



## REINFORCED CONCRETE COLUMNS SUBJECTED TO LATERAL LOADS

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### ABSTRACT

There are significantly large number of reinforced concrete buildings in the US and many parts of the world that lack essential seismic details. These buildings are vulnerable to major damage or even collapse in a strong earthquake. Typically, columns in such buildings have insufficient transverse reinforcement and exhibit lack of strength and ductility during strong ground shaking. This study evaluates and presents two different macro models to determine the lateral response of reinforced concrete columns. The models are applied to estimate the lateral load-deformation response of previously tested columns, and the predicted and experimental data are compared. One of the macro-models examined here considers total lateral deformation of column to be composed of three deformation components due to flexure, shear and reinforcement slip. These individual deformation components are computed separately and then combined according to a set of rules based on comparison of column's predicted flexural and shear strengths. The second approach considers the interaction of axial deformations and concrete compression softening to relate and couple flexural and shear deformations. Based on the comparison of predicted results with experimental data, conclusions are drawn to improve response estimation by either of the models. Implementation of suggested analytical procedure produces the response comparable with experimental results.

### Introduction

There is a large inventory of reinforced concrete buildings in the US and other parts of the world that are not designed according to modern seismic design provisions. These buildings are often characterized by low lateral displacement capacity and rapid degradation of shear strength and hence are vulnerable to severe damage or even collapse during strong ground motions. Reconnaissance of damage observed during the past earthquakes suggests that poorly designed reinforced concrete columns are the most critical elements to sustain damage leading to a potential building collapse. Typically, these columns have insufficient and widely spaced transverse reinforcement and lack essential seismic reinforcement details resulting in non-ductile behavior. The need to assess their vulnerability to earthquake damage and hence suggesting the desired level of retrofit requires evaluation of the expected behavior in terms of strength and deformation capacity. This can be achieved by estimating the load-deformation response considering all potential failure mechanisms associated with axial, flexure and shear behavior.

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There are a number of studies investigating structural response of non-ductile columns following varying approaches. This paper presents a comparative study of the application of two such macro models developed to evaluate response of lightly reinforced concrete columns.

### Displacement Component Model

The model developed by Setzler and Sezen (2008) is based on the idea that a typical fixed-ended reinforced concrete column, when subjected to earthquake loading, undergoes lateral deformation which is comprised of three components; flexural deformations, reinforcement slip deformations and shear deformations. These component deformations are illustrated in Fig. 1a. The model simulates lateral load-deformation response by estimating these deformation components individually and then combining them according to a set of rules specified for each category. The classification of the columns into categories is based on comparison of their predicted shear and flexural strengths. Component deformation models and total response model under monotonically increasing lateral load can serve as a response envelope or primary curve for respective component cyclic responses. Details of the model can be found in Setzler (2005).

### Deformation Components

In this model, flexural deformations are determined by performing traditional fiber section analysis in one dimensional stress field. This analysis takes into account the enhancement in the strength and ductility of the concrete due to confinement but ignores concrete behavior in tension. Following the moment-curvature analysis, the flexural deformations can be calculated by integrating curvature over the length of the column. The idealized curvature distribution for a cantilever column is assumed to have a linear curvature distribution up till yield point and after yielding, the inelastic curvatures are lumped over plastic hinge length. The plastic hinge length is taken as one half of the total section depth per the recommendations of Moehle (1992).

The flexural deformations as determined through fiber section analysis cannot account for the end rotations that are caused by reinforcement slip. These deformations can be as large as 25 to 40% of the total lateral deformations (Sezen 2002) and hence cannot be ignored. Therefore, slip deformations must be accounted for separately and added to the other deformation components (flexure and shear) to accurately model total drift of a column (Fig. 1a). Lateral displacements due to reinforcement slip are calculated in this study through a model which was proposed by Sezen and Moehle (2003) and further developed by Sezen and Setzler (2008). The model approximates the bond stress as bi-uniform function with different values for elastic and inelastic steel behavior (Fig. 2a). Slip at the loaded end of the reinforcing bar can be calculated by integrating bi-linear strain distribution over the development length as follows,

$$slip = \int_0^{l_d+l'_d} \varepsilon(x)dx \quad \left( l_d = \frac{f_s d_b}{4u_b} ; l'_d = \frac{(f_s - f_y) d_b}{4u'_b} \right) \quad (1)$$

where  $l_d$  and  $l'_d$  correspond to the development lengths for elastic and inelastic portions of the bar, respectively.  $f_s$  is stress at loaded end of the bar,  $f_y$  and  $d_b$  are yield stress and diameter of the bar, respectively. The slip is assumed to occur in tension bars only and cause the rotation about the neutral axis

The basis of the shear model used in this research is the model developed by Patwardhan (2005), which uses Modified Compression Field Theory, MCFT (Vecchio and Collins 1986). Patwardhan (2005) proposed a piecewise linear model defining key points in the lateral force-shear deformation envelope through a parametric study implementing MCFT through a computer program Response-2000. In this study, pre-peak non linear shear force-shear deformation response is obtained indirectly from Response-2000 by integrating shear strain distribution over the height of the column for each load step. After the peak strength has reached, the shear strength is assumed to remain constant at its peak value until the onset of shear strength degradation. Thereafter, shear strength decreases linearly with increasing shear deformations to the point of axial load failure, where lateral strength is assumed to be zero. The shear deformation model used is illustrated in Fig. 2b. Critical points on the shear response envelope are defined with set of following equations (Gerin and Adebar 2004, Elwood and Moehle 2005a),

$$\Delta_{v,u} = \left( 4 - 12 \frac{v_n}{f'_c} \right) \Delta_{v,n} \quad (2)$$

$$\Delta_{v,f} = \Delta_{ALF} - \Delta_{f,f} - \Delta_{s,f} \geq \Delta_{v,u} \quad (3)$$

$$\frac{\Delta_{ALF}}{L} = \frac{4}{100} \left( 1 + \tan^2 \theta \right) \left( \tan \theta + P \left( \frac{s}{A_{sv} f_{yv} d_c \tan \theta} \right) \right)^{-1} \quad (4)$$

where,  $v_n$  is the shear stress at peak strength,  $f'_c$  is the concrete compressive strength,  $\Delta_{v,n}$  is the shear displacement at maximum strength determined from Response-2000,  $\Delta_{ALF}$  is the total displacement at axial load failure,  $\Delta_{f,f}$  and  $\Delta_{s,f}$  are the flexural and slip displacements, respectively,  $\theta$  is the angle of the shear crack,  $P$  is the axial load,  $A_{sv}$  is the area of transverse steel with yield strength  $f_{yv}$  at spacing  $s$ , and  $d_c$  is the depth of the core concrete, measured to the centerlines of the transverse reinforcement. In the derivation,  $\theta$  is assumed to be 65 degrees.

## Total Response

The proposed procedure models each of flexure, slip and shear deformation by a spring subjected to the same force and the total response is the sum of the responses of each spring (Fig. 1b). Each deformation component can simply be added to obtain the total response up to the peak strength of the column. However, for post-peak behavior, the column is classified into one of the five categories based on a comparison of its shear, yield and flexural strengths and deformation components are combined as per set of rules specified for each category. (Setzler and Sezen, 2008). The flexural strength,  $V_p$  is the lateral load corresponding to the peak moment sustainable by the column during flexural analysis. The shear strength of the column  $V_n$ , is calculated by the expression developed by Sezen and Moehle (2004) for lightly reinforced concrete columns.

The peak response is limited by the lesser of the shear strength ( $V_n$ ) and the flexural strength ( $V_p$ ), however post peak response is assumed to be governed by the limiting mechanism (i.e., flexure or shear). Category I column ( $V_n < V_y$ ) fails in shear while the flexural behavior remains elastic ( $V_y$  is the lateral strength corresponding to first flexural yielding). Category II column ( $V_y \leq V_n < 0.95V_p$ ) also fails in shear, however inelastic flexural deformation occurring

prior to shear failure affects the post-peak behavior. Shear deformations continue to increase after the peak shear strength is reached, but the flexure and shear springs are locked at their peak strength values. In Category III column ( $0.95V_p \leq V_n \leq 1.05V_p$ ), the shear and flexural strengths are nearly identical. It is not possible to predict conclusively which mechanism will govern the peak response. Shear and flexural failure are assumed to occur “simultaneously,” and both mechanisms contribute to the post-peak behavior. Category IV column ( $1.05V_p < V_n \leq 1.4V_p$ ) may potentially fail in the flexure, however inelastic shear deformations affect the post-peak behavior and shear failure may occur as the displacements increase. The shear strength in Category V column ( $V_n > 1.4V_p$ ) is much greater than the flexural strength and column fails in flexure while shear behavior remains elastic. The implementation of this model on test specimens is illustrated in subsequent sections. The details of the column classification can be found in Setzler (2005).

### **Axial-Shear-Flexure Interaction (ASFI) Approach**

Axial-Shear-Flexural Interaction (ASFI) approach has recently been developed by Mostafaei and Kabeyasawa (2007) for the displacement-based analysis of reinforced concrete elements such as beams, columns and shear walls by considering interaction between axial, shear and flexural mechanisms. This macro-model based approach consists of two models evaluating axial-flexural and axial-shear responses simultaneously to obtain total response of elements subjected to axial, flexural and shear loads. The axial-flexural and axial-shear mechanisms are coupled in average stress-strain field considering axial deformation interaction, softening of concrete compression strength while satisfying compatibility and equilibrium conditions.

Axial-flexural behavior in ASFI approach is simulated by employing section analysis or fiber model in one-dimensional stress field in a conventional way, except that concrete behavior in tension is also incorporated in the analysis. Another deviation from standard flexural section analysis practice comes from the consideration of cracked concrete behavior in the analysis. Conventionally, the flexural section analysis is performed independently where compressive stress-strain relationships for the concrete are employed driven from its response in standard cylinder test which simulate uniaxial compression. However, the strain conditions for the concrete in the web of a reinforced concrete beam or column subjected to shear are significantly different from those in a cylinder test. The concrete in standard cylinder test is subjected to only small tensile strains primarily due to Poisson’s effect, whereas, the concrete in diagonally cracked web is subjected to substantial tensile strains. As a result, the concrete in diagonally cracked web is weaker and softer than the concrete in a cylinder. To account for these influences, axial-flexural model in ASFI approach employs constitutive model for cracked concrete in compression. Studies indicate that the principal compressive stress in the concrete is not only a function of principal compressive strain but also of the co-existing principal tensile strain, such that compressive strength and stiffness of the concrete decrease as the tensile strains increase. This phenomenon, known as compression softening is one of the interaction terms for axial-shear mechanism that is incorporated into axial-flexural model coupling both mechanisms. In ASFI approach, concrete compression softening is employed in flexural analysis by softening concrete response in uniaxial compression. Axial strain due to flexure is the second interaction term in ASFI approach that connects axial-flexure mechanism with axial-shear mechanism.

Axial-shear model in ASFI simulates shear mechanism by employing Modified Compression Field Theory and considering one integration point in in-plane stress condition. It is a displacement-based evaluation approach suitable for response estimation of reinforced concrete membrane elements subjected to normal and shear stresses. MCFT is essentially a smeared rotating crack model in which cracked concrete is treated as a new orthotropic material with its unique stress-strain characteristics. The critical aspect of MCFT is the consideration of local stress-strain conditions at cracks ensuring that the tension in the concrete can be transmitted across the crack and shear stress on the surface of the crack does not exceed maximum shear provided by the aggregate interlock. Thus, load deformation response of the members loaded in shear can be estimated by considering compatibility of average strains for concrete and reinforcement, equilibrium relationships involving average stresses in concrete and reinforcement and appropriate stress-strain relationships for reinforcement and diagonally cracked concrete. Complete details of ASFI approach can be found in Mostafaei (2006).

### Implementation of Analytical Models

Both of the above mentioned models are implemented using four test columns (Sezen and Moehle 2006, and Sezen 2002). These tests render useful data in terms of experimental force-displacement responses for each of the flexure, slip, and shear components individually, as well as for the overall response. The columns represent lightly reinforced columns that have shear and flexural strengths very close to each other. These are 18 inch square columns with fixed ends at top and bottom having height of 116 inches. The longitudinal reinforcement consists of eight No. 9 bars. No. 3 column ties with 90-degree end hooks were spaced at 12 in. over the height of the column. Specimens-1 and -4 were subjected to a constant 150 kip axial load, Specimen-2 was tested under a constant 600 kip axial load, and Specimen-3 had an axial load varying from 600 kip in compression to 60 kip in tension to simulate the range of forces experienced by an exterior building column in an earthquake. The columns were tested under unidirectional cyclic lateral loading, except for Specimen-4, which was tested under monotonic loading. Average concrete strength was 3077 psi, and the yield strength of longitudinal and lateral steel was 59 and 63 ksi, respectively.

In order to compare the analytical aspects of the two models, same material constitutive laws for concrete and reinforcing steel are employed. These are presented in Fig. 4. (Mander et al. 1988, Roy and Sozen 1964). In the figure,  $f_{cc}$  is the peak confined concrete stress calculated according to Mander et al. In ASFI approach, the concrete compression softening is incorporated into the response with compression softening factor  $\beta$  defined as,

$$\beta = \frac{1}{0.8 - 0.34 \frac{\varepsilon_{c1}}{\varepsilon_{co}}} \leq 1.0 \quad (5)$$

where  $\varepsilon_{c1}$  is principal concrete tensile strain obtained from axial-shear model of ASFI approach.

### Analysis Results and Discussion

In Figs. 4 through 7, results for implementation of both models are presented. Before analyzing total lateral force-deformation response, it is helpful to study individual response

components of both models to better understand their capabilities. Moment-curvature response from fiber section analysis was very similar with the exception of two aspects. Both models configure confined core and unconfined cover concrete separately with their respective stress-strain relationships and strain limits while employing confinement effects for the core. Setzler and Sezen model, however, does not consider concrete strength in tension, as is done typically, primarily for sake of simplicity. In ASFI approach, axial-flexure and axial-shear models interact with each other through axial strain compatibility and employ same constitutive laws for materials. In addition, in fiber section analysis, ASFI approach considers softening effect in concrete compression strength due to increasing level of shear strains. This effect is incorporated in the flexural analysis by calculating compression softening factor  $\beta$  (Eq. 5) at each load step. For all specimens,  $\beta$  is equal to 1 for most part of the analysis. That is the reason that moment-curvature responses from both models are nearly identical. The minor difference is due to considering concrete tensile behavior in one approach and not considering in the other. However, for the columns where flexural and shear strengths are not similar and shear is predominant, ASFI approach shall result in softened moment-curvature response.

Figs. 4 and 5 present experimental and predicted load–flexural displacement and load–slip displacement responses for specimens 1 and 2. Both models capture experimental response well and produce similar curves. This is due to the fact that moment-curvature relationships are very similar and same models are employed for determining flexural and slip displacements. The only difference in flexural model is the use of different plastic hinge lengths in plastic hinge model. If moment curvature responses are predicted differently by these approaches, which shall be the case for other categories of columns, flexural and slip responses shall be different even if same component displacement models are employed. In such cases, although ASFI approach might be able to predict total force-total displacement response very well, it is likely that the approach shall underestimate flexural and slip deformation components. In this analysis, this aspect could not be verified because the test columns in this study had either nearly identical flexural and shear strengths (specimen 1 and 4) or had higher shear strength than flexural strength (specimen 2 and 3).

Fig. 6 shows experimental and calculated lateral force-shear displacement response for the specimens. Both models employ MCFT for determining shear deformations but in a different way. ASFI approach employs MCFT directly into the analysis in each loading step considering the interaction between axial, flexural and shear mechanisms. It follows an iterative scheme to satisfy the equilibrium and compatibility conditions. Setzler and Sezen model also employs MCFT, indirectly, for determining shear deformations. It takes the average shear strain distribution along the length of the column from Response-2000 program and then integrates them to get shear displacements prior to peak load. The details for post peak shear response are described above as well and in Fig. 2. Comparison of the predicted responses in Fig. 6 shows that ASFI approach captures the peak shear load very well, but stiffness and deformation at peak load are not predicted well. Setzler and Sezen model does a better job in capturing the initial stiffness of the response and predicts peak shear strength fairly well.

Fig. 7 compares the experimental and predicted total lateral load-displacement responses. Both models predict the response equally well. In ASFI approach, deformation up till peak load are added together satisfying equilibrium and compatibility conditions and considering

interaction of compression softening and axial displacements. For post-peak response, the same conditions are satisfied while keeping stiffness of shear and pullout model as constants. The compression softening factor is also frozen at the value corresponding to the peak load. A very important aspect of the post-peak response in ASFI approach is reduction in compression strengths of the longitudinal bars to account for buckling or slip of compression bars. These stresses are reduced when unconfined cover concrete fibers reach approximately 30% of maximum strength. The compressive bars stresses are then reduced linearly in accordance with the slope of post-peak compression strength of the confined concrete. In this study, the model for reduction in compression bars stresses is illustrated in Fig. 3d. As per Setzler and Sezen (2008) model, Specimen-1 has flexural and shear strengths of 70.4 and 69.0 kips, respectively. According to their criterion, this column is classified as category III column. Setzler and Sezen model captures initial response very well up to the peak strength. The post-peak deformations are slightly over predicted, but slope of the degrading response follow the experimental data well. ASFI approach predicts a peak shear strength of 67 kips at displacement of 2.00 inches which agrees well with the experimental data. Predictions for post peak response are slightly better with slope of degrading curve following the experimental results.

Compression steel stresses were reduced as per the model given in Fig. 3d. According to this model, compression stresses start to decrease when concrete fibers adjacent to compression bars reach their peak strength. When this happens, corresponding strain in the relevant steel layer can be calculated from flexural strain distribution across the cross section depth. This strain is  $\epsilon_{sp}$  as shown in the figure. This point can fall anywhere on typical stress-strain relationship for steel depending upon the level of flexural strain. Steel stresses follow their usual constitutive stress-strain relationship until strain reaches this limit. Then compression stresses in reinforcement follow new path defined by line joining peak stress point to residual strength point having slope  $m$ , which is the same for descending branch of concrete compression strength. The figure shows that the post peak response improves remarkably after considering stress reduction in compression bars.

Specimen-2 is placed in category IV having shear and flexural strength of 93 kips and 72 kips respectively as per Setzler and Sezen model. Flexural behavior predicted by Setzler and Sezen model agrees with the experimental data. ASFI approach estimates the initial and peak responses equally well. Specimen-3, in the positive direction is under high axial load has shear and flexural strength of 93 and 72 kips, respectively as per Setzler and Sezen model making it a category IV column. In the negative direction, it has approximately equal shear and flexural strength around 56 kips, placing it in category III. Setzler and Sezen model estimates high initial stiffness. Peak strength under low axial load is estimated well but overestimated under high axial load. Post peak predictions are slightly off in the positive direction and overestimated in the negative direction. ASFI approach estimates the peak load as 69 kips at deformation of 1.00 inch, which is very close to the experimental data. Predicted initial response is less stiff. The response under tensile load could not be evaluated with the ASFI approach. Specimen-4 is identical to Specimen-1, except for lateral loading scheme. The initial stiffness by Setzler and Sezen model is overestimated, but peak strength is predicted fairly well. The ASFI model also predicts stiff initial response, and underestimates post peak response.

## Conclusions

The application of two macro models is examined for displacement-based analysis of previously tested shear critical columns subjected to lateral loads. One model considers total lateral deformation of columns stemming out from three components namely flexure, shear and bond slip. The other model, ASFI approach, consists of axial-flexure and axial-shear models connected together through interaction of concrete strength degradation and axial deformations. In this study ASFI approach predicted displacement at peak very well, but is expensive in calculations and requires iterative procedures and nested loops to achieve convergence of the results. On the other hand, Setzler and Sezen model is efficient in use due to its simplicity and ease of application. Both models perform well for estimating structural performance of lightly reinforced concrete columns including ultimate strengths and deformations, mode of failures and post peak responses.

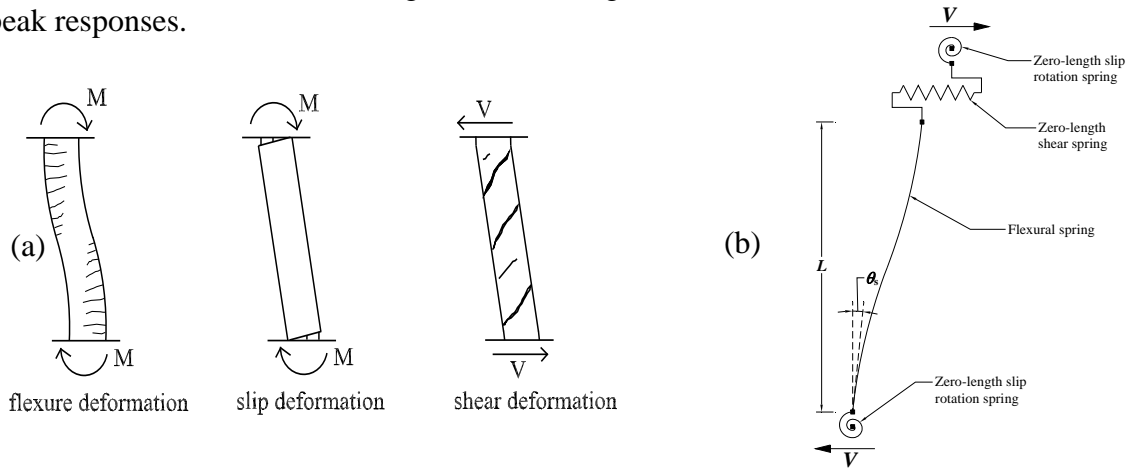
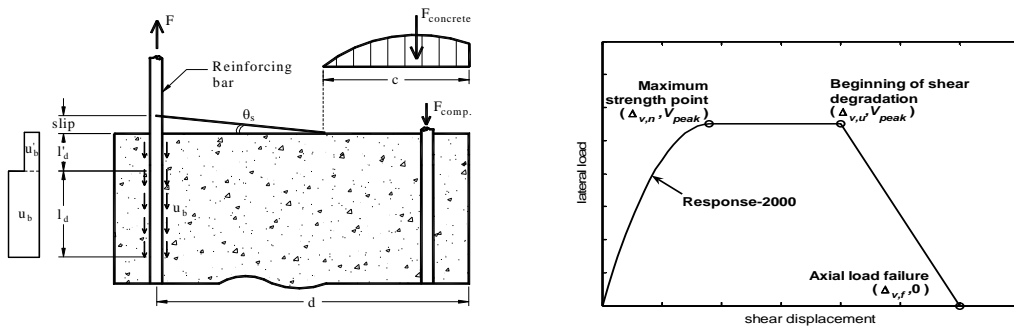


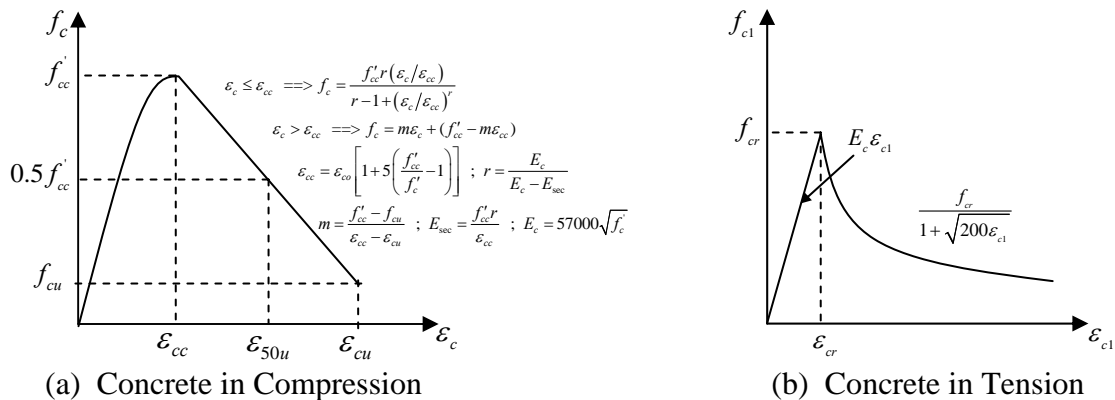
Figure 1. Deformation Components of a Reinforced Concrete Column and Spring Model.



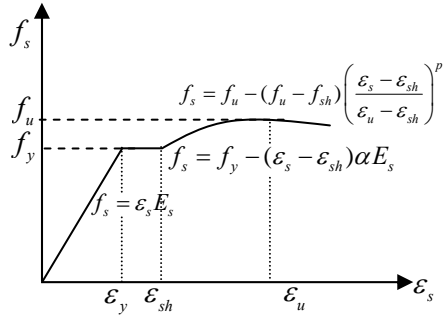
(a) Model for Slip Rotation/Deformations

(b) Model for Shear Deformations

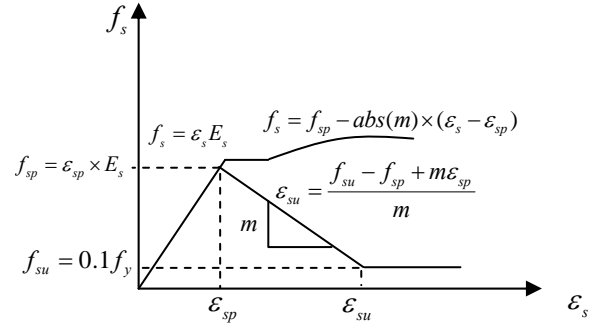
Figure 2. Models for component deformations







(c) Reinforcing Steel



(d) Compression Bars Stress Reduction

Figure 3. Material Constitutive Relationships

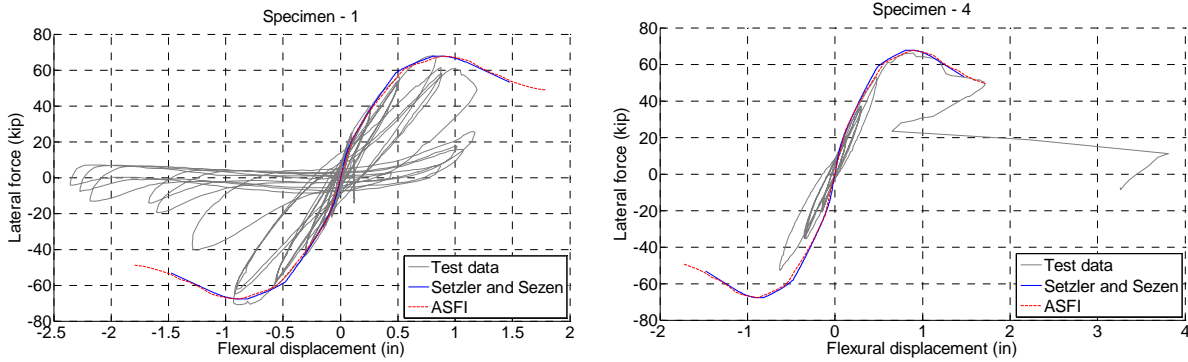


Figure 4. Comparison of Lateral Load-Flexural Displacement Responses.

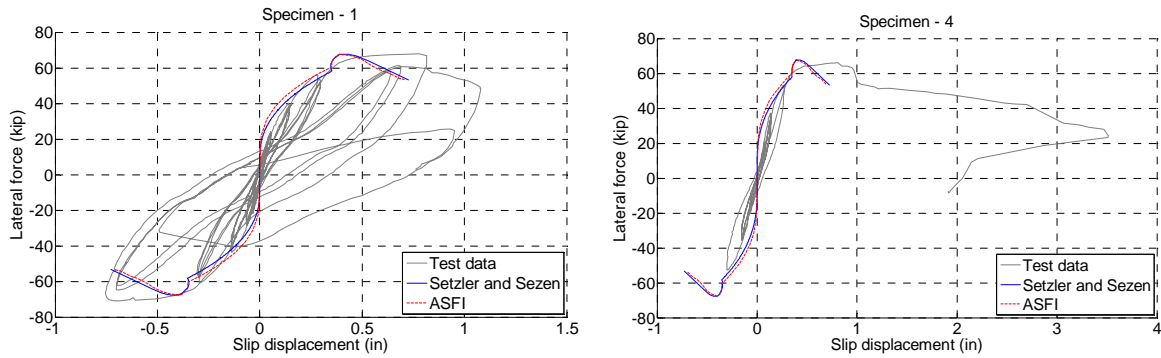


Figure 5. Comparison of Lateral Load-Slip Displacement Responses.

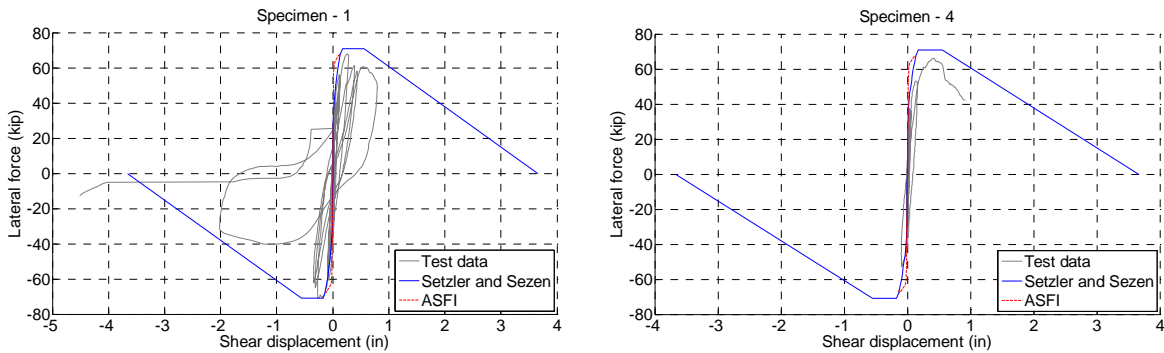


Figure 6. Comparison of Lateral Load-Shear Displacement Responses.

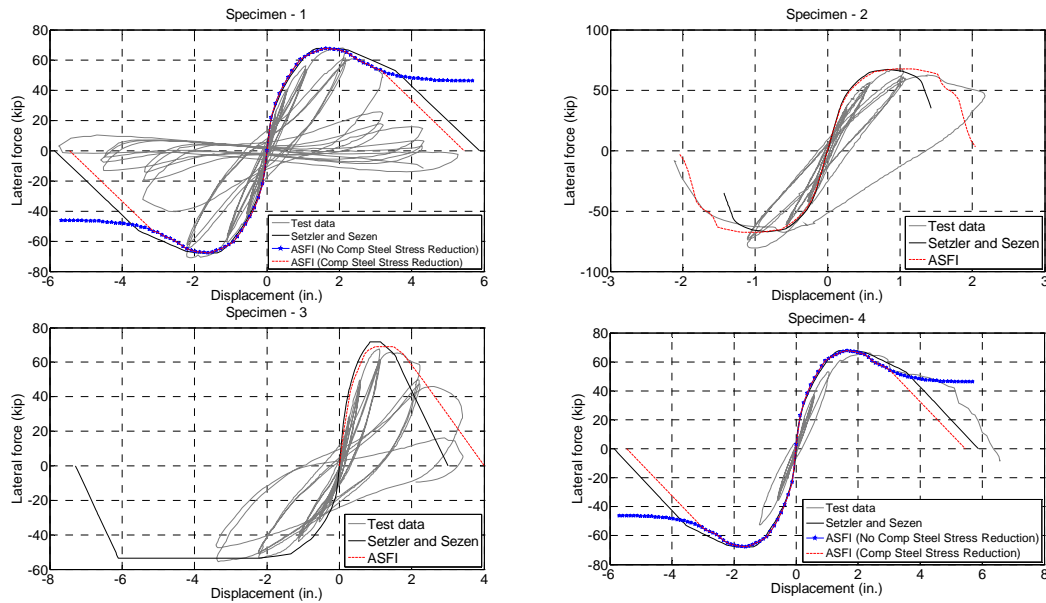


Figure 7. Comparison of Total Estimated Responses and Experimental Data.

## References

- Elwood, J.K., Moehle, J.P., 2003. Shake Table Tests and Analytical Studies on the Gravity Load Collapse of Reinforced Concrete Frames. *PEER Report 2003/01*, Univ. of California, Berkeley,
- Elwood, K. J., Moehle, J.P., 2005. Drift Capacity of Reinforced Concrete Columns with Light Transverse Reinforcement. *Earthquake Spectra*, 21 (1), 71-89.
- Gerin, M., Adebar, P., 2004. Accounting for Shear in Seismic Analysis of Concrete Structures. *13<sup>th</sup> World Conference on Earthquake Engineering*, Vancouver, Paper No. 1747
- Mander, J.B., Priestley, J.N., Park, R., 1988. Theoretical Stress-Strain Model for Confined Concrete. *ASCE Journal of Structural Engineering*. 114 (8), 1804-1825.
- Mostafaei, H., 2006. Axial-Shear-Flexure Interaction Approach for Displacement-Based Evaluation of Reinforced Concrete Elements. *PhD dissertation*, University of Tokyo, Tokyo, Japan.
- Mostafaei, H., Kabeyasawa, T., 2007. Axial-Shear-Flexure Interaction Approach for Reinforced Concrete Columns, *ACI Structural Journal*, 104(2), 218-226.
- Patwardhan, C., 2005. Strength and Deformation Modeling of Reinforced Concrete Columns. *M.S. Thesis*. The Ohio State University, Columbus, Ohio.
- Roy, H.E.H., Sozen, M.A., 1964. Ductility of Concrete. *International Symposium on Flexural Mechanics of Reinforced Concrete - Proceedings*. Miami, Fla, 213-235.
- Setzler, E.J., 2005. Modeling the Behavior of Lightly Reinforced Concrete Columns Subjected to Lateral Loads. *M.S. Thesis*. The Ohio State University, Columbus, Ohio.
- Setzler, E.J., Sezen, H., 2008. Model for the Lateral Behavior of Reinforced Concrete Columns Including Shear Deformations. *Earthquake Spectra*. 24 (2) 493-511
- Sezen, H., 2002. Seismic Behavior and Modeling of Reinforced Concrete Building Columns. *Ph.D. Dissertation*. University of California, Berkeley.
- Sezen, H., Moehle, J.P., 2003. Bond-Slip Behavior of Reinforced Concrete Members. *fib-Symposium: Concrete Structures in Seismic Regions*. CEB-FIP, Athens, Greece.
- Sezen, H., Moehle J.P., 2006. Seismic Tests of Concrete Columns with Light Transverse Reinforcement. *ACI Structural Journal*, 103 (6) 842-849.
- Sezen H., Setzler E.J., 2008. Reinforcement Slip in Reinforced Concrete Columns. *ACI Structural Journal*. 105(3), 280-289
- Vecchio, F.J., Collins, M.P., 1986. The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear. *ACI Journal*, 83 (2) 219-231.