Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 1043

GROUND MOTION PREDICTION EQUATION FOR YIELD STRENGTH AND INELASTIC DISPLACEMENT SPECTRA

Y. Bozorgnia[1], M.M. Hachem[2] and K.W. Campbell[3]

ABSTRACT

This paper presents the results of a comprehensive ground motion prediction equation (GMPE, or "attenuation" relationship) developed for inelastic response spectra. Inelastic spectra for over 3100 horizontal ground motions recorded in 64 worldwide earthquakes are used to develop the GMPE, which is used in both deterministic and probabilistic hazard analyses to directly generate inelastic spectra. Our analysis reveals that over a wide structural period range, the magnitude scaling for an inelastic system is higher than that for an elastic system, especially for ductility levels greater than 2 and magnitudes greater than 6.5. Both deterministic and probabilistic hazard analyses show that the "equal displacement rule", to estimate inelastic displacement, is valid for small to moderate magnitudes and/or for low ductility levels. However, it underestimates inelastic deformation even for long period structures if the earthquake magnitude is large and the structure needs to sustain a large ductility. We used a modified version of a standard probabilistic seismic hazard analysis (PSHA) package to directly generate probabilistic hazard for inelastic response. Examples of the PSHA results are presented in this paper.

Introduction

We developed a large database of inelastic response spectra using a subset of the PEER NGA strong motion database (Bozorgnia, et al., 2010a). Our selected database includes over 3100 horizontal records from 64 earthquakes with moment magnitudes ranging from 4.3–7.9 and rupture distances ranging from 0.1–199 km. Using this comprehensive database of "constant ductility" spectra, we developed ground motion prediction equations (GMPEs) for different levels of displacement ductility ranging from one (i.e., elastic response) to eight. For each ductility level, the GMPE correlates inelastic spectra to earthquake magnitude, fault mechanism, fault distance, local soil conditions, and basin (sediment) depth.

The subject structural model is a single-degree-of-freedom (SDF) inelastic system with

[1]Executive Director, Pacific Earthquake Engineering Research Center (PEER), University of California, Berkeley, CA 94720
[2] Senior Structural Engineer, Skidmore, Owings and Merrill LLP, San Francisco, CA 94111
[3] Senior Vice President, EQECAT, Inc., Oakland, CA

an elastic-perfectly-plastic force-deformation relationship. The key parameters of the inelastic system are defined as follows: yield strength F_y; yield deformation u_y; and maximum deformation u_{max} (relative to the ground). The system ductility is defined as $\mu = u_{max}/u_y$. We used a 5% viscous damping ratio.

Using the developed GMPEs, we deterministically and probabilistically predict yield strength and inelastic displacement spectra. Examples of comparisons of responses of elastic and inelastic systems over wide range of system and ground motion parameters are also presented in this paper.

Deterministic Prediction of Inelastic Spectra

The details of development of the GMPEs for inelastic spectra, including the tables of the regression coefficients, are found in Bozorgnia, et al. (2010a, 2010b). In this paper we present samples of the results.

Figure 1 presents deterministic prediction of yield strength spectra for a strike-slip fault with moment magnitude 7.5 at rupture distances 1 and 10 km from the seismic source at a site with shear-wave velocity V_{S30} =760 m/sec. The figure shows the predicted strength at various levels of displacement ductility. In general, similar to the cases presented in Figure 1, given the desired level of ductility, and fundamental parameters such as magnitude, site-to-source distance, local site condition and sediment depth, one can directly predict the inelastic strength demanded by the earthquake ground motion. It is important to note that in such a prediction, no simplifying assumption has been made to approximate inelastic strength demands from those of elastic response.

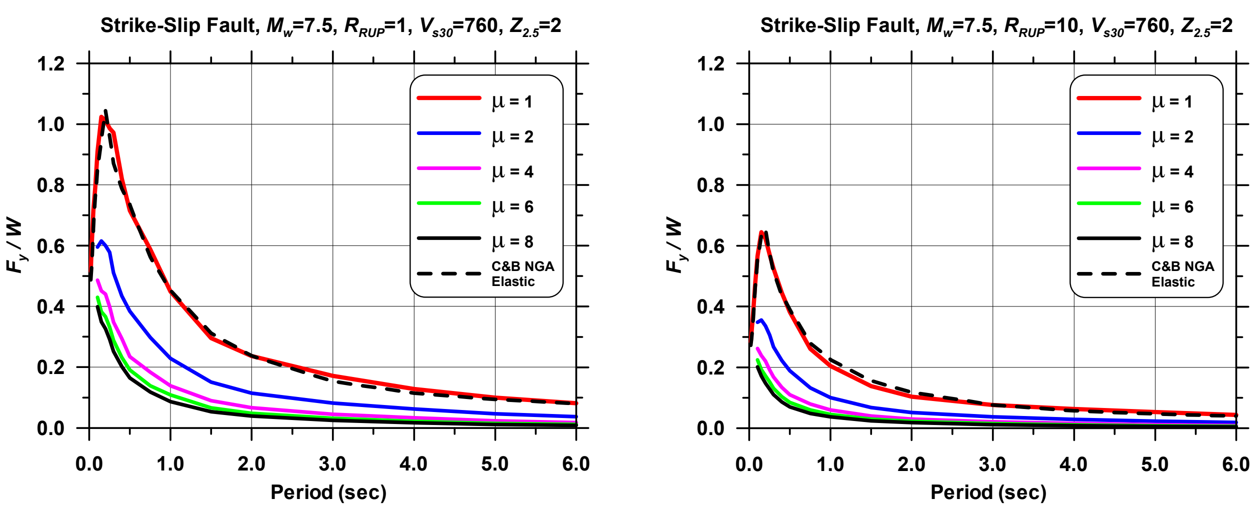

Figure 1. Median inelastic spectra for F_y/W at a site with V_{s30}=760 m/s located at rupture distances R_{RUP}=1 km (left), and 10 km (right) from a strike-slip fault for magnitude 7.5. Each case has spectra for ductility μ=1, 2, 4, 6 and 8. For comparison, median elastic NGA spectra predicted by Campbell & Bozorgnia (2008) are also plotted.

As expected, it is evident from Figure 1 that by allowing a moderate level of ductility (e.g., μ=2), a substantial reduction in the strength demand is resulted in.

Samples of the deterministic predictions of the maximum inelastic deformation are presented in Figure 2. This figure shows median values of the maximum displacements at a site located at 10 km from a strike-slip fault for moment magnitudes 6.5 and 7.5. The displacement spectra plotted in this figure are for μ=1, 2, 4, 6, and 8. For comparison, displacement spectra for elastic spectra predicted by the Campbell and Bozorgnia (2007, 2008) NGA relationships are also plotted in Figure 2. There is a difference between the displacement predicted for μ=1 and that by the NGA relationship. This is mainly due to smoothing of, and applying other constraints on, the regression coefficients in the NGA elastic GMPE (Campbell and Bozorgnia, 2007, 2008), especially at long periods. Figure 2 also shows that the shape of elastic and inelastic displacement spectra is a function of magnitude. By increasing magnitude, the spectral peak shifts to longer periods, as larger magnitude earthquakes tend to be richer in long-period ground motions than smaller events.

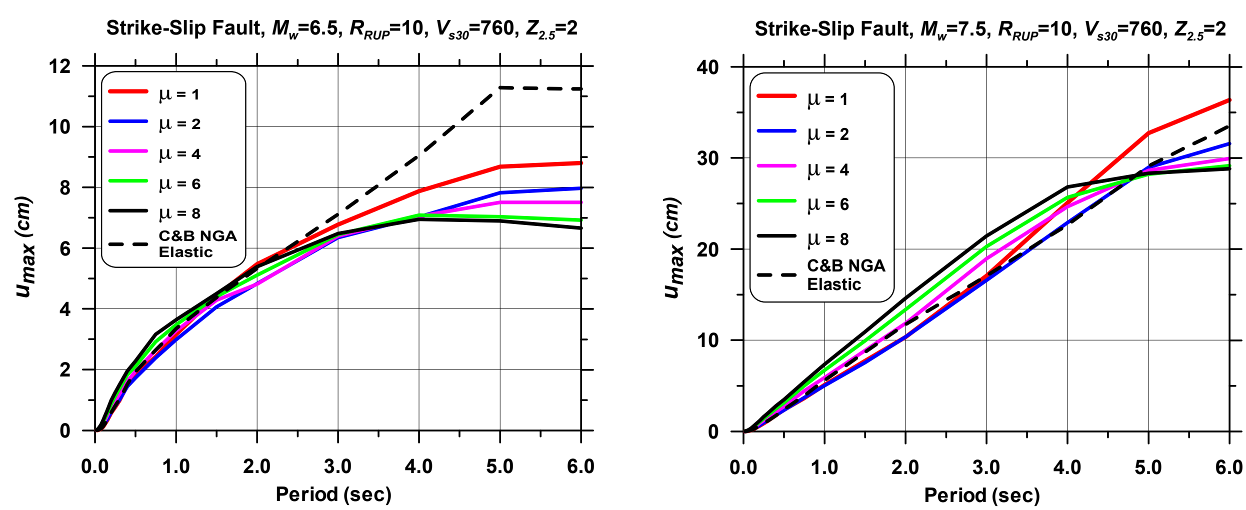

Figure 2. Median displacement spectra (u_{max}) at a site with V_{s30}=760 m/s located at rupture distance 10km from a strike-slip fault of magnitudes 6.5 (left) and 7.5 (right). Each frame has spectra for ductility μ=1, 2, 4, 6 and 8, and NGA elastic prediction by Campbell & Bozorgnia (2008).

To examine the relative differences between the spectra of inelastic and elastic displacements, the ratio of the maximum inelastic displacement over the displacement for μ=1 is plotted in Figures 3 for various magnitudes and ductility levels. This figure is for a distance of 10 km from the earthquake source. The figure shows that for periods longer than some "crossing period" the maximum displacement is less sensitive to ductility, and in fact the maximum displacement decreases with increasing ductility, and as a special case, the elastic displacement is larger than the inelastic displacement.

An important revelation in Figure 3 is that the crossing period is a function of earthquake magnitude, as it shifts to longer periods by increasing magnitude. The crossing period is around

0.5-1 sec, 3-4.5 sec, and 5-5.5 sec for magnitudes 5.5, 6.5, and 7.5, respectively. For periods shorter than the crossing period, the ratio of inelastic over elastic displacements increases with increasing the ductility, and shorter periods correspond to a wider difference between elastic and inelastic displacements.

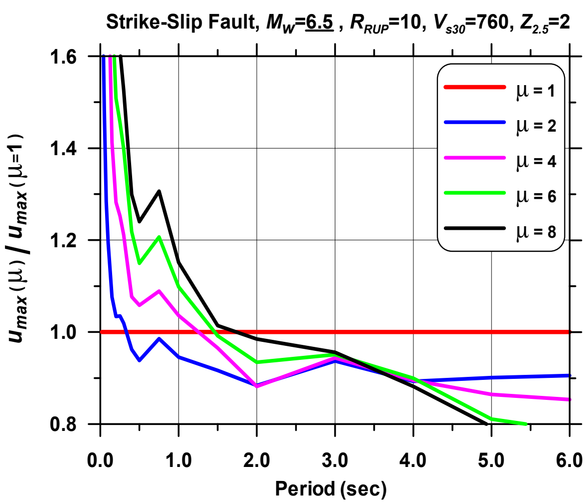

Figure 3. Ratio of median displacement for a ductility μ over that for μ=1, for magnitude 6.5 (left), and 7.5 (right) for R_{RUP}=10km.

We can definitely confirm the equal displacement rule of Veletsos-Newmark-Hall (Newmark and Hall, 1982); however, its period range of validity is a function of earthquake magnitude and ductility level. We should be especially careful in using the equal displacement rule if the earthquake magnitude is large and the structure needs to sustain a large ductility. For example for a magnitude of 7.5 and μ=6, the equal displacement rule can result in an unconservative estimate of the inelastic displacement over a wide period range shorter than about 3.0 sec. On the other hand, for the same earthquake, we can comfortably use the equal displacement rule for μ=2 and periods longer than about 0.5 sec.

An important difference between characteristics of elastic and inelastic systems is that over a wide period range, *magnitude scaling is higher at higher ductility levels,* and as a special case, magnitude scaling for inelastic systems (especially for μ>2) is higher than that for elastic systems (Bozorgnia, et al., 2010b). Consequently, it is generally unconservative to use the magnitude scaling model for an elastic system and apply it to an inelastic system, as is the common practice when performing seismic hazard analyses for elastic response spectra and using these results to predict inelastic response spectra. The degree of such an unconservatism depends on the level of ductility (Bozorgnia, et al., 2010b).

Probabilistic Prediction of Inelastic Spectra

In order to directly perform a probabilistic seismic hazard analysis (PSHA) on inelastic spectra, the developed GMPE for inelastic spectra is used as part of a PSHA package. One such PSHA computer package is OpenSHA (Field, *et al.*, 2003; OpenSHA, 2008). To demonstrate the concept, we used OpenSHA to carry out inelastic PSHA for a site in San Francisco, California, located about 15 km from the San Andreas Fault. Examples of PSHA results are presented in Figure 4. This figure shows the probability of exceedance in 50 years for the maximum inelastic deformation for period 1.0 and 3.0 sec. This figure reveals that the maximum deformation of an inelastic system is higher than that for an elastic system, even for long structural periods, especially for low probability (rare) events and large ductility demands. Therefore, for a relatively low probability of exceedance and large levels of ductility it is unconservative to carry out a PSHA for an elastic system and assume the constant displacement rule to estimate maximum displacement of an inelastic system.

In practice, one can use the hazard curves at a site, such as those presented in Figure 4, to directly estimate the maximum deformation demand on an inelastic system. This can be carried out by estimating the period and the global available ductility of the system, choosing the desired hazard level (probability of exceedance), and reading the estimate of maximum deformation from the hazard figure.

It should be noted that the effects of directivity pulses were not explicitly considered as part of our analysis.

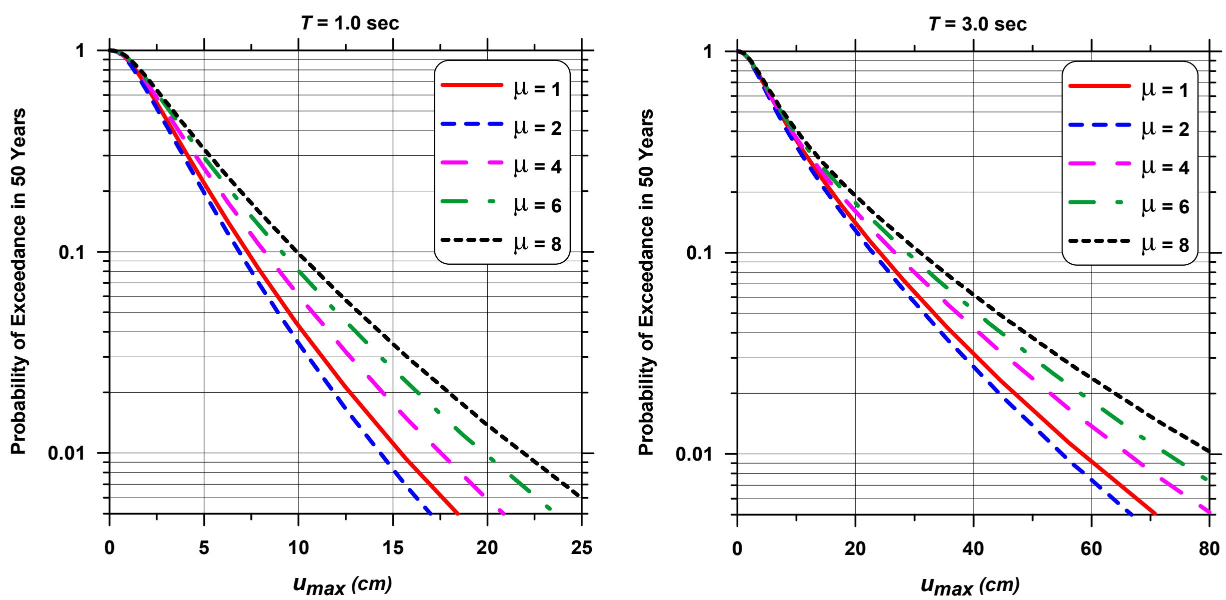

Figure 4. Results of PSHA for maximum deformation at a site in San Francisco, California, for ductility μ=1, 2, 4, 6 and 8, and for periods 1.0 sec (left), and 3.0 sec (right), for V_{s30}=760 m/sec, $Z_{2.5}$=1 km.

Summary and Conclusions

We developed a comprehensive ground motion prediction equation (GMPE, or "attenuation" relationship) for inelastic spectra, and we used this GMPE to directly perform deterministic and probabilistic seismic hazard analyses for inelastic response spectra. We used our inelastic GMPE in OpenSHA, an open source PSHA computer package, to perform a hazard analysis for inelastic response spectra for a site in San Francisco, California.

The newly developed GMPE for inelastic spectra and its implementation in hazard analyses have revealed that the period range of validity of "equal-displacement" rule is a function of earthquake magnitude and level of ductility. For small and moderate events, and/or for low ductility levels, the constant displacement rule works well over a wide period range. Inelastic displacement will, however, be underestimated if we use the equal displacement rule, even at long periods, for structures that need to sustain a large degree of ductility during a large magnitude earthquake.

We also observed that "magnitude scaling" of inelastic spectra, i.e., increase of spectral ordinates with increasing earthquake magnitude, is higher than that for elastic spectra, especially for $\mu>2$ and magnitudes greater than 6.5. It is, therefore, generally unconservative to use magnitude scaling of an elastic system and apply it to an inelastic system, especially for $\mu>2$.

Acknowledgements

This study was sponsored by the Pacific Earthquake Engineering Research Center (PEER). Any opinions, findings, conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect those of the funding agency.

References

Bozorgnia, Y., M.M. Hachem, and K.W. Campbell, 2010a. Ground motion prediction equation ("attenuation relationship") for inelastic response spectra. *Earthquake Spectra*, **26**, 1-23.

Bozorgnia, Y., M.M. Hachem, and K.W. Campbell, 2010b. Deterministic and probabilistic predictions of yield strength and inelastic displacement spectra. *Earthquake Spectra,* **26**, 25-40.

Campbell, K. W., and Y. Bozorgnia, 2007. *Campbell-Bozorgnia NGA ground motion relations for the geometric mean horizontal component of peak and spectral ground motion parameters*, PEER Report No. 2007/02, Pacific Earthquake Engineering Research Center, University of California, Berkeley, 238 pp.

Campbell, K. W. and Y. Bozorgnia, 2008. NGA ground motion model for the geometric mean horizontal component of PGA, PGV, PGD and 5% damped linear elastic response spectra for periods ranging from 0.01 to 10 s. *Earthquake Spectra*, **24**, 139-171.

Field, E.H., Jordan, T.H. and Cornell, C.A., 2003. OpenSHA: A developing community-modeling environment for seismic hazard analysis", *Seismological Research Letters,* **74**, 406-419.

Newmark, N.M. and W.J. Hall, 1982. *Earthquake Spectra and Design*. Monograph, Earthquake Engineering Research Institute, Berkeley, California.

OpenSHA, 2009. *Open Seismic Hazard Analysis*, http://www.opensha.org/