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EVALUATION OF VOLUMETRIC THRESHOLD STRAIN CONSIDERING NOISY FEEDBACK SIGNALS FROM SIMPLE SHEAR DEVICE

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ABSTRACT

Statistical methods are presented to help evaluate cyclic test results in the small amplitude range. We utilize several statistical methods to extract from noisy feedback signals meaningful response parameters at very small strain levels. Previous work has shown that the uncertainty in the estimation of vertical strains is much greater than that for shear strains. Hence, we focus in this article on the procedures for estimation of vertical strain. Kernel regression with the Nadaraya-Watson estimator and a Gaussian kernel was utilized in evaluating vertical strain response. The calculated response is dependent on the bandwidth, which is user-selected and takes the form of a kernel regression parameter. We select the bandwidth by minimizing the difference between the bias and 95% confidence interval range. We utilize these statistical methods to infer shear strain amplitudes and vertical strains for cyclic simple shear tests conducted at low levels of applied shear strains on dry soil specimens. We identify the threshold shear strain for a soil material as the largest level of shear strain where the 95% confidence interval range on vertical strain spans the null value.

Introduction

Dynamic soil testing at small strains is required to measure several critical parameters such as maximum shear modulus G_{max} (e.g., Zeng and Ni, 1999; Youn et al., 2008) and threshold shear strain γ_{tv} (e.g., Vucetic, 1994). Many soil test machines, such as cyclic simple shear, cyclic triaxial, or cyclic torsional shear, are designed to measure large strain properties such as shear strength and liquefaction characteristics, and may have a limited ability to reliably measure small strain properties as displacement and force signals approach system noise levels. Recent improvements in control system technologies hold the potential to extend the range of displacements and frequencies that are reliably controlled and measured (e.g., Duku et al., 2007). Yee et al. (2010, in review) describe procedures for evaluating cyclic shear strain amplitudes and vertical strains from feedback signals that might visually appear to be noise-dominated. This conference paper summarizes some of the key findings from that work, especially focusing on the vertical displacement estimates, which have the greater uncertainty. The results are then applied to a data

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set to illustrate how the volumetric threshold shear strain can be evaluated in consideration of the measurement noise.

Simple Shear Device

We utilize the Digitally-Controlled Simple Shear (DC-SS) device described by Duku et al. (2007). The DC-SS device features servohydraulic actuation and true digital control, and is capable of applying broadband (earthquake-like) horizontal displacement demands on soil specimens in two directions and with minimal cross coupling between the motions. Duku et al. (2007) found that the principal source of the errors in the feedback signals from the DC-SS device is noise introduced by the analog-to-digital (A/D) conversion of the feedback signal. As such, this noise is independent of the specimen response (i.e., the specimen does not experience the noise portion of the signal).

Vertical Measurements

Quantification of Noise

In the tests with the DC-SS device discussed here, three LVDTs are used to measure vertical displacement and potential rotation of the specimen top cap, which is free to displace (constant volume is not enforced). Figure 1(a) shows vertical displacements from a constant height test with displacement histograms from sample feedback signals of LVDTs v2, v3, and v4 are shown in Figure 1(b), (c), and (d) respectively. The noise for LVDTs v2 and v3 is normally distributed with zero mean and standard deviations of 0.0007 mm and 0.0006 mm respectively, whereas the noise for LVDT-v4 takes on one of two values with a standard deviation of 0.0014 mm. LVDT v4 has lower resolution than the others due to the type of analog-to-digital converter channel used. LVDT-v4 uses a 12-bit channel whereas LVDTs-v2 and v3 use 16-bit channels, thereby producing higher resolution signals for the fixed range of displacement.

The standard deviations for the vertical LVDTs are higher than the horizontal LVDT because the horizontal LVDTs are connected to a scaling amplifier which increases the signal resolution. This higher resolution allows the data acquisition system to record more data near the mean whereas the resolution levels for the vertical LVDTs will show relatively more scatter. Additionally, the variation of the signal did not change significantly for the sampling rates considered. Test data was obtained from a sampling rate of 0.001s. For a sampling rate of 0.01s, the standard deviations did not change significantly.

Mean Vertical Displacements from Noisy Signals

We estimate the mean specimen vertical displacement by smoothing out the noise effects with nonparametric regression. Nonparametric regression can track complex displacement patterns occurring over a wide range of values without the constraint of an assumed functional form.

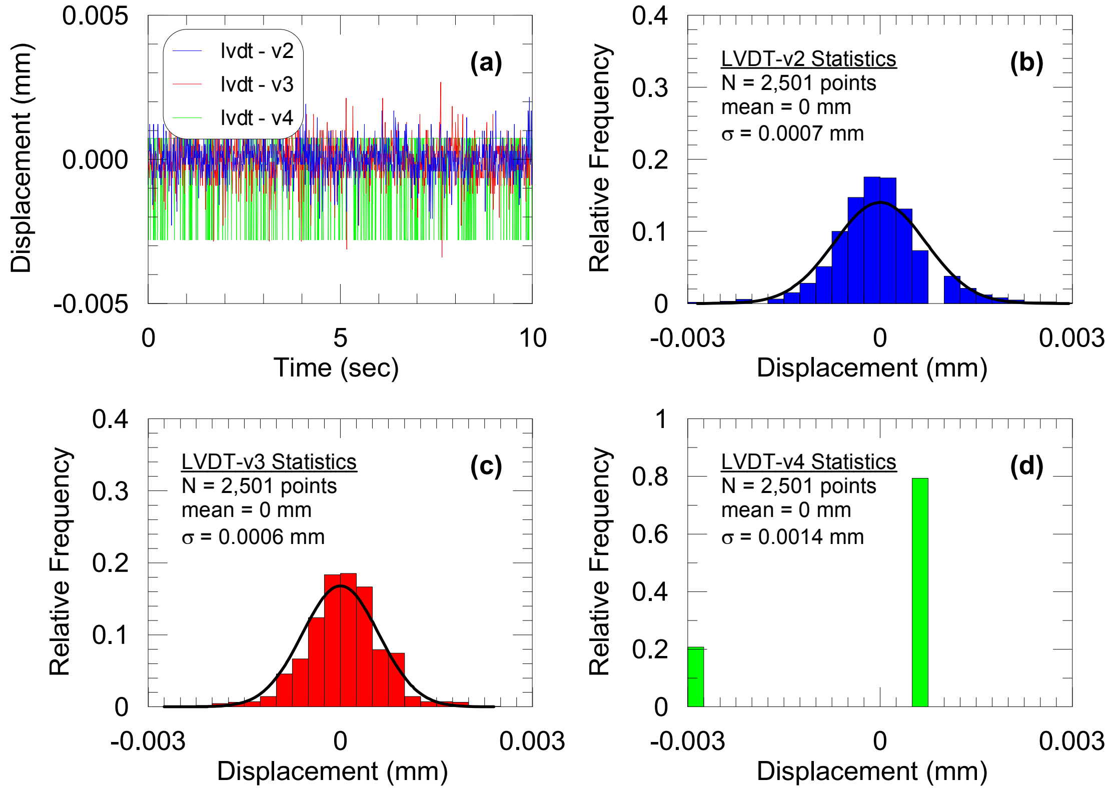

Figure 1. (a). Sample vertical displacement output, and distributions of sample vertical signal noise with normal probability distribution fit to the data for (b) lvdt-v2, (c) lvdt-v3, and (d) lvdt-v4.

Kernel regression (or smoothing) is a type of nonparametric regression that utilizes locally weighted averages of the data, in which the weights are defined by a kernel. The kernel is dependent on the selected bandwidth and kernel function. The bandwidth and kernel function determine the weights assigned to data points used to estimate the mean response for a particular point in time. We utilize the Nadaraya-Watson kernel regression function estimator (Nadaraya 1964, Watson 1964) with a Gaussian kernel function:

$$\hat{m}_h(x) = \frac{\sum_{i=1}^{N} K_h(x - x_i) y_i}{\sum_{i=1}^{N} K_h(x - x_i)} \tag{1}$$

where h = bandwidth (length of time window); N = number of data points within bandwidth; and $K_h(\mu)$ denotes the kernel and is taken as:

$$K_h(\mu) = K(\mu/h)/h \tag{2}$$

where K denotes the Gaussian kernel function for generic operator u:

$$K(u) = (\sqrt{2\pi})^{-1} e^{-u^2/2} \tag{3}$$

For application with DC-SS data, all three vertical LVDT signals are considered simultaneously. By doing this, the Nadaraya-Watson estimator essentially evaluates a smoothed average displacement. Point-wise confidence intervals for kernel regression are defined by (Härdle 1990):

$$\hat{m}_h(x) \pm z_{1-\alpha/2} \sqrt{\frac{\|K\|_2^2 \hat{\sigma}^2(x_i)}{nh\hat{f}_h}} \tag{4}$$

where $z_{1-\alpha/2} = (1-\alpha/2)$, quantile of the standard normal distribution, $\|K\|_2^2 = \int_{-\infty}^{\infty} [K_h(x)]^2 dx$, $\hat{\sigma}^2(x) = n^{-1}\sum_{i=1}^{n} W_{hi}(x)(y_i - \hat{m}_h(x))^2$, $W_{hi}(x) = K_h(x-x_i) \big/ \hat{f}_h(x)$, and $\hat{f}_h(x) = n^{-1}\sum_{i=1}^{n} K_h(x-x_i)$. Thus, the general procedure to calculate mean vertical displacements from noisy signals is to:

1. Select a bandwidth (a suggested selection criteria is described in the subsequent section).
2. Calculate the kernel for each time step (Eq. 2 and 3).
3. Calculate the kernel regression function estimator (Eq. 1) by applying the kernel to each displacement data point.
4. Calculate the point-wise confidence intervals using the kernel and kernel regression function estimator (Eq. 4).

Bandwidth Selection

A critical consideration in the application of nonparametric regression is bandwidth selection (Härdle 1990). A large bandwidth incorporates more data into the regression, which decreases the uncertainty in the mean estimate (i.e., narrows the confidence interval). However, a larger bandwidth may also increase bias, which we define as the difference between the ideal estimate and the computed estimate. To select a bandwidth that balances bias and variability, we compare the average point-wise 95% confidence interval range against the average root mean square error. The average root mean square error is calculated by:

$$\varepsilon_{RMS,v} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{1}{3}\left[\left(y_{i,LVDT-v2} - \hat{m}_{h,i}\right)^2 + \left(y_{i,LVDT-v3} - \hat{m}_{h,i}\right)^2 + \left(y_{i,LVDT-v4} - \hat{m}_{h,i}\right)^2\right]} \tag{5}$$

where n = number of data points; $y_{i,LVDT-v2}$, $y_{i,LVDT-v3}$, and $y_{i,LVDT-v4}$ = signal data from LVDT-v2, LVDT-v3, and LVDT-v4; and $\hat{m}_{h,i}$ = nonparametric regression estimate at time index i.

Minimizing the difference between the average 95% confidence interval range and the average root mean square error results in a bandwidth that balances variability with loss of accuracy by ensuring that at least half of the bias is taken into account in the confidence interval. For the common test frequency of 1 Hz, we find an average bandwidth of 0.3 sec. However, for certain flat signal profiles, the aforementioned approach produces wide and unreasonable bandwidths. This is because for flat signals, the average root mean square error is equivalent to zero, which necessitates more data points to reduce the confidence interval range. Therefore, the DC-SS selected bandwidth has a maximum equivalent to the length of one full loading cycle. This cap is reasonable as it allows the capture of cycle-to-cycle behavior, typical of most soil experiments.

Volumetric Threshold Shear Strain

Volumetric threshold shear strain, γ_{tv}, is defined as the amplitude of shear strain in cyclic loading tests below which no volume change occurs and is a critical parameter in evaluating earthquake phenomena such as seismic compression. Threshold strains were evaluated by Vucetic (1994) and Hsu and Vucetic (2004) by simply plotting vertical strain versus shear strain on a semi-log plot (linear scale for vertical strain), and the threshold strain was taken as the shear strain where vertical strains visually appear to be zero, or were projected to be zero. This approach is sensitive to the scaling of the y-axis and is subjective. The procedures described by Yee et al. (2010, in review), some of which are summarized above, offer the potential to establish a more robust definition of γ_{tv} for a given device.

We consider data obtained from DC-SS tests of cyclic volume change in an unsaturated, non-plastic silty sand material with 10% fines content known as Newhall#2. Figure 2 synthesizes the results of tests on numerous soil specimens on this same material. Vertical displacement results are shown for vertical strain at 15 cycles ($\varepsilon_{v,N=15}$) as a function of the applied cyclic shear strain amplitude (γ_c). Confidence intervals (95%) are shown both for the shear and vertical strain readings. The shear strain intervals were calculated using a statistical processing technique described in Yee et al. (2010) and are narrow and not clearly visible on the plot. Also shown is the histogram of noise from the vertical sensors expressed as an equivalent normal distribution with zero mean and standard deviation = 0.001 mm along with the 95% confidence interval on a zero mean reading for a typical signal duration (25 sec). Note that all of the data points at shear strains $\gamma_c < 0.06\%$ fall within the range of the zero-mean noise histogram, hence simple visual inspection of these data would not provide satisfactory results.

We propose that data with vertical displacements having a confidence interval that spans zero may be reasonably interpreted as being below the volumetric threshold shear strain. Similarly, data points whose confidence intervals do not include zero are considered to be above threshold. Using these criteria, the data in Figure 2 indicate a volumetric threshold shear strain of $\gamma_{tv} = 0.03\%$ for this Newhall#2 soil material.

Figure 3 shows the variation of vertical strain with shear strain for the Newhall #2 material with 10% fines content over a relatively wide strain range. Figure 3(a), drawn with a linear vertical strain axis, illustrates the potential error associated with assigning a volumetric threshold shear strain based on simple visual interpretation of data plotted in this form, which would result in a γ_{tv} value of approximately 0.1%. Figure 3(b), drawn with log axes, shows a linear trend in the vertical strain-shear strain plot. However, the data points at vertical strains below approximately 0.0025% are not significantly different from zero, and hence fall below γ_{tv} per our definition.

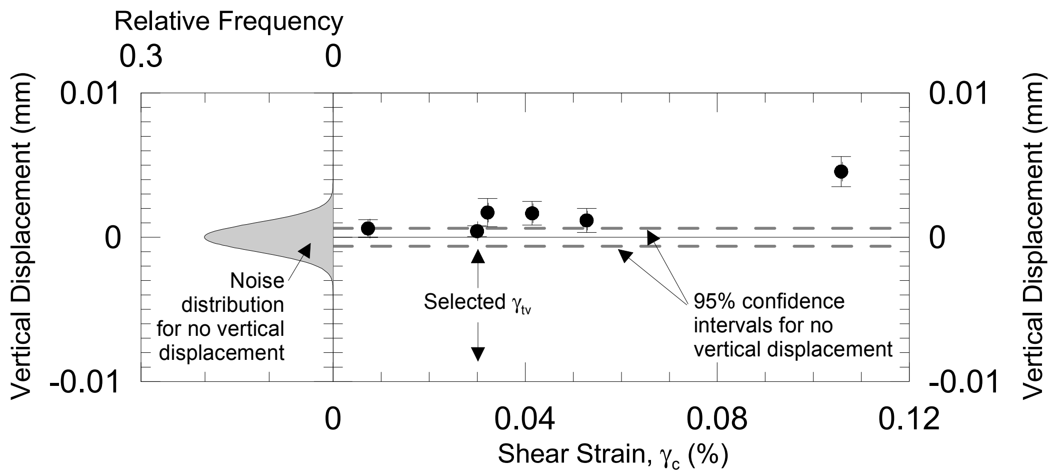

Figure 2. Plot of sample test data (N=15) with error bars against vertical noise signal distribution (no vertical movement).

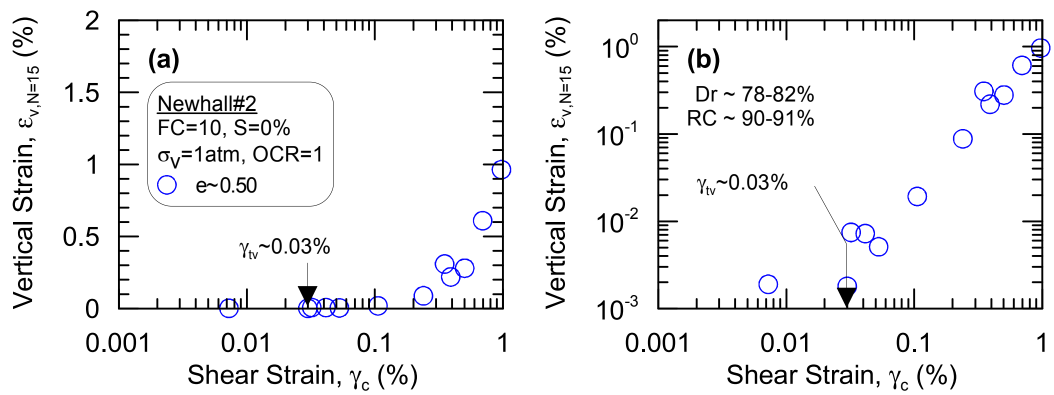

Figure 3. (a) Sample horizontal feedback signal noise and (b) distribution of sample horizontal feedback signal noise with normal fit to the data.

Conclusions

We summarize critical aspects of statistical methods designed to facilitate interpretation of cyclic test results in the small amplitude range. At these small strains, feedback signals can visually appear to be dominated by noise for many testing devices that are not designed specifically for small strain testing. Previous work by Yee et al. (2010) describes procedures for evaluation of shear strain amplitudes and their confidence intervals. For vertical strains, Kernel regression with the Nadaraya-Watson estimator and a Gaussian kernel was utilized for signal processing. The calculated response is dependent on the bandwidth, which is expressed as a kernel regression parameter. Values for this parameter are selected by minimizing the difference between the bias and 95% confidence interval range.

Applying these methods to DC-SS tests of cyclic volume change of a dry soil material enables a relatively robust assessment of volumetric threshold shear strain relative to approaches described in previous literature. Tests where the 95% confidence interval of the vertical response spans zero displacement is interpreted to be below the threshold shear strain. These criteria estimate a volumetric threshold shear strain of 0.03% for a dry sand material with 10% low-plasticity fines.

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