# INFLUENCE OF ORIENTATION OF RECORDED GROUND MOTION COMPONENTS ON THE LONGITUDINAL REINFORCING STEEL AREA IN CONCRETE FRAME ELEMENTS WITHIN THE CONTEXT OF LINEAR RESPONSE HISTORY ANALYSIS 

K. G. Kostinakis ${ }^{1}$, A. M. Athanatopoulou ${ }^{2}$ and I. E. Avramidis ${ }^{3}$


#### Abstract

The present paper aims to investigate the influence of the orientation of recorded horizontal ground motion components on the longitudinal reinforcing steel areas in R/C buildings within the context of linear response history analysis. For this purpose two single-story buildings are studied for seven recorded bi-directional ground motions applied along the horizontal structural axes. The ground motions are represented by: (a) the recorded accelerograms; (b) the recorded accelerograms transformed to other sets of orthogonal axes forming with the initial ones an angle $\theta=30^{\circ}, 60^{\circ}, \ldots, 360^{\circ}$ and (c) the principal (uncorrelated) components of ground motion. For each orientation the longitudinal steel area at all critical cross sections is calculated using three methods of selecting the set of internal forces which are needed in order to compute the required reinforcement. The results show that the reinforcing steel area calculated by two of the applied methods is significantly affected by the orientation of the recorded ground motion components, while the third one, which takes into account all possible orientations of seismic motion, leads to results which do not depend on the orientation of the seismic input.


## Introduction

Modern seismic codes (ASCE 41/06, NEHRP, FEMA356, EAK2003) suggest the linear time history analysis as one of the methods that can be used for the seismic analysis and design of R/C structures. According to this method a spatial model of the structure is analyzed using simultaneously imposed consistent pairs of earthquake records along each of the two horizontal structural axes (with a few exceptions, the vertical component of the ground motion is allowed to be ignored as its influence on seismic response is considered negligible). In most strong-motion databases the horizontal components of the ground motion are given along the orientation they were recorded for. Thus, the orientation of the recorded seismic components is predetermined by the orientation of the recording instrument, which is in general arbitrary. However, it has been

[^0]shown (Kostinakis 2009) that the structural response is strongly affected by the recording angle of the ground motion and that the recording angle that yields the maximum response does not coincide with the orientation for which the accelerograms are recorded.

The aim of the present paper is to investigate the influence of the orientation of recorded horizontal ground motion components on the reinforcement steel areas of $\mathrm{R} / \mathrm{C}$ frame elements, within the framework of linear response history analysis. As existing seismic codes do not clearly specify how to select the set of internal forces for which the sections' longitudinal steel area must be calculated, three different methods of selection are applied. The first method, which is proposed as the most rational one, utilizes the normal stresses in every relevant cross section and accounts for the critical angle of the seismic excitation, i.e., the angle that yields the maximum value of each response quantity of interest. In an attempt to interpret the seismic code provisions two other methods of selecting the sectional forces are introduced. As numerical example, two single-story buildings subjected to 7 strong earthquake ground motions are analyzed. The analyses results show that the reinforcement steel area calculated by two of the applied methods is significantly affected by the orientation of the recorded ground motion components, while the third method leads to results which do not depend on the orientation of the seismic input.

## Principal Directions of Horizontal Seismic Components

In most strong-motion databases the horizontal components of the ground motion are given along the orientation they were recorded for. Thus, the orientation of the recorded seismic components is predetermined by the orientation of the recording instrument (accelerograph), which, is in general arbitrary (Fig. 1).

Let $\alpha_{x}(\mathrm{t}) \kappa \alpha \mathrm{l} \alpha_{\mathrm{y}}(\mathrm{t})$ represent the recorded ground accelerations at the position of the accelerograph along the axes $x$ and $y$ respectively. The same ground motion can be represented by components $\alpha_{x(\theta)}(\mathrm{t})$ and $\alpha_{\mathrm{y}(\theta)}(\mathrm{t})$ along another set of horizontal axes, which is defined by the angle $\theta$ with regard to the accelerograph axes $x$ and $y$ (Fig. 1). In other words, if the accelerograph had another orientation (e.g. $x(\theta), y(\theta)$ ) it would record the acceleration time histories $\alpha_{x(\theta)}$ and $\alpha_{y(\theta)}$. These components can be computed (Penzien 1975) with the aid of $\alpha_{x}$ and $\alpha_{\mathrm{y}}$ by using Eq. 1 :

$$
\binom{\alpha_{x(\theta)}(\mathrm{t})}{\alpha_{\mathrm{y}(\theta)}(\mathrm{t})}=\left[\begin{array}{ll}
\cos \theta & \sin \theta  \tag{1}\\
-\sin \theta & \cos \theta
\end{array}\right] \cdot\binom{\alpha_{\mathrm{x}}(\mathrm{t})}{\alpha_{\mathrm{y}}(\mathrm{t})}
$$

where $\alpha_{\mathrm{x}}(\mathrm{t}), \alpha_{\mathrm{y}}(\mathrm{t})$ are the recorded horizontal acceleration time histories along the axes x and y and the $\alpha_{\mathrm{x}(\theta)}(\mathrm{t}), \alpha_{\mathrm{y}(\theta)}(\mathrm{t})$ are the components of the transformed record when rotated counterclockwise by an angle $\theta$ (Fig. 1).


Figure 1. Recording angle of the ground motion and orientation of building structural axes.
In general, the two components $\alpha_{x}, \alpha_{y}$ or $\alpha_{x(\theta)}, \alpha_{y(\theta)}$ are correlated. The correlation factor $\rho$ is given by Eq. 2:

$$
\begin{equation*}
\rho=\frac{\sigma_{x y}}{\left(\sigma_{x x} \sigma_{y y}\right)^{1 / 2}} \quad \text {, with } \quad \sigma_{i j}=\frac{1}{\mathrm{~s}} \int_{0}^{\mathrm{s}} \alpha_{\mathrm{i}}(\mathrm{t}) \cdot \alpha_{\mathrm{j}}(\mathrm{t}) \mathrm{dt} ; \mathrm{i}=\mathrm{x}, \mathrm{y}, \tag{2}
\end{equation*}
$$

where $\sigma_{x x}, \sigma_{y y}$ are quadratic intensities of $\alpha_{x}(t)$ and $\alpha_{y}(t)$ respectively; $\sigma_{x y}$ is the corresponding cross-term; $s$ is the duration of the motion.

There is, however, a specific set of horizontal orthogonal axes, defined by the angle $\theta_{0}$ (Fig. 1), along which the correlation coefficient $\rho$ between the horizontal components of the ground motion is zero (Penzien 1975). The axes specified by angle $\theta_{0}$ represent the principal directions of the ground motion. The angle $\theta_{0}$ is called critical angle of ground motion and can be computed (Penzien 1975) by Eq. 3:

$$
\begin{equation*}
\tan 2 \theta_{0}=\frac{2 \sigma_{x y}}{\sigma_{x x}-\sigma_{y y}} \tag{3}
\end{equation*}
$$

## Maximum Response under Bi-directional Excitation

The structure is subjected to bi-directional horizontal seismic motion consisting of the accelerograms $\alpha_{\mathrm{x}(\theta)}(\mathrm{t})$ and $\alpha_{\mathrm{y}(\theta)}(\mathrm{t})$. As the direction of the seismic motion is unknown, they can form any angle $\theta^{s}$ with the $X$ and $Y$ structural axes (Fig. 2a). We consider two orientations of the seismic excitation:

- Excitation ' $\alpha 0$ ': The accelerograms $\alpha_{x(\theta)}(\mathrm{t})$ and $\alpha_{y(\theta)}(\mathrm{t})$ are applied simultaneously along the axes $X$ and $Y$ respectively, i.e. the angle of seismic incidence is $\theta^{s}=0^{\circ}$ (Fig. 2b). A typical response quantity R is denoted as $\mathrm{R},{ }_{a 0}$.
- Excitation ' $\alpha 90$ ': The accelerograms $\alpha_{x(\theta)}(\mathrm{t})$ and $\alpha_{y(\theta)}(\mathrm{t})$ are applied simultaneously along the axes Y and X respectively, i.e. the angle of seismic incidence is $\theta^{s}=90^{\circ}$ (Fig. 2c). A typical response quantity R is denoted as $\mathrm{R}, \alpha 90$.


Figure 2. Excitations ' $\alpha \theta$ ’ (a), ‘ $\alpha 0$ ’ (b) and ' $\alpha 90$ ’ (c).
It has been proved (Athanatopoulou 2005) that the maximum value of a response parameter for any angle $\theta$ of seismic incidence is given, as a function of time, by the relation:

$$
\begin{equation*}
\mathrm{R}_{0}(\mathrm{t})=\left[\mathrm{R},{ }_{\alpha 0}^{2}(\mathrm{t})+\mathrm{R},{ }_{\alpha 90}^{2}(\mathrm{t})\right]^{1 / 2} \tag{4}
\end{equation*}
$$

The plot of the function $\pm \mathrm{R}_{0}(\mathrm{t})$ provides the maximum/minimum value of the required response parameter as well as the time instant $\mathrm{t}_{\mathrm{cr}}$ at which the maximum/minimum occurs.

$$
\begin{equation*}
\max \mathrm{R}=+\mathrm{R}_{0}\left(\mathrm{t}_{\mathrm{cr}}\right) \quad \text { and } \quad \min \mathrm{R}=-\mathrm{R}_{0}\left(\mathrm{t}_{\mathrm{cr}}\right) \tag{5}
\end{equation*}
$$

The corresponding critical angles $\theta_{\mathrm{cr} 1}$ (maximum value) and $\theta_{\mathrm{cr} 2}$ (minimum value) are given by the relations (Athanatopoulou 2005):

$$
\begin{equation*}
\theta_{\mathrm{cr} 1}=\tan ^{-1}\left(\frac{\mathrm{R},{ }_{\alpha 90}\left(\mathrm{t}_{\mathrm{cr}}\right)}{\mathrm{R},{ }_{\alpha 0}\left(\mathrm{t}_{\mathrm{cr}}\right)}\right) \quad \text { and } \quad \theta_{\mathrm{cr} 2}=\theta_{\mathrm{cr} 1}-\pi \tag{6}
\end{equation*}
$$

The value of any other response parameter $\mathrm{R}^{\prime}$ at the time instant $\mathrm{t}_{\mathrm{cr}}$ for incident angle $\theta_{\text {cri }}$ $(\mathrm{i}=1,2)$ is computed by the relation:

$$
\begin{equation*}
\mathrm{R}^{\prime}\left(\theta_{\mathrm{cri}}, \mathrm{t}_{\mathrm{cr}}\right)=\mathrm{R}^{\prime},_{\alpha 0}\left(\mathrm{t}_{\mathrm{cr}}\right) \cdot \cos \theta_{\mathrm{cri}}+\mathrm{R}^{\prime},_{\alpha 90}\left(\mathrm{t}_{\mathrm{cr}}\right) \cdot \sin \theta_{\mathrm{cri}} \tag{7}
\end{equation*}
$$

## Methods of Selecting the Sectional Forces

## Method of Extreme Stresses ( $\mathbf{M S}_{\text {ex }}$ )

According to this method, which is proposed as the most rational one, two response history analyses, under bi-directional excitation for incident angles $\alpha=0^{\circ}$ (Fig. 2b) and $\alpha=90^{\circ}$ (Fig. 2c), are performed. The time histories of the response quantities $\mathrm{N}(\mathrm{t}),{ }_{\alpha 0}, \mathrm{M}_{\xi}(\mathrm{t}),{ }_{\alpha 0}$ and $\mathrm{M}_{\eta}(\mathrm{t}), \alpha 0$, as well as of $\mathrm{N}(\mathrm{t}), \alpha 90, \mathrm{M}_{\xi}(\mathrm{t}),{ }_{\alpha 90}, \mathrm{M}_{\eta}(\mathrm{t}),{ }_{\alpha 90}$ at any relevant cross section are computed.

Then, the time histories of the normal stresses $\left(\sigma_{A}(t),{ }_{\alpha 0}, \sigma_{B}(t),{ }_{\alpha 0}, \sigma_{C}(t),{ }_{\alpha 0}, \sigma_{D}(t),{ }_{\alpha 0}\right.$ and $\sigma_{A}(t),{ }_{\alpha 90}$, $\left.\sigma_{B}(\mathrm{t}),{ }_{\alpha 90}, \sigma_{C}(\mathrm{t}),{ }_{\alpha 90}, \sigma_{\mathrm{D}}(\mathrm{t}),{ }_{\alpha 90}\right)$ at the four corners $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D of a rectangular cross section are calculated. Finally, using Eqns. 4, 5 and 6, the maximum and minimum values of the stresses, the associated critical incident angles $\theta_{\mathrm{cr} 1}$ and $\theta_{\mathrm{cr} 2}$, as well as the time instant $\mathrm{t}_{\mathrm{cr}}$ are determined. The sectional forces corresponding to these maximum and minimum values of normal stresses are used for design purposes. The design combinations for any relevant rectangular cross section are presented in Table 1.

Table 1. Combinations of internal forces for method $\mathrm{MS}_{\mathrm{ex}}$.

| $\max _{\mathrm{A}}$ | $\mathrm{N}, \max \sigma \mathrm{A}$ | $\mathrm{M}_{\xi}, \max \sigma \mathrm{A}$ | $\mathrm{M}_{\mathrm{\eta}}, \max \boldsymbol{A}$ |
| :---: | :---: | :---: | :---: |
| $\min ^{\text {A }}$ | N, minoA | $\mathrm{M}_{\xi}$, minoA | $\mathrm{M}_{\eta}$, minoA |
| $\max ^{\text {a }}$ | $\mathrm{N},{ }_{\text {max }} \mathrm{B}$ | $\mathrm{M}_{\xi}$, maxob | $\mathrm{M}_{\eta}$, maxob |
| $\min ^{\text {b }}$ | $\mathrm{N},{ }_{\text {minoB }}$ | $\mathrm{M}_{\xi}$, minoB | $\mathrm{M}_{\mathrm{n}}$, minoB |
| $\max _{\mathrm{C}}$ | $\mathrm{N}, \operatorname{maxoC}$ | $\mathrm{M}_{\xi}, \operatorname{maxaC}$ | $\mathrm{M}_{\eta}$, maxaC |
| min $\sigma_{C}$ | $\mathrm{N},{ }_{\text {minoC }}$ | $\mathrm{M}_{\xi}, \operatorname{minoC}$ | $\mathrm{M}_{\mathrm{\eta}}$, minoC |
| max ${ }_{\mathrm{D}}$ | $\mathrm{N}, \max ^{\text {a }}$ D | $\mathrm{M}_{\xi}, \max \sigma \mathrm{D}$ | $\mathrm{M}_{\mathrm{y}}$, maxad |
| min $\sigma_{\text {D }}$ | N, mincD | $\mathrm{M}_{\xi}$, minoD | $\mathrm{M}_{\mathrm{n}}$, minoD |

## Method of Maximum Absolute Forces for Angle $\boldsymbol{\alpha}=\mathbf{0}^{\mathbf{0}}\left(\mathbf{M F}_{\text {abs }} \mathbf{0}\right)$

According to this method the acceleration loads $\alpha_{x(\theta)}(\mathrm{t})$ and $\alpha_{\mathrm{y}(\theta)}(\mathrm{t})$ are applied simultaneously along the structural axes X and Y , respectively (excitation ' $\alpha 0$ ') (Fig. 2b) as codes specify. The maximum absolute values of the response parameters $N(t),{ }_{\alpha 0}, M_{\xi}(t),{ }_{\alpha 0}$ and $M_{\eta}(t),{ }_{\alpha 0}$ are used for design purposes. The sign of each parameter can be positive or negative. Any combination of these values can be considered as an unfavourable combination of the sectional internal forces. Hence, the eight unfavourable combinations of sectional internal forces presented in Table 2 are produced

Table 2. Combinations of internal forces for method $\mathrm{MF}_{\mathrm{abs}} 0$.

| $\max \left\|\mathrm{N},{ }_{\text {a }}\right\|$ | $\max \left\|\mathrm{M}_{\xi,, 00}\right\|$ | $\max \mid \mathrm{M}_{\mathrm{n},{ }_{\text {, }}{ }^{\prime} \mid}$ |
| :---: | :---: | :---: |
| $\max \left\|\mathrm{N},{ }_{\text {a }}\right\|$ | $\max \left\|\mathrm{M}_{\xi, \alpha 0}\right\|$ | $-\max \left\|M_{\eta, a 0}\right\|$ |
| $\max \left\|\mathrm{N},{ }_{\text {a }}\right\|$ | $-\max \left\|\mathrm{M}_{\xi, 00}\right\|$ | $\max \left\|\mathrm{M}_{\mathrm{n}, 0,0}\right\|$ |
| $\max \left\|\mathrm{N},{ }_{\text {a }}\right\|$ | $-\max \left\|\mathbf{M}_{\xi, 00}\right\|$ | $-\max \left\|M_{\eta, a 0}\right\|$ |
| $-\max \left\|\mathrm{N},{ }_{\text {a }}\right\|$ | $\max \left\|\mathrm{M}_{\xi,, 00}\right\|$ | $\max \left\|\mathrm{M}_{\mathrm{n}, 0.0}\right\|$ |
| $-\max \left\|\mathrm{N},{ }_{\alpha 0}\right\|$ | $\max \left\|\mathrm{M}_{\xi, 00}\right\|$ | $-\max \left\|M_{\eta, 00}\right\|$ |
| $-\max \left\|\mathrm{N},{ }_{0}\right\|$ | $-\max \left\|\mathbf{M}_{\xi, 00}\right\|$ | $\max \left\|\mathrm{M}_{\mathrm{n}, 0,0}\right\|$ |
| $-\max \left\|\mathrm{N},{ }_{\alpha 0}\right\|$ | $-\max \left\|\mathbf{M}_{\xi, \alpha 0}\right\|$ | $-\max \left\|M_{\eta, a 0}\right\|$ |

## Method of 30\% Rule (M30)

This method stems from FEMA356 and ASCE/41-06 provisions. According to this method two response history analyses, for uni-directional inputs $\alpha_{x(\theta)}(t)$ and $\alpha_{y(\theta)}(t)$ along the structural axes X and Y, respectively are performed. The time histories of the response quantities
$\mathrm{N}(\mathrm{t}),{ }_{\mathrm{x}}, \mathrm{M}_{\xi}(\mathrm{t}), \mathrm{x}$ and $\mathrm{M}_{\eta}(\mathrm{t}), \mathrm{x}$, as well as $\mathrm{N}(\mathrm{t}), \mathrm{y}, \mathrm{M}_{\xi}(\mathrm{t}), \mathrm{y}, \mathrm{M}_{\eta}(\mathrm{t}), \mathrm{y}$ at any relevant cross section are computed and their maximum absolute values are determined. Then the $30 \%$ directional combination rule is applied. The sets of internal forces for design purposes according to this method for any relevant cross section are presented in Table 3.

Table 3. Combinations of internal forces for method M30.

|  |  |  |
| :---: | :---: | :---: |
| ax $\mathrm{N}^{1}$ | $\max \left\|\mathrm{M}_{6}\right\|-03 \mathrm{max} \mid \mathrm{M}_{\text {col }}$ | $\max \left\|\mathrm{M}_{n}, \mathrm{x}\right\|-0.3 \max \left\|\mathrm{M}_{\mathrm{n}, \mathrm{y}}\right\|$ |
| $-\max \|\mathrm{N}, \mathrm{x}\|+0.3 \max \|\mathrm{~N}, \mathrm{y}\|$ | $-\max \left\|\mathbf{M}_{\xi, \mathrm{x}}\right\|+0.3 \max \left\|\mathbf{M}_{\xi, \mathrm{y}}\right\|$ | -max ${ }^{\text {a }}+0.3 \mathrm{max}$ |
| $-\max \|\mathbf{N}, \mathrm{x}\|-0.3 \max \|\mathbf{N}, \mathrm{y}\|$ | $-\max \left\|\mathbf{M}_{\xi, \mathrm{x}}\right\|-0.3 \mathrm{max}\left\|\mathrm{M}_{\xi, \mathrm{y}}\right\|$ | $-\max \left\|\mathrm{M}_{\mathrm{n}, \mathrm{x}}\right\|-0.3 \mathrm{max} \mid \mathrm{M}_{1}$ |
| $0.3 \mathrm{max}\|\mathrm{N}, \mathrm{x}\|+\mathrm{max}\|\mathrm{N}, \mathrm{y}\|$ | $0.3 \max \left\|\mathrm{M}_{\xi, \mathrm{x}}\right\|+\max \mid \mathrm{M}_{\xi}$ | 3max $\left\|\mathbf{M}_{\eta, x}\right\|+\max \left\|\mathbf{M}_{\eta, y}\right\|$ |
| $0.3 \max \left\|\mathbf{N},{ }_{x}\right\|-\max \|\mathrm{N}, \mathrm{y}\|$ | $0.3 \max \left\|\mathbf{M}_{\xi, \mathrm{x}}\right\|-\max \left\|\mathrm{M}_{\xi, \mathrm{y}}\right\|$ | $0.3 \max \left\|\mathrm{M}_{\mathrm{\eta}, \mathrm{x}}\right\|-\max \left\|\mathrm{M}_{\eta, y}\right\|$ |
| -0.3max $\|\mathrm{N}, \mathrm{x}\|+\max \|\mathrm{N}, \mathrm{y}\|$ | -0.3max $\left\|\mathrm{M}_{\xi, \mathrm{x}}\right\|+\max \mid \mathrm{M}_{\xi}$, , | -0.3max $\left\|\mathrm{M}_{\mathrm{y}, \mathrm{x}}\right\|+\max \mid \mathrm{M}_{\mathrm{y}}$, |
| - Nax | -0 |  |

## Applications

Two structural models are considered in this study. Each model represents a single-story reinforced concrete building (Fig. 3). The first model is a torsionally balanced system ( $\mathrm{E}_{\mathrm{s}}=0, \mathrm{E}_{\mathrm{s}}$ is the structural eccentricity), while for the second one it is considered that the Mass Centre CM is located on the X -axis at a distance $\mathrm{E}_{\mathrm{s}}$ from the Rigidity Centre CR. For the mass eccentricity is chosen: $\mathrm{e}_{\mathrm{s}}=\mathrm{E}_{\mathrm{s}} / \mathrm{L}=0.2$. The beam and column dimensions, as well as the mass and the material properties are listed in Fig. 3, where $f_{c}=$ concrete strength, $f_{y}=y i e l d$ strength of the reinforcing steel and $E_{c}=$ the concrete modulus of elasticity.


Figure 3. Structural model (CM: Mass Centre; CR: Rigidity Centre).

Each one of the two models was analyzed for the two horizontal components of each ground motion shown in Table 4. Ground motions were recorded on site class D of FEMA356. The accelerograms were scaled so as to match the design spectrum of the Greek Seismic Code (EAK2003) for Peak Ground Acceleration PGA=0.36g and behavior factor $\mathrm{q}=3.5$. The scaling was performed according to the procedure suggested in FEMA356. Each ground motion was represented by: i) the two horizontal recorded components; ii) the recorded components transformed to other sets of axes forming an angle $\theta=30^{\circ}, 60^{\circ}, \ldots, 360^{\circ}$ with respect to the initial ones and (c) the recorded accelerograms transformed to the principal directions of the ground motion. For all these cases the two horizontal accelerograms were imposed along the structural axes and the longitudinal steel area at all critical cross sections was calculated using the three aforementioned methods.

Table 4. Ground motions recorded on soil type D of FEMA356.

| No | Date | Earthquak e name | Station name | Station number | Component (deg) | PGA (g) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & 17 / 1 / 1 \\ & 994 \end{aligned}$ | Northridge | Los Angeles, Hollywood Storage Bldg. | 24303 | 360 | 0.358 |
|  |  |  |  |  | 90 | 0.231 |
| 2 | $\begin{aligned} & 17 / 1 / 1 \\ & 994 \end{aligned}$ | Northridge | Santa Monica City Hall | 24538 | 360 | 0.370 |
|  |  |  |  |  | 90 | 0.883 |
| 3 | $\begin{aligned} & 18 / 10 / \\ & 1989 \end{aligned}$ | Loma Prieta | Gilroy \#3, Sewage Treatment Plant | 47381 | 0 | 0.555 |
|  |  |  |  |  | 90 | 0.367 |
| 4 | $\begin{aligned} & 18 / 10 / \\ & 1989 \end{aligned}$ | Loma Prieta | Agnews, Agnews State Hospital | 57066 | 0 | 0.172 |
|  |  |  |  |  | 90 | 0.159 |
| 5 | $\begin{aligned} & 15 / 10 / \\ & 1979 \end{aligned}$ | Imperial Valley | Calexico, Fire Station | 5053 | 225 | 0.275 |
|  |  |  |  |  | 315 | 0.202 |
| 6 | $\begin{aligned} & 24 / 4 / 1 \\ & 984 \end{aligned}$ | Morgan Hill | Gilroy \#4, 2905 Anderson Rd | 57382 | 270 | 0.224 |
|  |  |  |  |  | 360 | 0.348 |
| 7 | $\begin{aligned} & 24 / 4 / 1 \\ & 984 \end{aligned}$ | Morgan Hill | Gilroy \#3, Sewage Treatment Plant | 47381 | 0 | 0.194 |
|  |  |  |  |  | 90 | 0.200 |

In order to better quantify the differences among the results produced for the 12 orientations of the recorded ground motions, the relative variation is defined as:

$$
\begin{equation*}
R V_{\theta}=\frac{\mathrm{A}_{\mathrm{s}, \theta}-\mathrm{A}_{\mathrm{s}, 0}}{\mathrm{~A}_{\mathrm{s}, 0}} \cdot 100(\%) \tag{8}
\end{equation*}
$$

where $A_{s, \theta}$ or $A_{s, 0}$ : the required reinforcement area for recording angle $\theta$ or $\left(0^{\circ}\right)$.
Fig. 4 illustrates the variation of the reinforcement steel ratio and the $\mathrm{RV}_{\theta}$ with regard to the recording angle for column C13 (bottom) of the mass eccentric system ( $\mathrm{e}_{\mathrm{s}}=0.2$ ) under earthquake record No 3. The black vertical line corresponds to the principal directions of the ground motion. From the figure it is apparent that the required reinforcement is strongly affected by the recording angle when methods $\mathrm{MF}_{\text {abs }} 0$ and M 30 are used. However, it is important to notice that the required reinforcement is not influenced by the orientation of the recorded ground motion when method $\mathrm{MS}_{\mathrm{ex}}$ is used.


Figure 4. a) Influence of the orientation of recorded ground motion components on the reinforcement steel ratios ( $\rho$ ) and b) $\mathrm{RV}_{\theta}(\%)$ for column C13 (bottom) of the mass eccentric system ( $\mathrm{e}_{\mathrm{s}}=0.2$ ) under earthquake record No 3

Furthermore, to facilitate comparisons, the Maximum Relative Variation MRV, the Maximum Relative Variation with regard to the principal directions ( $\mathrm{MRV}_{\mathrm{pr}}$ ) and the Relative Variation with regard to method $\mathrm{MS}_{\mathrm{ex}}\left(\mathrm{RV}_{\mathrm{MSex}}\right)$ for every structural element and earthquake record are introduced:

$$
\begin{align*}
& \mathrm{MRV}_{, \mathrm{i}}=\frac{\max \mathrm{A}_{\mathrm{s}, \mathrm{i}}-\min \mathrm{A}_{\mathrm{s}, \mathrm{i}}}{\min \mathrm{~A}_{\mathrm{s}, \mathrm{i}}} \cdot 100(\%)  \tag{9}\\
& \mathrm{MRV}_{\mathrm{pr}, \mathrm{i}}=\frac{\max \mathrm{A}_{\mathrm{s}, \mathrm{i}}-\min _{\mathrm{s}, \mathrm{i}}^{\mathrm{pr}}}{\min _{\mathrm{s}, \mathrm{i}}^{\mathrm{pr}}} \cdot 100(\%)  \tag{10}\\
& \mathrm{RV}_{\mathrm{MSex}}=\frac{\min _{\mathrm{s}, \mathrm{i}}-\mathrm{A}_{\mathrm{s}}^{\mathrm{MSex}}}{\mathrm{~A}_{\mathrm{s}}^{\mathrm{MSex}}} \cdot 100(\%) \tag{11}
\end{align*}
$$

where i: method $\mathrm{MF}_{\mathrm{abs}} 0$ and M 30 ; $\operatorname{maxA}_{\mathrm{s}, \mathrm{i}}$ and $\min \mathrm{A}_{\mathrm{s}, \mathrm{i}}$ : the maximum and the minimum reinforcement area produced by method i for any recording angle $\theta$, respectively. Moreover, $\mathrm{A}_{\mathrm{s}, \mathrm{i}}{ }^{\mathrm{pr}}$ and $\mathrm{A}_{\mathrm{s}}{ }^{\text {MSex }}$ are the reinforcement area for the principal components of the ground motion produced by method i and the reinforcement area produced by the method $\mathrm{MS}_{\mathrm{ex}}$, respectively.

Tables 5 and 6 present the average values of MRV, $\mathrm{MRV}_{\mathrm{pr}}$ and $\mathrm{RV}_{\mathrm{MSex}}$ for all the earthquake records considered. The results are tabulated separately for the two methods $\left(\mathrm{MF}_{\text {abs }} 0\right.$ and M30) used and for the two buildings examined.

Table 5. Average values of $M R V$ and $M R V_{p r}$ for all earthquake records considered.

|  | MRV |  |  |  | MRV ${ }_{\text {pr }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{e}_{\mathrm{s}}=0$ |  | $\mathrm{e}_{\mathrm{s}}=0.2$ |  | $\mathrm{e}_{\mathrm{s}}=0$ |  | $\mathrm{e}_{\mathrm{s}}=0.2$ |  |
|  | $\begin{gathered} \mathrm{MF}_{\mathrm{abs}} \\ 0 \end{gathered}$ | M30 | $\begin{gathered} \mathrm{MF}_{\mathrm{abs}} \\ 0 \end{gathered}$ | M30 | $\begin{gathered} \mathrm{MF}_{\mathrm{abs}} \\ 0 \\ \hline \end{gathered}$ | M30 | $\begin{gathered} \mathrm{MF}_{\mathrm{abs}} \\ 0 \\ \hline \end{gathered}$ | M30 |
| BX1(a) | 85.40 | 85.40 | 39.79 | 38.67 | 3.00 | 3.00 | 13.79 | 3.18 |
| BX4(a) | 84.24 | 84.24 | 72.41 | 72.37 | 2.89 | 2.89 | 3.86 | 2.94 |
| BY1(a) | 85.39 | 85.39 | 98.41 | 98.41 | 83.19 | 83.19 | 89.29 | 89.29 |
| BY4(a) | 84.22 | 84.22 | 79.81 | 79.81 | 81.95 | 81.95 | 73.28 | 73.28 |
| C1(b) | 14.45 | 24.57 | 47.65 | 71.05 | 6.76 | 5.79 | 39.81 | 59.66 |
| C2(b) | 15.55 | 24.33 | 41.61 | 45.77 | 6.80 | 3.32 | 33.38 | 34.33 |

Table 6. Average values of $\mathrm{RV}_{\mathrm{MSex}}$ for all earthquake records considered.

| section |  | BX1(a) | BX4(a) | BY1(a) | BY4(a) | C1(b) | C2(b) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{e}_{\mathbf{s}}=\mathbf{0}$ | $\mathrm{MF}_{\text {abs }} 0$ | -43.69 | -43.43 | -43.69 | -43.43 | 9.79 | 8.41 |
|  | M 30 | -43.69 | -43.43 | -43.69 | -43.43 | -25.15 | -25.51 |
| $\mathbf{e}_{\mathbf{s}}=\mathbf{0 . 2}$ | $\mathrm{MF}_{\text {abs }} 0$ | -28.50 | -40.64 | -47.09 | -41.95 | -23.00 | -16.84 |
|  | M 30 | -24.11 | -37.09 | -47.09 | -41.95 | -37.34 | -25.87 |

From Table 5 it is apparent that MRV can attain large values (up to $98.41 \%$ for beam BY1(left joint) and $71.05 \%$ for column C1(bottom)) depending on the structural element and on the mass eccentricity of the building. Of particular interest is the fact that the MRV of the beams is much larger than that of the columns for the torsionally balanced system. However, with increasing the mass eccentricity of the building the values of the columns' MRV tend to increase. Moreover, Table 5 indicates that MRV $_{\text {pr }}$ is not significant for beams BX1 (left joint) and BX4 (left joint). This observation is valid for the vast majority of the beams which are parallel to the X structural axis regardless of the building's mass eccentricity. On the other hand, beams which are parallel to the Y axis appear to have large values of $\mathrm{MRV}_{\mathrm{pr}}$. Concerning the columns of the torsionally balanced system, it is shown that the application of the uncorrelated seismic components along the structural axes of the building leads to reinforcement which is close to the maximum reinforcement for any recording angle. However, when $\mathrm{e}_{\mathrm{s}}=0.2$, the principal seismic components produce reinforcement steel areas which are smaller (up to 59.66\% for C 1 (bottom)) than the maximum required reinforcement. It must be noticed that the above conclusions are valid for both methods $\mathrm{MF}_{\text {abs }} 0$ and M 30 used to determine the longitudinal reinforcement steel areas.

From Table 6 it can be deduced that the minimum reinforcement steel area produced by methods $\mathrm{MF}_{\text {abs }} 0$ and M 30 can be small enough (up to $47.09 \%$ for beam BY1 (left joint) and
$37.34 \%$ for column C 1 (bottom)) with regard to the reinforcement determined by method $\mathrm{MS}_{\mathrm{ex}}$. Concerning the columns, it is shown that method M30 produces smaller values of $\mathrm{RV}_{\mathrm{MSex}}$ than those determined by method $\mathrm{MF}_{\text {abs }} 0$.

## Conclusions

The conclusions derived from the present study can be summarized as follows:

- The reinforcement steel area depends on the orientation of the ground motion when methods $\mathrm{MF}_{\text {abs }} 0$ and M30 are used. Thus, ignoring the influence of the recording angle may lead to a significant underestimation of seismic demands. Particular attention must also be paid to the fact that the influence of the recording angle on the columns' response and reinforcement tends to be stronger as the mass eccentricity of the building increases.
- Method $\mathrm{MS}_{\mathrm{ex}}$, which is here proposed as the most rational one, leads to results that are not influenced by the orientation of the recorded ground motion.
- For the majority of structural elements neither the recording correlated accelerograms nor the corresponding uncorrelated ones lead to conservative results, if they are applied along the structural axes.


## References

ASCE 41/06. American Society of Civil Engineers, 2009. Seismic Rehabilitation of Existing Buildings.
Athanatopoulou, A.M., 2005. Critical orientation of three correlated seismic components, Engineering Structures 27, 301-12.

EAK 2003, Ministry of Environment, Planning and Public Works, 2003. Greek Code for Earthquake Resistant Design of Structures, Greece

FEMA 356, Federal Emergency Management Agency, 2000. Prestandard and commentary for the seismic rehabilitation of buildings, Washington, DC.

FEMA 440, Federal Emergency Management Agency, 2004. Improvement of nonlinear static seismic analysis procedures, Washington, DC.

Kostinakis K.G., Athanatopoulou A.M., Avramidis I.E., 2009. Influence of the orientation of seismic records on structural response, IABSE Symposium, Bangkok, Thailand, paper No 051-02-01.

NEHRP (FEMA 450), Federal Emergency Management Agency, 2003. Recommended provisions for seismic regulations for new buildings and other structures, Part 1-provisions, Washington, DC.

Penzien J., and Watabe M., 1975. Characteristics of 3-D Earthquake Ground Motions, Earthquake Eng. Struct. Dyn., 3, 365-373.

SAP2000, Computers and Structures Inc, Berkeley.


[^0]:    ${ }^{1}$ Phd Candidate, Dept. of Civil Engineering, Aristotle University of Thessaloniki, Thessaloniki. Greece
    ${ }^{2}$ Associate Professor, Dept. of Civil Engineering, Aristotle University of Thessaloniki, Thessaloniki. Greece
    ${ }^{3}$ Professor, Dept. of Civil Engineering, Aristotle University of Thessaloniki, Thessaloniki. Greece

