at period T_1 , $\overline{\ln S_a(T_1)}$ is the mean of logarithm of the spectral acceleration of all n records at T_1 , and n is the total number of selected records.

Figure 1 illustrates the importance of the correlation in controlling the variability of the spectral accelerations for a ground-motion set. In an extreme case where the spectral values are assumed to be perfectly-correlated (let $\rho = 1$), the spectra (numerically “simulated”) are a set of parallel lines, as shown in Fig 1(a). Although the median and standard deviation of the set closely resemble the statistical distribution of the prescribed seismic demand, shown in Fig 1(b), obviously, the set of perfectly-correlated records can not represent the true variability of the ground motion. Similarly, a randomly-correlation record set (let $\rho = 0$) can also fit reasonably well to the prescribed mean and standard deviation curves, as shown in Fig 1 (c) (d), but the spectral distribution can not possibly represent any real earthquake scenario. More realistic representation of the spectral variability can be simulated using the theoretical correlation, as shown in Fig. 1 (e)(f). Therefore, we propose the following guideline for ground-motion selection and modification: *The selected ground-motion set should preserve the median, the standard deviation of the spectral distribution, and also the correlation structure between spectral values at different periods.* Accordingly, we term the median, the standard deviation and the correlation structure spectral shape as “aleatory variability vector” of the ground motion.

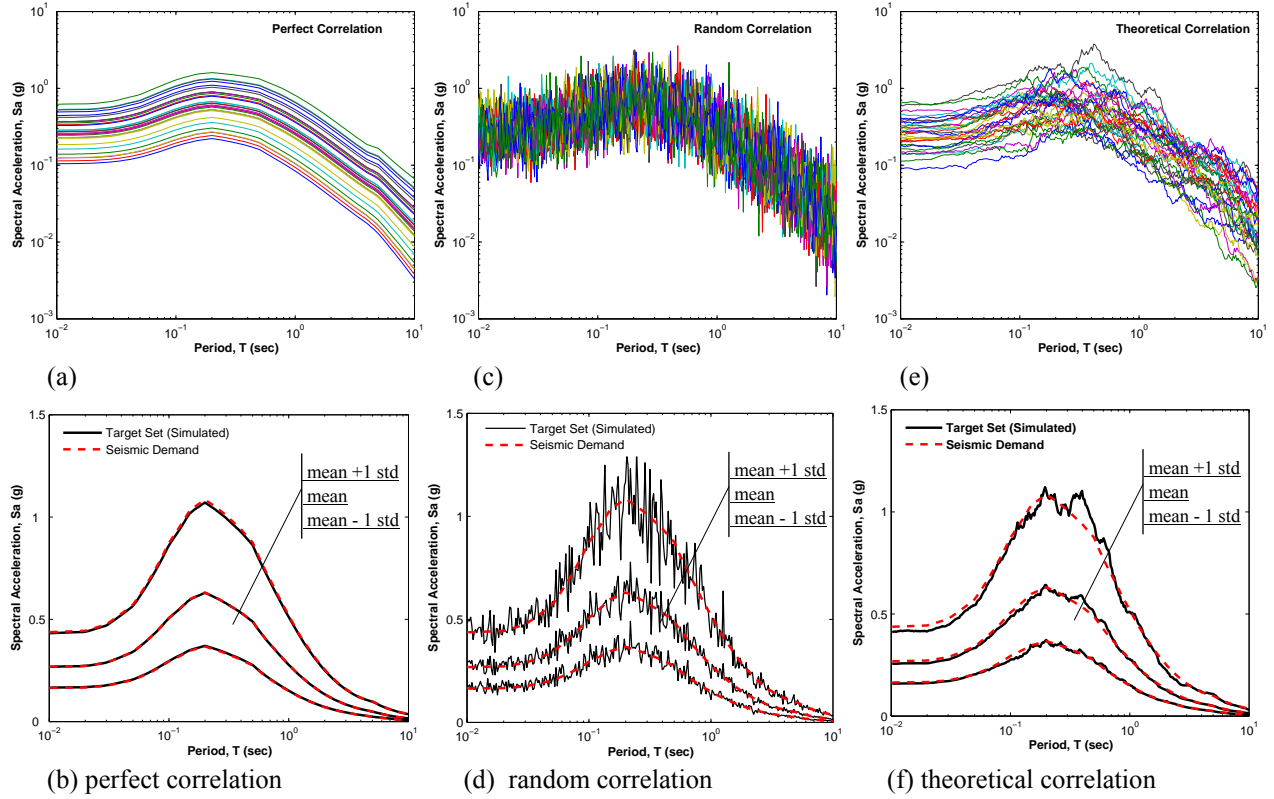


Figure 1: Simulated spectrum acceleration set using perfect correlation (a)(b), random correlation (c)(d), and theoretical correlation (e)(f). The mean and mean \pm standard deviation curves of the seismic demand are given by Campbell-Bozorgnia (2008).

Ground Motion Selection and Modification Method

Given the aleatory variability vector of the design scenario earthquake, great challenge remains to find the optimal combination of a set of n records and their corresponding scale factors to best approximate the seismic demand. For example, set aside the scale factors, there are about 3×10^{18} possible combinations to select a set of 7 records out of a database of 1500 records in total; The possible combinations explode to a staggering number of 1.5×10^{158} if the attempt is to select 100 records out of the database of same size. Therefore, a mathematically rigorous optimal solution to this problem is not feasible. Instead, we propose a fast, innovative algorithm based on correlated random target generation, described as follows:

Step (1) Developing the Correlated Target Spectrum Set

Using multivariate random generation algorithm, a set of n “target” spectra can be generated from multivariate normal distribution with the specified mean value $\mu_{\ln S_a}$ and covariance matrix $\text{Cov}_{\ln S_a}$ of the log spectral acceleration.

$$\ln Sa^{\text{target}} = \text{mvnrnd}(\mu_{\ln S_a}, \text{Cov}_{\ln S_a}, n); \quad (2)$$

The covariance matrix between log spectral acceleration can be obtained from the correlation coefficients at T_i and T_j ($i, j = 1, 2, \dots$) as follows,

$$\text{Cov}_{\ln S_a} = \text{Cov}_{\ln S_a(T_i), \ln S_a(T_j)} = \rho_{\ln S_a(T_i), \ln S_a(T_j)} \sigma_{\ln S_a(T_i)} \sigma_{\ln S_a(T_j)} \quad (3)$$

We calculate the mean $\mu_{\ln S_a^{\text{target}}}$ and the standard deviation $\sigma_{\ln S_a^{\text{target}}}$ of the generated target set, and then the residuals between the target set and the specified values,

$$\begin{aligned} \mathcal{R}_1 &= \sum_i \left[\mu_{\ln S_a^{\text{target}}}(T_i) - \mu_{\ln S_a}(T_i) \right]^2; & \mathcal{R}_2 &= \sum_i \left[\sigma_{\ln S_a^{\text{target}}}(T_i) - \sigma_{\ln S_a}(T_i) \right]^2; \\ \mathcal{R}_3 &= \sum_i \sum_j \left[\rho_{\ln S_a^{\text{target}}(T_i), \ln S_a^{\text{target}}(T_j)} - \rho_{\ln S_a(T_i), \ln S_a(T_j)} \right]^2 \end{aligned} \quad (4)$$

In principal, there are multiple ways to define the total residual as a functional form of residuals \mathcal{R}_1 , \mathcal{R}_2 and \mathcal{R}_3 . Since the objective of the proposed method is to capture the variability of ground motions, matching the mean and the standard deviation are regarded as equally important.

$$\mathcal{R}_{\text{total}} = \mathcal{R}_1 + \mathcal{R}_2 \quad (5)$$

By repeating this step for limited times (based on our experience, a couple of hundred times are enough), we can identify an optimal set of n spectra that have the smallest residual via Eq. (5). The optimal set will be chosen as the target spectrum set. The hypothesis underlying this process is that an optimal target set will lead to a better record set by the procedure outline in Step 3.

Step (2). Specify the Search Criterion and Limits for Searches

Besides the spectral shape, the ground-motion characteristics important to the seismic response of the facility may also include the significant duration, number of strong shaking cycles, near-field directivity effects and pulse sequencing etc. It is necessary to specify the ranges of parameters over which searches are to be conducted and other limits and restrictions on the searches. These may include: earthquake magnitude range; type of faulting; distance range; V_{S30} range; significant duration range; whether records are to exclude, include, or be limited to pulse records; limits on the scale factor f ; and restrictions on directional component (i.e., arbitrary Fault Normal FN or Fault Parallel FP components; FN components only; FP component only; or FN and FP components in pair). If three dimensional analyses are to be conducted requiring pairs of horizontal components, ordinarily FN and FP components in pairs would be searched for and scaled by the same factor. The screening process would reduce the ground-motion database to a smaller “selection bin” with the specified characteristics.

Step (3). Find the Record Set that Best Match the Target Set

For *each* target spectrum obtained from step (1), we select and linearly scale *each* record within the “selection bin” and find the one that can provide the closest match to the target spectrum. We use the weighted sum of squared errors (WSSE) between the logarithms of the target spectrum and the scaled record spectrum as the metric to measure the closeness of the match for *each* target spectrum and scaled record:

$$\text{WSSE} = \sum_i w(T_i) \left[\ln \left(Sa^{\text{target}}(T_i) \right) - \ln \left(f Sa^{\text{record}}(T_i) \right) \right]^2 \quad (6)$$

where parameter $w(T_i)$ is a weight function that allows assigning relative weights to different parts of the period range of interest, providing greater flexibility in the selection of records. Arbitrary weight functions may be specified, although the simplest case is to assign equal weight to all periods in the period range of interest (i.e. $w(T_i)=1.0$). Parameter f in the above equation is a linear scale factor applied to the entire response spectrum of the recording. Accordingly, the scale factor f can be determined by minimizing the WSSE defined in the above equation as

$$\ln f = \sum_i w(T_i) \ln \left(Sa^{\text{target}}(T_i) / Sa^{\text{record}}(T_i) \right) \quad (7)$$

Substituting Eq. (7) back into (6) can result in a WSSE value for the scaled record. Similarly, WSSEs are calculated for all records in the “selection bin” to match this target spectrum. The scaled record that renders the minimal WSSE is the one that best matches the spectral shape of the target spectrum over the specified period range of interest. Please note that the linear scaling will not change the spectral shape plotted in a logarithm scale, and thus preserved the relative frequency content of the original record. Repeat the process for *each* individual target spectrum will result in a set of n scaled records that has the closest spectral shape to each target spectrum individually.

Step (4). Evaluate the Selected Record Set

Calculate the aleatory variability vector of the identified set of n scaled records from step (3), and compare it with the prescribed values to find the residuals, similar to Step 1:

$$\mathbf{R}_1 = \sum_i \left[\mu_{\ln S_a^{\text{scaled}}}(T_i) - \mu_{\ln S_a}(T_i) \right]^2; \quad \mathbf{R}_2 = \sum_i \left[\sigma_{\ln S_a^{\text{scaled}}}(T_i) - \sigma_{\ln S_a}(T_i) \right]^2;$$

$$\mathbf{R}_3 = \sum_i \sum_j \left[\rho_{\ln S_a^{\text{scaled}}(T_i), \ln S_a^{\text{scaled}}(T_j)} - \rho_{\ln S_a(T_i), \ln S_a(T_j)} \right]^2 \quad (8)$$

$$\mathbf{R}_{\text{total}} = \mathbf{R}_1 + \mathbf{R}_2 \quad (9)$$

Since step (1) generates the correlated target spectrum set by random realization, the steps (1)-(4) can be iterated (repeated) until a satisfactory scaled-record set that minimizes the total residual via Eqs. (9) can be reached. In practice, we found that one iteration is usually sufficient to generate a rather desirable set. It is worth pointing out that the proposed algorithm is simple, fast, and is *linear* with respect to the size of selected record and the database. It only takes less than two minutes on a personal PC to complete searching and scaling 30 records from a database of about 7000 records. The above procedure is easy to implement, and it remains great flexibility to incorporate other features such as specifying the desirable scale factor range to avoid excessive scaling. At the present time, we stipulate that it is *not* necessary to restrict the scaled record set derived only from unique ground motion record, i.e., the same ground-motion record can be selected with different scale factors as long as the scaled spectrum provides the best match to the target. Further refinement to include this restriction can be easily achieved by eliminating the identified record in step (3) one by one from the “selection bin”.

Predicting the Distribution of Nonlinear Structural Response

The efficiency of the GSM scheme is demonstrated in this section to predict the nonlinear response of buildings under a scenario earthquake. The structural model utilized in this study is a modern 20-story reinforced concrete perimeter frame building designed according to 2003 International Building Code and ASCE7-02. The finite element model was developed using OpenSees, and the same structural model was utilized by PEER GSM Working Group to conduct benchmark tests on various ground-motion selection and modification methods (Haselton eds 2009, termed as Building “C” therein). The building represents a typical high-rise ductile frame system with the fundamental period of 2.63 sec, and the second-, third- and fourth-mode periods of 0.85, 0.46 and 0.32 sec, respectively. Previous studies show the building response is moderately nonlinear and is sensitive to the second (or higher) mode under shaking.

The seismic demand is a scenario earthquake of magnitude $M_w=7$, strike-slip faulting, with rupture distance $R_{rup}=10$ km, and the average of shear wave velocity in the first 30 m of the site $V_{s30}=400$ m/s. The mean and standard deviation of the scenario earthquake are determined using the Next Generation Attenuation Model (Campbell and Bozorgnia, 2008) , and the correlation coefficients follow Baker and Jayaram (2008). Based on the procedure described in the above section, three ground-motion sets are determined independently from PEER strong motion database rotated to fault-normal and fault-parallel components. The generated ground-motion sets consist of 30, 100 and 200 scaled records, respectively.

Besides the shape of the spectrum, ground-motion characteristics important to the response of the nonlinear system may also include the earthquake magnitude, fault mechanism, rupture distance, significant duration etc. To take into the account of the ground-motion characteristics, we limit the selection bin to records within magnitude $M_w=6-8$, and rupture distance $R_{rup} = 0-30$ km. No restriction is imposed on the range of scale factors, fault mechanics, the significant duration D_{5-95} and the site condition V_{s30} . The average and standard deviation of M_w , R_{rup} , D_{5-95} and V_{s30} obtained from each selected ground-motion set are summarized in Table 1. In general, the characteristics of the ground-motion sets are found to be *compatible* with the specified earthquake scenario. It is also noted that the same record can be selected more than once as long as the scaled spectrum best matches the shape of the target. Figures 2 (a) (b) (d) (e) show the spectral distribution of the selected ground-motion sets against the scenario earthquake (labeled as “seismic demand”) for the 100- and 200-record sets. Since the GSM procedure optimizes the total residual to fit both mean and the standard deviation curves, the calculated mean and standard deviations from the scaled spectra set closely resemble the distribution of the prescribed seismic demand.

Nonlinear numerical analyses were performed to investigate the structural response under the scenario earthquake using the scaled acceleration time history sets. To simplify the analysis, the maximum interstory drift ratio (MIDR) is chosen as a single engineering demand parameter (EDP) to represent the building performance. Figures 2(c) (f) plot the cumulative distribution functions (CDF) of the MIDRs from the 100- and 200-record sets, where the empirical CDF can be well-fitted using a lognormal distribution function.

Table 2 summarizes the regressed lognormal MIDR distribution for all three ground-

motion sets. Reduced relative errors are observed by increasing the size of the ground-motion sets, indicating the trend of convergence. Very similar MIDR distributions are obtained from the 100- and 200-record set, where the difference is less than 1% for mean MIDRs, and less than 8% for MIDRs at *mean+2 standard deviations* level (corresponding to 2.28% of exceedance). Based on pervious studies, the inelastic structural response is sensitive to the response spectral shape over a period range from the third-mode period, T_3 , to twice of the first-mode period, $2T_1$. It is noted that the large errors incurred using the 30-record set are mainly due to under-estimated spectral distribution over the period range of importance (0.46-5.26 sec) for the structure. Since the fitness to the targets is specified over the entire period range (0.01-10 sec) during selection process, more consistent results can be expected by improving the fitting over the period range of importance using weighting factors as mentioned in Step (3).

Figure 3 further compares the correlation coefficients regressed from Next Generation Attenuation model (Chiou and Youngs, 2008) and those obtained from the selected 100- and 200-record sets. Excellent agreement between these cases indicates that the GSM method preserves the correlation structure of the scenario earthquake in the selected record sets. The feature is particularly important in the seismic analysis of nonlinear broadband systems.

Table 1. Summary of the ground-motion characteristics

Ground-motion Set	Scale Factors	M_w	R_{rup} (km)	D_{5-95} (sec)	V_{s30} (m/s)
Scenario	—	7.0	10	14.1 [§]	400
30 records	1.54 [#] (1.28 [*])	6.9 (0.5)	12.8 (8.3)	18.4 (10.5)	410 (169)
100 records	2.00 (2.56)	6.9 (0.4)	13.8 (8.0)	16.1 (9.8)	452 (257)
200 records	1.59 (1.51)	6.9 (0.5)	13.3 (7.9)	18.3 (12.1)	426 (261)

bold data show the average value;

* data in parenthesis show the standard deviation;

§ the predicted mean value of the significant duration from Kempton and Stewart (2006).

Table 2. Summary of the lognormal MIDR distribution from three ground-motion sets

Ground-motion set	MIDR (mean)	Standard Deviation	MIDR (mean+1 std)	MIDR (mean+2 std)
30 records	0.004566 (-9.5% *)	0.575693	0.008120 (-21%)	0.014440 (-32%)
100 records	0.005012 (-0.6%)	0.761029	0.010728 (3.6%)	0.022963 (7.8%)
200 records	0.005045	0.717648	0.010341	0.021195

* Numbers in parenthesis show the relative errors of each set *w.r.t.* the 200-record set

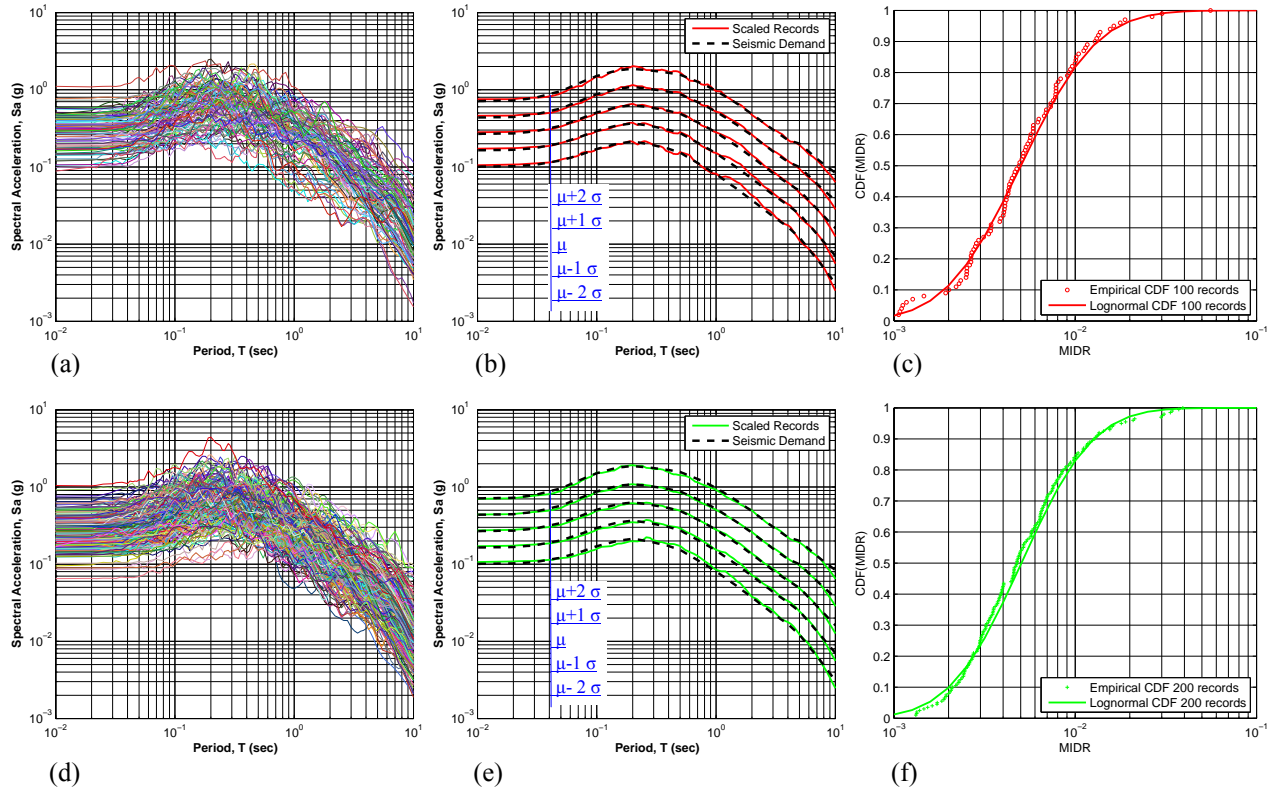


Figure 2: Spectral distribution, statistics of selected ground motions, and cumulative distribution of MIDR. (a) (b) (c) for 100-record set, (d) (e) (f) for 200-record set.

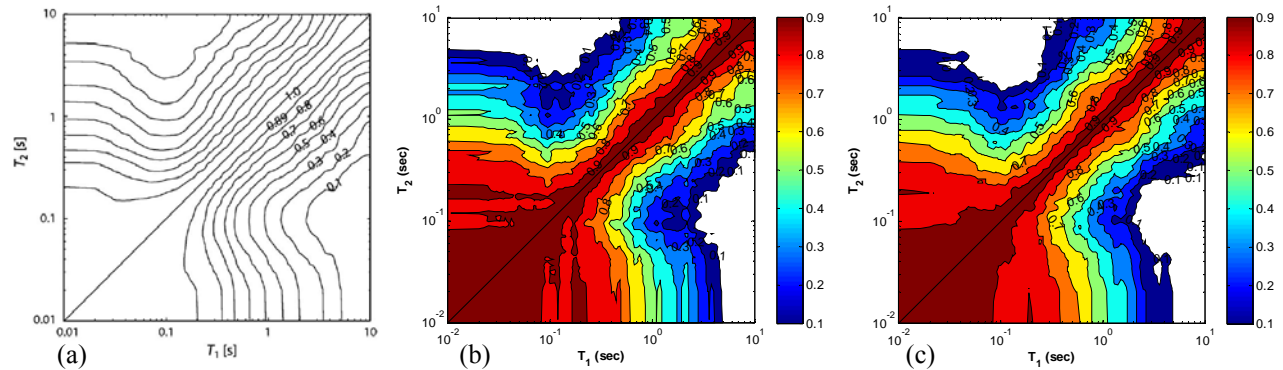


Figure 3: (a) Contours of empirical correlation coefficients from NGA model, adopted from Baker and Jayaram (2008); (b) correlation coefficients from 100-record set, and (c) from 200-record set.

Conclusions

A new ground-motion selection and modification (GMSM) method was proposed in this paper to generate “scenario-compatible” ground-motion set that realistically represents the characteristics and aleatory variability of a scenario earthquake. The resulted ground motions set can preserve the statistical distribution (mean, standard deviations) and correlations of response spectra, and other characteristics of the recordings such as earthquake magnitude, distance, and

site characteristics etc. The numerical analyses of a 20-story RC frame structure demonstrated excellent capacity of the proposed method in the study of full distribution of nonlinear responses.

Earthquake-induced liquefaction and deformation in earth structures are broadband systems that particularly suitable for validation and application of the GSM method proposed. Studies are underway to apply the GSM method to these nonlinear systems. Restrictions and limitations of the method are also to be explored in the future studies.

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