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ESTIMATING BASE SHEAR VERSUS ROOF DRIFT CURVES USING EARTHQUAKE-RESPONSE DATA

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ABSTRACT

The base shear versus roof drift relationship is perhaps the most efficient way to define the expected or observed behavior of a building subject to earthquake ground motion. These relationships, also known as global capacity curves or pushover curves, are often estimated from incremental nonlinear static analyses of the numerical models of the buildings loaded following a prescribed pattern of lateral forces. It is demonstrated that one could generate these characteristic curves, particularly for the dominant fundamental structural response, using actual building response data recorded during earthquakes. Practical understanding of building structural dynamics and certain signal processing tools recently developed are used in the process. An approach based on the Empirical Mode Decomposition (EMD) is developed to process acceleration data recorded in a building during an earthquake in order to extract the fundamental structural response, the equivalent of fundamental mode response in linear systems. The fundamental response data is then used to generate an estimate of the dominant global lateral-load versus drift curve, expressed in terms of equivalent spectral acceleration and spectral displacement, for the structure.

Introduction

Base shear versus roof drift curves, also known as capacity curves, describe the global relationship between lateral-load and displacement of building structural systems. Assuming that the dominant response could be represented in the form of a modal response and for which the participation factor and the effective mass ratio could be estimated, the capacity curve of a structure can be converted to an equivalent spectral acceleration versus spectral displacement curve.

Typically building capacity curves are estimated following a nonlinear static analysis procedure, commonly known as pushover analysis, in which incremental, piece-wise elastic analyses are carried out. According to Freeman et al. (1999), the term pushover refers to the analytical procedure that identifies the sequential yielding of structural elements and the internal redistribution of forces when a structure is subjected to increasing external lateral loads with a pre-set proportioning over the height of the structure. Capacity curves estimated using numerical analysis software depend on idealizations of the structure and its response to strong ground

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shaking. However, a capacity curve that is developed using measured response, such as the roof acceleration recorded during an actual strong ground shaking, will provide more realistic results as it is based on the observed behavior of the structure.

Freeman et al. (1999) presented a method to estimate the capacity curve of a building using response data recorded during earthquakes. The method uses relative roof displacement derived from acceleration recorded at the ground floor and roof level of a building. The data are manually reviewed to estimate sequential peak cyclic force-displacement relationship of individual vibration modes of building. The fundamental mode data are used to develop the capacity curve. Use of displacement data simplifies the visual detection of the fundamental mode response. However, deriving relative displacement response from acceleration data could introduce uncertainties due to errors in the acceleration record. Additionally, it has been observed that use of displacement data complicates accurate decomposition of vibration response into physically sensible components using the approach described in this paper. Accordingly, in the approach presented herein, the data that is most readily available from typical instrumented buildings is used: roof acceleration record.

The nonlinear behavior of a building during a strong earthquake limits the use of classical signal processing tools, such as Fourier transform-based techniques, to analyze its response data. To overcome the complications originating from the nonlinear behavior of a building subjected to strong earthquake motion and the nonstationary characteristics of its response, an approach based on the Empirical Mode Decomposition (EMD) method (Huang et al. 1998) has been developed.

Hilbert-Huang Transform

Hilbert-Huang Transform (HHT), developed by Huang et al. (1998), is a signal processing tool to analyze nonlinear and nonstationary signals. HHT is a two step process consisting of EMD followed by a Hilbert Spectral Analysis (HSA). The EMD decomposes the signal into a set of Intrinsic Mode Functions (IMFs). Each IMF characterizes a simple oscillatory mode component of the original signal, or a single-component that could behave in a nonlinear fashion. IMFs can be thought of as analogous to response in distinct vibration modes in modal analysis of linear systems. Hilbert transform is applied to each IMF to obtain the corresponding analytic signals. The derived analytic signals contain amplitude and phase information of the components of the original signal. The phase data are then used to derive the instantaneous frequency of each of the analytic signals. HHT presents essential characteristics of the original signal, in particular, the energy-frequency-time relationship of each characteristic component that make up the original signal. The details of the process are explained in Huang et al. (1998) for general application and in Luna (2009) for study of building response data. At the end of the process, the original data set is decomposed into IMF components and a final residual:

$$acc(t) = \sum_{j=1}^{n} IMF_j(t) + r(t) \tag{1}$$

Then one can combine each IMF component IMF_i with its Hilbert transform pair, $hIMF_i$, to obtain the corresponding analytic signal $C_j(t)$ (Gabor 1946)

$$C_j(t) = IMF_j(t) + i\, hIMF_j(t) = A_j(t)e^{i\theta_j(t)} \tag{2}$$

$A_j(t)$ and $\theta_j(t)$ are the instantaneous amplitude and the instantaneous phase, respectively, and are

$$A_j(t) = \sqrt{IMF_j^{\,2}(t) + hIMF_j^{\,2}(t)} \quad ; \qquad \theta_j(t) = \frac{\arctan\left(hIMF_j(t)\right)}{IMF_j(t)} \qquad (3)$$

If $IMF_j(t)$ is a narrow band or single-component signal, such as a proper IMF obtained from an EMD process, then the instantaneous frequency of the signal can be defined as

$$\omega_j(t) = \frac{d}{dt}\theta_j(t) \qquad (4)$$

If the EMD of building response data proceeds ideally, each resulting IMF is a single-component signal representing a particular characteristic oscillatory response mode.

A Method to Estimate Building Capacity Curves Using Earthquake-Response Data

The building acceleration recorded at the roof level during an earthquake is used to estimate the capacity curve of the building. The acceleration data contain contributions from different oscillatory modes of the building. High and low frequency noise may also be present. The objective of the developed method is to extract the fundamental response of a building from its roof acceleration data, and to present this response in the form of a capacity curve.

The procedure is based on the assumption that the fundamental mode dominates the response of the structure. It is also assumed that the ratio of the second mode frequency to the first mode frequency is greater than two so that EMD process could extract the fundamental mode response properly (Luna 2009). This frequency ratio assumption is satisfied readily in typical buildings.

In the basic EMD approach, the acceleration data are processed using sifting to obtain a set of IMFs. In theory, each IMF corresponds to a distinct dynamic process involved in the total response. However, it has been observed that the sifting could result in split-representation of a mode over multiple IMFs. In other words, a given IMF extracted from the acceleration data through an otherwise reasonable EMD process may represent different response components at different times. Should it exist, the shift from one IMF to another can be tracked.

HSA is performed after extracting the IMFs from the acceleration response. From the instantaneous frequencies of the IMFs computed from HSA, the next step is to determine which IMF is meaningful and representative of the fundamental response of the building. If split-representation is evident, the relevant IMFs are determined. Simple guidelines can be followed to select the relevant IMF(s). Based on the number of stories of a building, a rough estimate of the fundamental period can be made for the pre-strong motion part of the response. The IMF(s) that has (have) approximately the same frequency as the estimated fundamental frequency is (are) determined. Additionally, a spectrogram, which is a nonstationary data processing tool that uses a limited time window-width Fourier spectral analysis (Huang et al. 1998), can be used to track the frequency of the dominant response of the building. A relevant IMF should have the same trend of frequency as the spectrogram. Later in an event when the amplitude of the earthquake motion has diminished, typically the building would be responding in its fundamental mode. A meaningful IMF should describe a softening system, i.e. the frequency should decrease with increase in displacement.

For the cases where the fundamental response shifts from one IMF to another, the move will not be that difficult to track. If there is a sudden and large change in the value of the instantaneous frequency of an IMF, the neighboring IMF should be checked for continuity of the frequency.

After selecting the IMF representing the fundamental response, the capacity curve can be obtained as follows. The selected IMF is superimposed on the original acceleration record. The parts where the IMF tends to flow with the acceleration record are identified. The amplitude of the IMF should not exceed the amplitude of the original acceleration data significantly, otherwise, it will not be considered. An empirically determined maximum of 10% exceedance in amplitude is recommended. From the meaningful cycles, the period T is approximated by the duration between the peaks, and the corresponding amplitude a_r is approximated by the average of the amplitudes of subsequent peaks. This procedure is done for all meaningful cycles. Assuming that the fundamental mode shape has been normalized at the roof, the following formulas are used to convert T and a_r into spectral acceleration S_a and spectral displacement S_d for each cycle considered in the analysis:

$$S_a = \frac{a_r}{PF} \quad ; \quad S_d = \frac{S_a}{\omega^2} \tag{5}$$

where PF is the estimated modal participation factor for the fundamental mode when the structure is assumed to be linear-elastic, and ω is the instantaneous angular frequency. The converted points are plotted sequentially with time to obtain the capacity curve in Acceleration-Displacement Response Spectrum (ADRS) format (Mahaney et al. 1993).

Capacity Curves of the Seven-Story Hotel Building in Van Nuys, California

The seven-story Van Nuys, CA hotel building is a 1960s vintage reinforced concrete frame structure. The building is approximately 60 ft x 150 ft in plan, and is approximately 65 ft high. The columns are spaced at 20 ft on center in the transverse (plan short) direction and about 19 ft in the longitudinal (plan long) direction. The floor system is a flat plate with thickness around 8-1/2 inches with perimeter spandrel beams. The building experienced two major earthquakes: 1971 San Fernando and 1994 Northridge. During the 1971 San Fernando earthquake, acceleration data were recorded by nine accelerometers. During the 1994 Northridge earthquake, 16 channels of acceleration data were recorded.

According to Gilmartin et al. (1998), the building suffered only minor structural damage during the 1971 San Fernando earthquake. However, the building suffered severe structural damage during the 1994 Northridge earthquake, particularly in the longitudinal direction between the fourth and fifth floors where five columns in the south elevation suffered shear failure.

Estimating the Capacity Curve in the Transverse Direction

1971 San Fernando Earthquake

The roof acceleration along the transverse direction recorded during the 1971 San Fernando earthquake is shown in Fig. 1. The maximum roof acceleration is 3.8 m/sec^2 which occurred at around 10 seconds into the record. The spectrogram shown in Fig 2 is generated using an eight-second window with seven seconds overlap. The spectrogram plot is normalized with respect to the peak value of the power spectral density from all time windows. A clear trend in the dominant frequency of the roof acceleration response can be seen in the spectrogram. Initially, the frequency is around 1.7 Hz and then gradually decreases to 0.7 Hz at around 10 seconds. After 10 seconds, the dominant frequency of the response stays around 0.7 Hz.

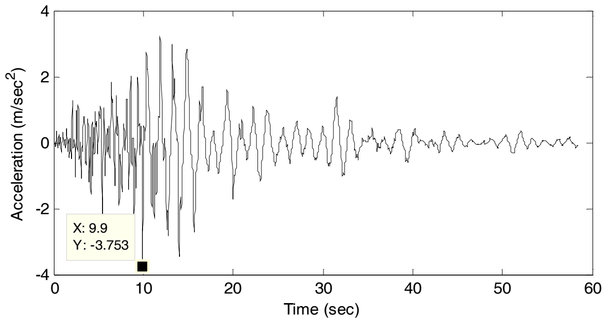

Figure 1. Roof acceleration (1971 San Fernando – transverse)

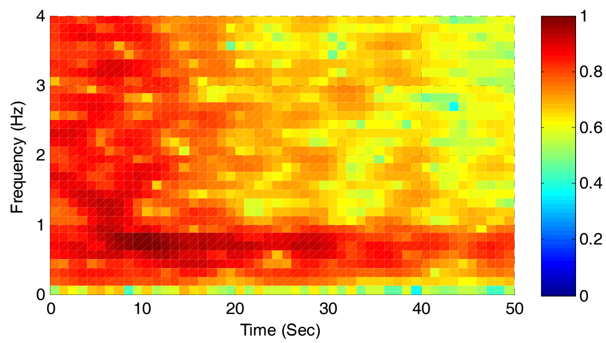

Figure 2. Normalized spectrogram of the roof acceleration (1971 San Fernando– transverse)

Ten IMFs are generated in the EMD of the roof acceleration data and are shown in Fig. 3. Fig. 4 shows an enlarged version of IMFs 3 and 4 which will be useful in identifying the fundamental mode response. The instantaneous frequencies of IMFs 3 and 4 are shown in Fig .5.

Looking at the instantaneous frequencies of the IMFs (Fig. 5), IMF 4 is a found to be good first choice as it has a frequency in the 1.4-1.7 Hz range earlier in time. The instantaneous frequency of IMF 4 follows the trend of frequency change in the spectrogram up to around 12 seconds. However, from 12 seconds to about 33 seconds, it is IMF 3 that has a frequency closer to the frequency range given by the spectrogram. This is a split-representation case where the fundamental response is captured by two different IMFs at different times. For the first 12 seconds, the fundamental response is captured by IMF 4; from 12 to around 33 seconds, the fundamental response is captured by IMF 3; and beyond 33 seconds, the fundamental response is captured alternately by IMF 3 and IMF 4 but the level of response is too low to be of use.

For a regular multi-story building, PF is generally between 1.3 and 1.4 (Freeman et al. 1975). Lepage (1997) calculated the PF for the fundamental mode of the Van Nuys building to be 1.3 in the transverse direction. The value of PF in the transverse direction is taken to be 1.3.

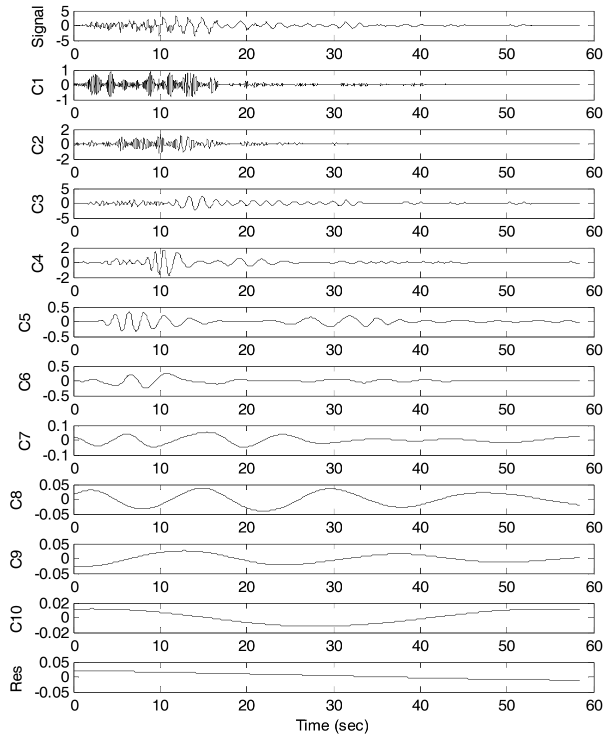

Figure 3. EMD of the roof Acceleration (1971 San Fernando – transverse)

Fig. 6 shows IMF 4 superimposed over the roof acceleration data and Fig. 7 shows IMF 3 superimposed over the roof acceleration data. The cycles where the IMF tends to flow along the roof acceleration data are selected. The cycles where the amplitude of the IMF is more than 10% the amplitude of the original acceleration data are ignored. The peaks of the selected cycles are marked with circles on Figures 6 and 7. The period of each cycle is calculated from the duration between peaks and the corresponding amplitude is approximated as the average of the amplitudes of subsequent peaks. The spectral acceleration and the spectral displacement, i.e., S_a and S_d, are calculated using Eq. 5. The coordinates of the peaks, period of each cycle, corresponding response amplitude, spectral acceleration, and spectral displacement are listed in Table 1. The points are plotted sequentially to obtain the capacity curve in the transverse direction shown in Fig.8. The capacity curve shown in Fig. 8 describes a softening system with period around 0.8 sec initially and elongating to around 1.5 sec. The results are comparable with those of Gilmartin et al. (1998).

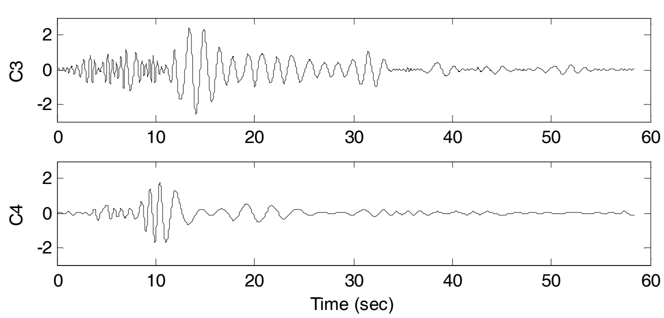

Figure 4. IMFs 3 and 4 (1971 San Fernando – transverse)

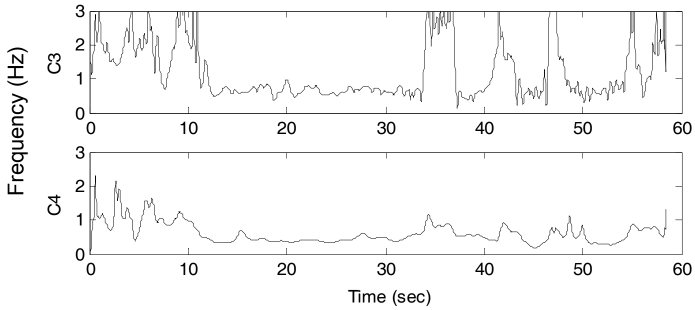

Figure 5. Instantaneous frequencies of IMFs 3 and 4 (1971 San Fernando – transverse)

Table 1. Selected peaks in IMF 3 and IMF 4 and the corresponding spectral displacement and acceleration in each cycle (1971 San Fernando– transverse direction).

Peak	Time (sec)	Acc (m/sec^2)	Period (sec)	Amp (m/sec^2)	S_d (cm)	S_a (g)
1	5.0	0.49				
2	5.8	0.30	0.8	0.40	0.5	0.03
3	9.5	1.39				
4	10.4	1.80	0.9	1.60	2.5	0.13
5	13.4	2.43				
6	14.9	2.40	1.5	2.42	10.6	0.19
7	16.4	1.28	1.5	1.84	8.1	0.14

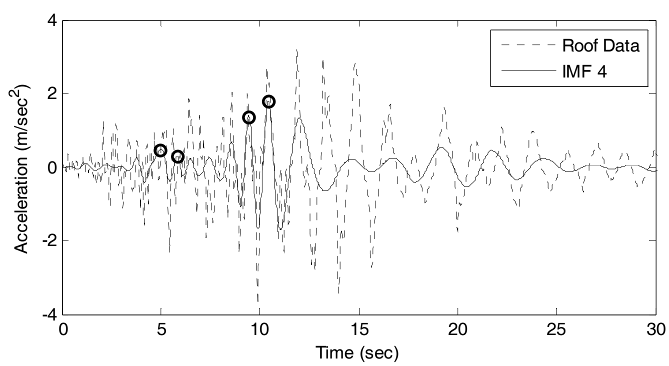

Figure 6. IMF 4 superimposed on the roof acceleration (1971 San Fernando– transverse)

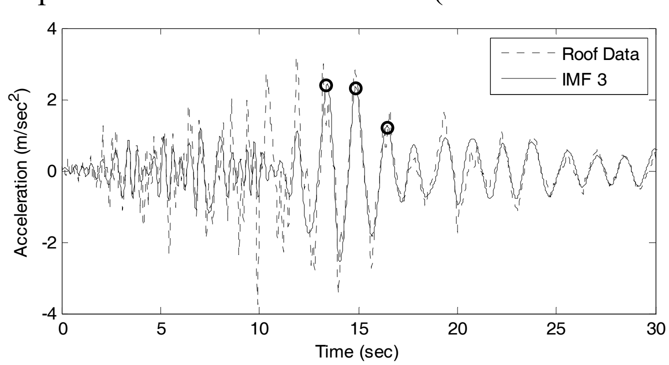

Figure 7. IMF 3 superimposed on the roof acceleration (1971 San Fernando – transverse)

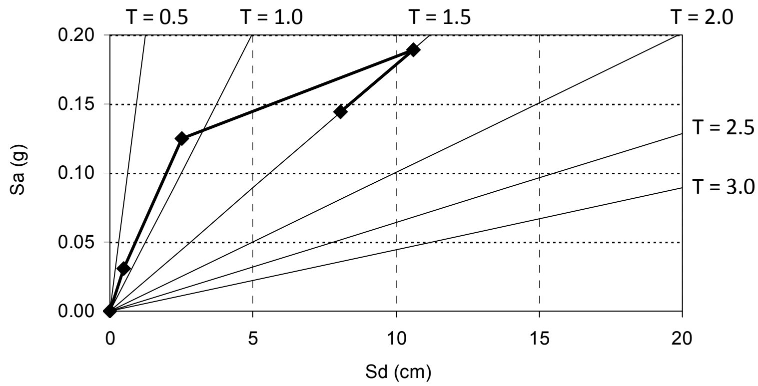

Figure 8. Capacity curve of the 7-story hotel estimated from 1971 San Fernando earthquake record – transverse direction.

1994 Northridge Earthquake

The capacity curve in the transverse direction during the 1994 Northridge earthquake is estimated following the same procedure described in the previous section. The capacity curve is compared with the one obtained from 1971 San Fernando record in Fig. 9.

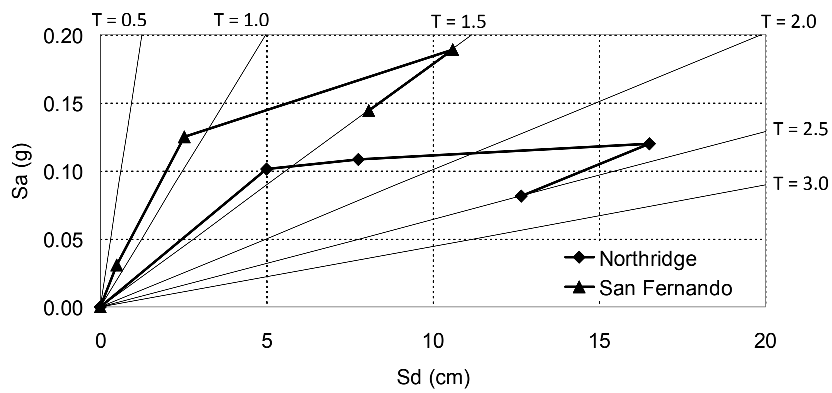

Figure 9. Comparison of the capacity curves of the 7-story hotel in the transverse direction.

The fundamental period started at 0.8 sec and elongated to around 1.5 sec during the San Fernando earthquake. During the Northridge earthquake, the fundamental period started at around 1.4 sec elongating to 2.5 sec by the end of the earthquake.

Capacity Curve in the Longitudinal Direction

Following the same procedure discussed above, the capacity curve in the longitudinal direction was estimated first by using the roof acceleration data recorded during the 1971 San Fernando and the 1994 Northridge earthquakes. An effective modal mass coefficient of 0.85 and a PF of 1.3 are used. In Fig. 10 the resulting curves are shown and compared with the capacity curve estimated by Lepage (1997) through numerical nonlinear static analysis of the structure.

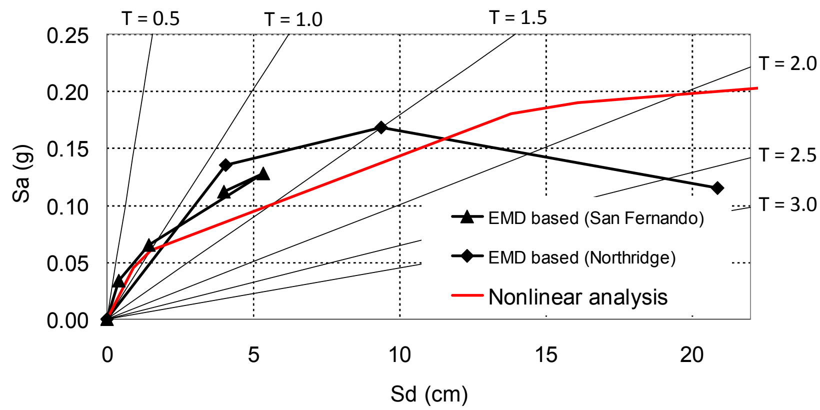

Figure 10. Capacity curves estimated using EMD-based approach compared with the one given by Lepage (1997) – longitudinal direction.

Freeman et al. (1999) reported several capacity curves from independent studies by different researchers. A plot of those capacity curves together with the curves estimated using the EMD based is shown in Fig.11. In agreement with Freeman et al. (1999), although there are variations in the different estimates of the capacity curves, one can state that the results fit into a broad band and are comparable with each other.

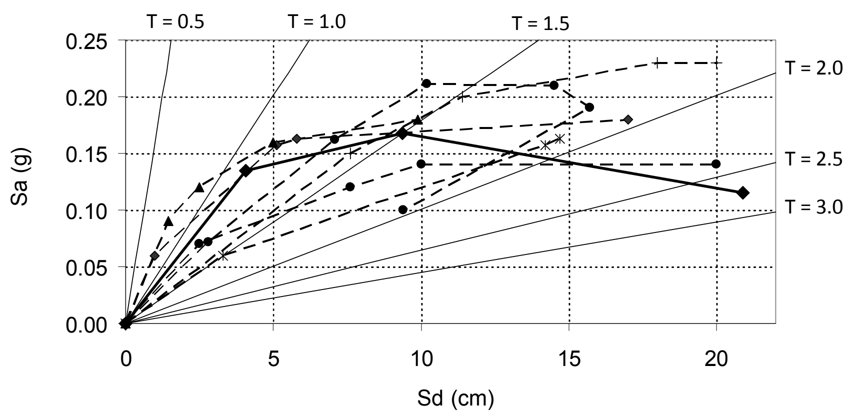

Figure 11. Capacity curve from the EMD based method (solid line) compared with the capacity curves presented by Freeman et al. (1999) (longitudinal direction).

Conclusion

The empirical mode decomposition-based response analysis method provides an efficient way to obtain the relationship between base shear and roof drift, namely, the capacity curve for a building if one has roof acceleration data for the building recorded in an earthquake. The procedure takes into account the nonlinear and nonstationary response of buildings during strong earthquakes. Using the obtained capacity curve one can observe whether and how much nonlinear response occurred during the earthquake.

The method can be integrated into structural health monitoring frameworks as it requires only minimal human interference. It can facilitate rapid evaluation of earthquake performance of buildings by providing essential structural behavior information based on the actual seismic response of the structure.

Capacity curves obtained from response data can be invaluable in improving the proportioning of similar buildings and assessing their earthquake resistance. They also enable improvement of load-displacement curves generated through the numerical pushover analyses.

The method requires basic knowledge about the building, such as the type of its lateral load resistance system and height of the structure, to identify the appropriate response components extracted from the roof acceleration data. Due to the nature of the EMD process, the method works best if the frequency of the tracked component of response is separated from other structural response frequencies by a factor of at least two. This limitation is not too restrictive if one is after the fundamental vibration mode-related response of typical buildings.

Acknowledgments

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