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# ESTIMATING SEISMIC DEMANDS ON GRAVITY-LOAD COLUMNS IN CONCRETE SHEAR WALL BUILDINGS

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## ABSTRACT

A simple procedure is presented for estimating the seismic demands on gravity-load columns in concrete shear wall buildings. The inelastic curvatures of concrete shear walls are assumed to vary linearly over a height related to the length of the wall, as is the shear strains in the wall due to yielding of vertical reinforcement in the presence of diagonal cracks. The gravity-load columns are assumed to have the same lateral displacement as the shear walls at the locations of the floor slabs. Gravity-load columns were found to exhibit a curvature profile very similar to that of the wall when no shear deformation of the wall existed. Wall shear deformation significantly increased maximum curvature demand on gravity-load columns. Damage of the columns due to loss of concrete cover and buckling of vertical reinforcement caused a concentration of curvature at the base of the columns that further increased the maximum curvature demand.

## Introduction

It has been a Canadian code requirement for about 25 years to check whether concrete gravity-load columns can tolerate the imposed deformations due to the design earthquake; but the way this has been done using linear analysis has significantly underestimated the demands over the plastic hinge region of concrete shear wall buildings. The seismic analysis of concrete buildings in Canada is usually done using response spectrum analysis (RSA) with a program such as ETABS. Such analyses can be used to estimate the top displacement of concrete shear wall buildings as long as an appropriate effective stiffness is used for the concrete wall; but cannot be used to estimate inelastic displacement profiles. Nonetheless, this method has been used by Canadian designers to estimate demands on gravity-load columns due to seismic deformations of the building.

Concern about the safety of gravity-load columns over the plastic hinge height of concrete shear walls, particularly elongated wall-like columns, has resulted in new design requirements being adopted in the August 2009 addendum of CSA A23.3-04 (Adebar et al., n.d). The revised clause includes a clear statement that the inelastic displacement profile of the concrete shear walls must be accounted for when estimating demands on gravity-load columns. The current paper describes a simplified procedure that can be used to conduct such nonlinear analyses, and presents results from some example analyses.

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## Shear Wall Deformation Profiles

Gravity-load columns are tied to concrete shear walls by the floor slabs, which are very flexible out-of-plane (bending); but essentially infinitely rigid in-plane. Assuming the predominate deformations of a building are in one of the two principle directions of the seismic force resisting system (consisting of shear walls arranged in two perpendicular directions), the lateral deformation of the gravity-load columns at the floor levels will be equal to the lateral displacements of the shear walls at those same levels. The displacement profile of a shear wall can be divided into flexural and shear portions. The displacements at the top of a wall are completely dominated by flexural deformations; however the shear deformations of a wall are very significant in the first few floors near the base of the wall. The flexural curvatures (slope change per unit height) of the wall are also maximum near the base. The scope of the current study is to estimate the demands on the gravity-load columns near the base of the wall where the flexural curvatures and shear strains of the wall are both maximum. These two wall deformations are discussed separately below.

### Wall Flexural Deformations

The maximum top displacement of a wall can be reasonably estimated using response spectrum analysis if an appropriate effective stiffness is used for the wall. This issue is examined in detail for high-rise concrete cantilever walls by Dezhdar and Adebar (2010). The relationship between the top wall displacements and the curvatures in the wall are reasonably well-known. The inelastic curvatures near the base of a cantilever shear wall typically vary linearly (Bohl and Adebar, 2010); but for simplicity, the inelastic curvatures are often assumed to be uniform over half the height that the inelastic curvatures actually vary linearly. The height over which the inelastic curvatures are assumed to be uniform, called the plastic hinge length $l_p$, typically varies from about 0.5 to 1.0 times the wall $l_w$. Thus the length of linear varying inelastic curvature denoted by $l_p$* typically varies from $l_w$ to $2l_w$.

While the inelastic curvature profile depends only on the maximum inelastic displacement demand at the top of the wall, the distribution of elastic curvatures depend on the lateral force profile. Concrete cantilever walls designed in Canada are expected to have maximum displacement demands that are from about 2 to about 5 times the yield displacement of the wall. Fig. 1 examines how the lateral force profile influences the total (elastic plus inelastic) curvature profile. The example shown in Fig. 1 is from a 20-storey shear wall subjected to a $1^{st}$ mode lateral force distribution, inverted triangular load, and a single point load at the top. The inelastic and elastic portions of the displacement at the top of the wall are equal (displacement ductility of 2.0). Fig. 1 demonstrates that the assumed elastic curvature profile in a cantilever concrete wall has little influence on the maximum total (elastic plus inelastic) curvatures of the wall.

### Wall Shear Deformations

Wall shear deformations are defined in the current study as the portion of wall lateral displacements in addition to that resulting from integrating curvatures over the wall height (flexural deformations). Shear strain in the wall and consequently shear deformation is significant when vertical reinforcement in the wall yields and there are diagonal cracks in the wall. This phenomenon becomes particularly important in the wall inelastic behavior region (i.e., first few stories above the base of the wall) where flexural displacements are small. Because shear strain is a consequence of yielding of vertical reinforcement in flexure, it is assumed to follow the same pattern as that of the wall inelastic curvature profile. That is, the shear strain is assumed to vary linearly from a maximum value at the base of the wall where the inelastic curvature is maximum, to zero at the point that inelastic curvatures equals zero. This simple model was developed by examining the results from a nonlinear finite element analysis of cantilever shear walls (Bazargani, P., 2011)

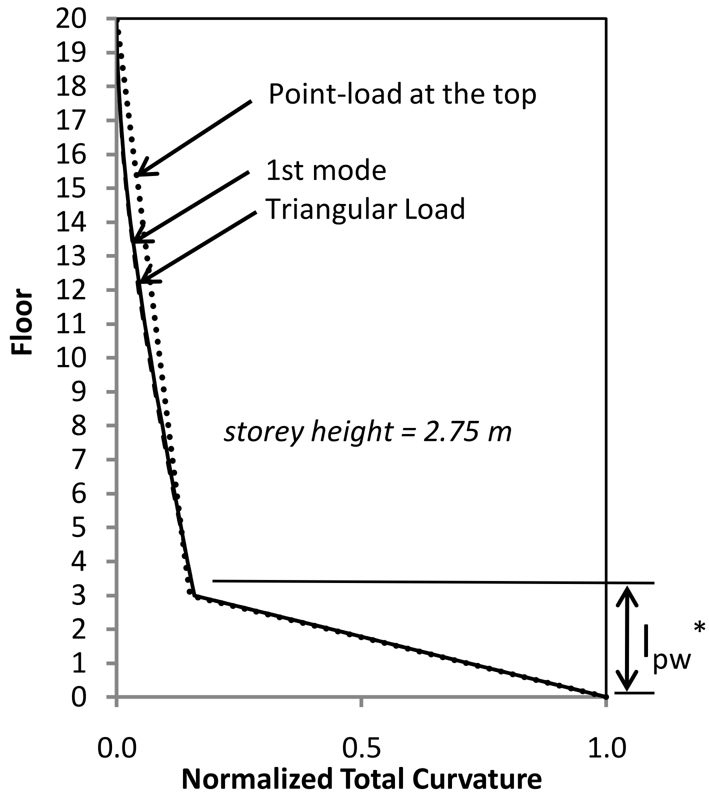


Figure 1. Total curvature profiles in the concrete cantilever walls in a 20-storey building with fundamental period of 1 s at displacement ductility of 2.0.

## Structural Model and Analysis Methodology

A nonlinear structural analysis algorithm was developed that given the moment-curvature response of the column and the imposed lateral displacement profile at floor levels, can be used to analyze the curvature demand throughout the height of the column. To analyze the column under a specified lateral displacement profile defined at floor levels, the column can be modeled as a cantilever member neglecting rotational stiffness provided by the slabs. Appropriate boundary conditions at the base and adequate number of floors above the base should be modeled to get an accurate estimate of the curvature demand. Target displacements are then increased with the correct proportions to carry out a pushover analysis. Modeling assumptions and structural analysis procedure are further discussed in the following sections.

## Moment-Curvature Response of Gravity-Load Columns

When a cantilever gravity column sustaining substantial axial compression is pushed with a horizontal point load at the top, bending moments along the height of the column induce a curvature profile that will result in the imposed slab displacement. At small tip loads, the column behaves almost as an elastic member distributing the curvatures linearly along its height. As the tip displacement (and the tip load) is increased, maximum bending moment at the base of the column becomes progressively greater causing the bottom part of the column to soften due to both softening of concrete and yielding of steel in compression (and possibly tension). This causes the lower parts of the column to become nonlinear while the parts near the top are still elastic which results in a curvature profile that is no longer linear; more curvature is concentrated at the base of the column and less in the upper parts. At the ultimate state just before failure, excessive curvatures at the base of the column can cause vertical steel bars to buckle under compression, concrete cover to spall off and steel bars to fracture in tension. Losing the concrete cover over a certain height and buckling of the outer layer of vertical reinforcement makes the base of the column even more flexible resulting in highly concentrated curvature in the damaged region. Damage is also in the form of diagonal cracking due to combined action of shear and flexure. Columns sustaining high axial loads are assumed to have no further drift capacity once maximum compressive concrete strain reaches 0.0035 (CSA A23.3-04, 10.1.3). Fig. 2 shows moment-curvature response of a 1220×305 mm column reinforced with 12 30M steel bars sustaining an axial load of 6000 kN ($0.4f_c'A_g$). Clear concrete cover was 40 mm.

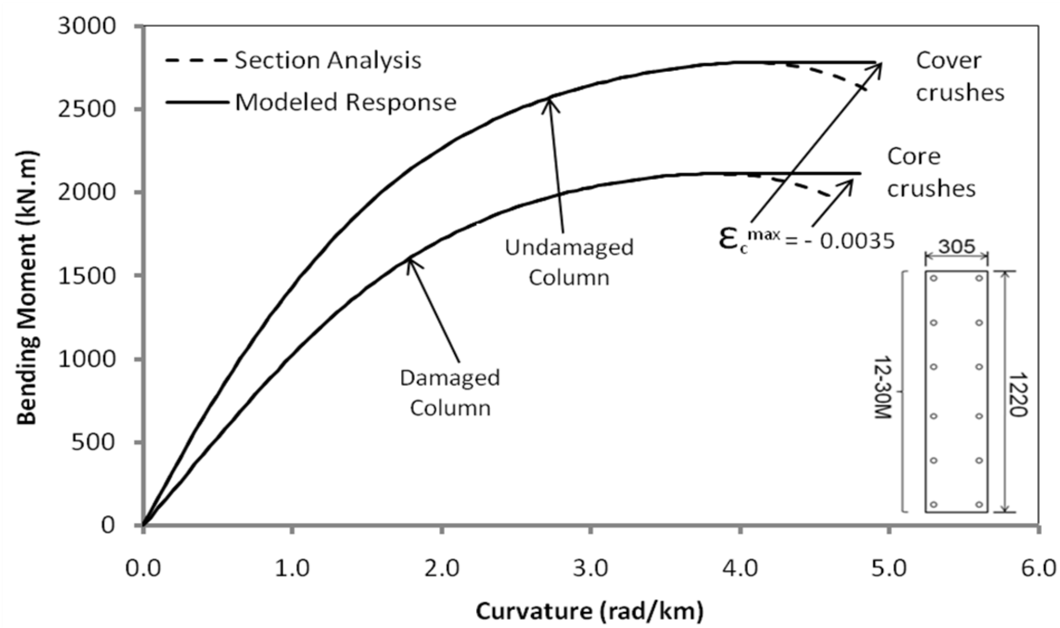


Figure 2. Moment-curvature response of gravity-load column, (note: damage of column includes spalling of concrete cover and buckling of outer reinforcement on compression face.)

If deformation capacity of gravity-load columns is to be studied, modeling curvature distribution in column's inelastic behaviour region is inevitable. A model is developed to account for inelastic curvature that gets accumulated in the plastic region and translates into significant inelastic rotation. This is done by assuming a certain curvature distribution along the height of the inelastic behaviour region and associating addition of curvature with certain stiffness.

As for the assumption on curvature distribution, it is best to select a model that is a close representation of how gravity-load columns behave when forced to deform laterally. For this purpose, two scenarios are considered (see Fig. 3). When the column cross-section does not experience damage in the inelastic behaviour zone and the section maintains its original dimensions, maximum bending moment occurs at the base of the column and inelastic curvatures are expected to be distributed approximately linearly over the column's plastic region. When the column suffers damaged in the inelastic behaviour zone (whether the damage is in the form of losing concrete cover, bar buckling under compression or bar fracturing under tension), the damaged section becomes much softer than the rest of the column causing even more curvature concentration in the damaged region. Sections in the damaged region have almost the same structural properties and a uniform crack pattern is observed over the height of the damaged zone. This suggests that assuming uniform inelastic curvature over the height of the damaged region is reasonable.

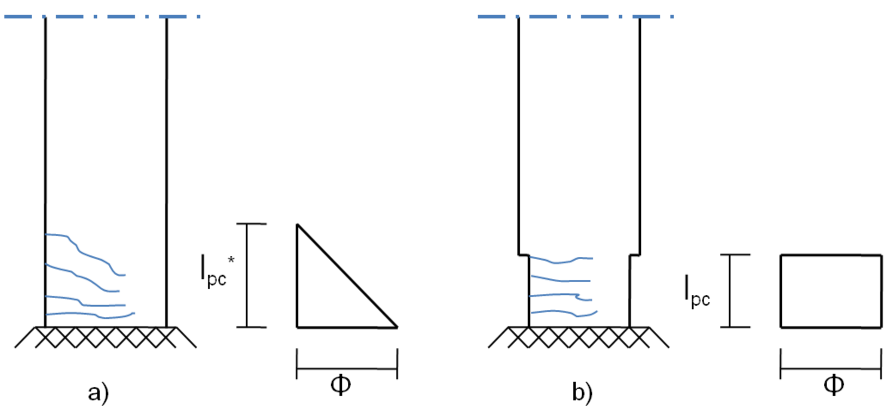


Figure 3. Assumptions of inelastic curvature due to concrete compression strains in: (a) undamaged column, (b) damaged column.

**Structural Analysis Procedure**

Fig. 4 shows a scheme of the idealized column structure for a 5-storey building modeled as a cantilever column pushed to certain lateral displacements ($\Delta$) at floor levels under action of slab forces ($P$). Storey forces produce bending moments ($M$) along the column height and from the bending moment diagram, curvature ($\Phi$) profile can be obtained using moment-curvature response of the column. Neglecting shear deformation of the column, curvatures can then be integrated using Eq. 1 to obtain the displacement profile along the height of the column.

$$\Delta(\mathrm{x}) = \int_0^x \Phi(\mathrm{x}).\,(\mathrm{H}-\mathrm{x})\ \mathrm{dx} \qquad (1)$$

The column can be divided into several equally-sized elements along the height of each storey to facilitate numerical integration. Curvature is considered to be constant over the height of the elements and is computed using bending moment at elements' mid-height. The problem in hand will then be finding the set of storey forces ($P$) which produce the target displacements at corresponding floor levels.

Floor displacements ($\Delta$) are considered to be a function of storey shear forces ($V$) as noted by Eq. 2. Note that $i$ and $j$ can assume any integer between 1 and 5.

$$\Delta_i = F_i(V_1, V_2, V_3, V_4, V_5) \tag{2}$$

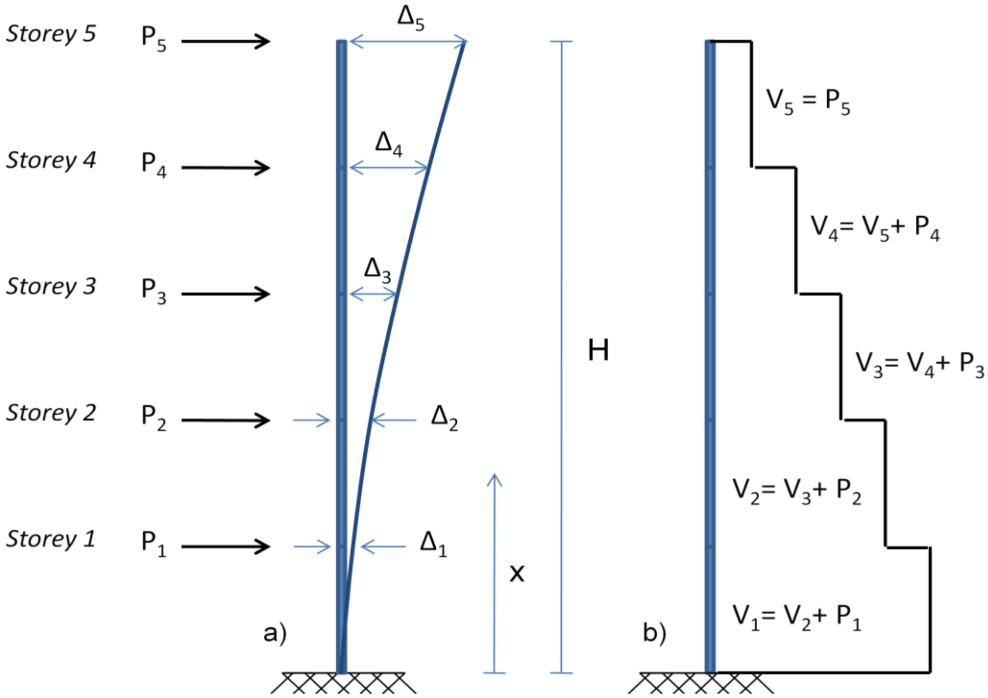


Figure 4. Idealized column structure: (a) storey forces and displacement profile, (b) shear force diagram.

First order Taylor series expansion (Eq. 3) is applied to the floor displacements ($\Delta_i$) and multi-variant Newton-Raphson iteration procedure is adapted to solve for the unknown storey shears ($V_i$) which will result in the target floor displacements. The following set of equations explains the iteration procedure.

$$\Delta_i^{t+1} = \Delta_i^t + \sum_{j=1}^{5} \frac{\partial Fi}{\partial Vj} |V^t (V_j^{t+1} - V_j^t) \tag{3}$$

Rearranging Eq. 3 gives,

$$\sum_{j=1}^{5} \frac{\partial Fi}{\partial Vj} |V^t\, V_j^{t+1} = \Delta_i^{t+1} - \Delta_i^t + \sum_{j=1}^{5} \frac{\partial Fi}{\partial Vj} |V^t\, V_j^t \tag{4}$$

In a matrix form, Eq. 4 can be written as

$$[F_{ij}][V_j]=[C_i] \tag{5}$$

Where $[F_{ij}]$ is a 5x5 matrix containing the derivatives of the storey displacements with respect to storey shear forces at step “t”, $[V_j]$ is the 5x1 vector of revised storey forces, and $[C_i]$ is the 5x1 vector of constants on the right hand side of Eq. 4. In each step, derivatives of storey displacements with respect to each floor shear force ($\frac{\partial Fi}{\partial Vj}$) are found numerically, that is, a unit (e.g. 1 kN) shift is applied to each storey shear force in turn and the variation in each floor displacement value is calculated (i.e. the fundamental definition of derivates).

To solve for the revised storey forces,

$$[V_j]= [F_{ij}]^{-1} [C_j] \tag{6}$$

Analysis is triggered at an arbitrary storey shear force vector $V_i^t$ (e.g. linearly varying storey shear forces along the height of the structure). Revised storey shear forces $V_i^{t+1}$ are then used to calculate the displacements in the next step and iteration process is carried on until acceptable convergence is achieved.

Analysis procedure just mentioned is valid only for the ascending part of the column moment-curvature response (see Fig. 2). In other words, for this algorithm to be able to find the storey forces resulting in the target displacements, column stiffness should be positive (i.e. additional bending moment needed for increase in curvature). Since for a uniform column fixed at the base maximum bending moment always occurs at the base of the column, this condition holds up until the element at the very bottom of the column reaches the column moment strength called the "yield" point after which column moment strength starts to decay due to compression softening of concrete. To model the strength decay portion of the column's flexural behaviour, a plastic model is used. The state of the column at "yielding" is recorded, that is, both the displacement and curvature profiles are stored. From thereafter, as the column is further pushed, to keep the maximum bending moment at the base constant at bending column strength, an inflection point is forced to occur at the base of the column. This ensures that no additional moment is added at the base while the fixed (zero rotation) boundary condition is satisfied. Based on the state of the column at yield, moment-curvature relation for each element is modified using a simple axis transformation technique and used to calculate elastic curvatures in excess of the state of the column at "yield"; stiffness of the elements adjacent to the base of the column is therefore taken into account. Additional floor displacements are now considered target displacements and additional storey forces and consequently additional curvatures are found to result in the desired displacement profile beyond the "yield" state.

In order to keep the maximum bending moment constant at the base of the column, the storey force at the first floor level is evaluated in terms of the rest of the storey forces ensuring no addition of moment at the base beyond the "yield" state of the column; maximum plastic curvature of the column is then treated as an independent variable. At the end of each pushover analysis step, bending moments and curvatures are all added up to obtain the total profiles along the height of the column.

Although the bending moment diagram of the column at the end of the analysis will differ from the real case (i.e. strength decay resulting in bending moment at the base of the column at failure being less than the column strength in reality), curvature profile is expected to represent the real case. Since the focus of this study is on quantifying curvature demand on gravity-load columns subject to specified lateral displacements at floor levels, obtaining the real bending moment profile is not of interest.

## Wall-Column Analysis Results

Pushover analysis results for the column cross-section shown in Fig. 2 tied to a 20-storey shear wall with storey height of 2.75m is presented in the following sections. The wall was 7.6m long with yield curvature of 0.35 rad/km and height of linearly varying inelastic curvature $l_p$* = 8.25m. Bilinear wall shear strains were added to the bilinear curvature distribution. To model column post-peak behavior, height of the linearly varying inelastic curvatures in the column $l_{pc}^*$ was assumed to be 1.2 m, which is equal to the column maximum cross sectional dimension. When damage was added to the base of the column, height of the constant inelastic curvatures ($l_{pc}$) was taken to be 0.305m equal to the column smallest dimension which is in turn equal to tie spacing when column is not detailed for seismic ductility. Column moment-curvature response is shown in Fig. 2. Note that the column "yields" due to compression strains at a curvature of 4.05 rad/km and fails at 4.95 rad/km.

**Effect of Wall Shear Strain on Column Curvature Demand**

Fig. 5 compares analysis results for the undamaged column in the presence and absence of wall shear deformation. When no wall shear deformation is included, the column tends to follow curvature distribution of the wall resulting in the maximum column curvature demand to be equal to the wall maximum curvature except for when the column nears "yielding" and failure where column curvature demand is slightly higher than maximum wall curvature. When significant wall shear deformation (e.g. max. shear strain of 0.003) is included, the maximum column curvature demand becomes increasingly larger than the wall maximum curvature.

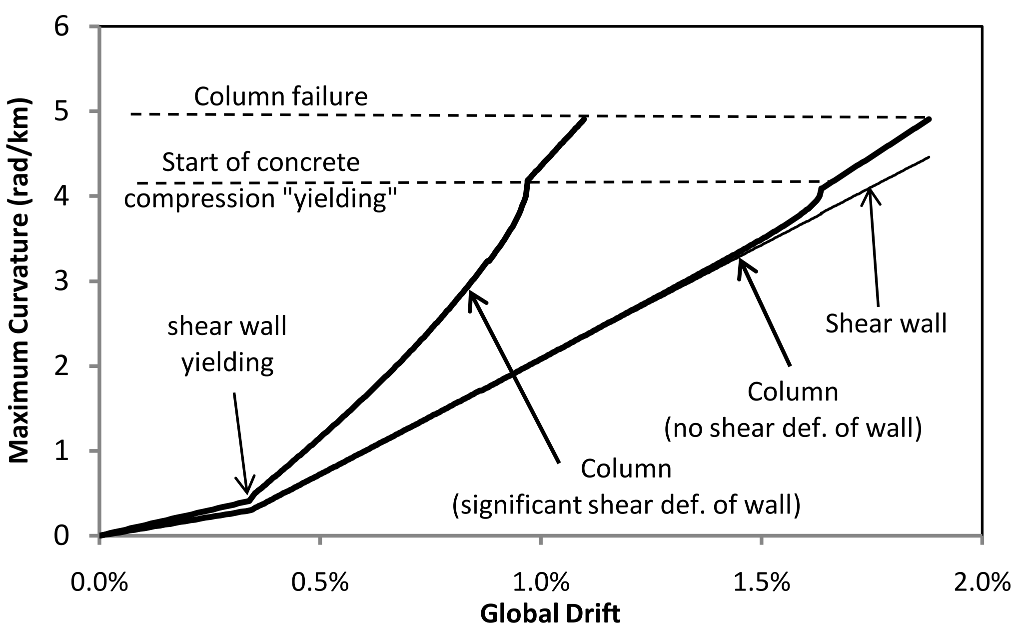


Figure 5. Pushover analysis results for undamaged column showing amplification of column maximum curvature demand due to significant shear deformation of wall.

Fig. 6 further explains the effect of wall shear deformation on curvature distribution along the height of the column. Wall shear deformation significantly increases the displacement at the first floor slab putting a high rotation demand on the column. Because the column cannot develop noticeable shear deformation, column curvatures tend to concentrate at the base to satisfy the increased rotation demand due to wall shear deformation which results in column curvature demand being much higher than wall maximum curvature.

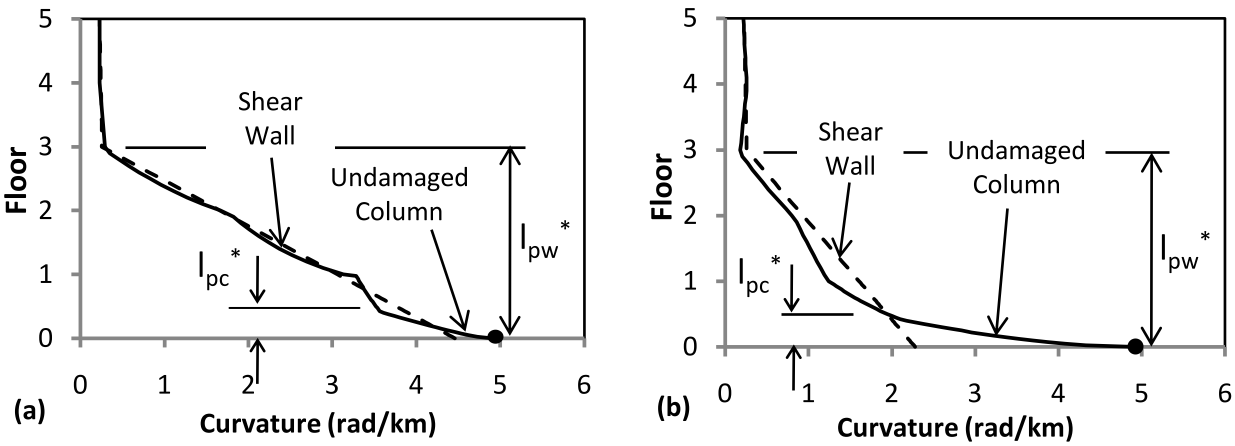


Figure 6. Curvature distribution of undamaged column in wall plastic hinge region at column failure: (a) no wall shear deformation, (b) significant wall shear deformation.

**Effect of Concentrated Damage at Base of Column**

Fig. 7 compares analysis results for the damaged column to that of the undamaged column shown in Fig. 5.

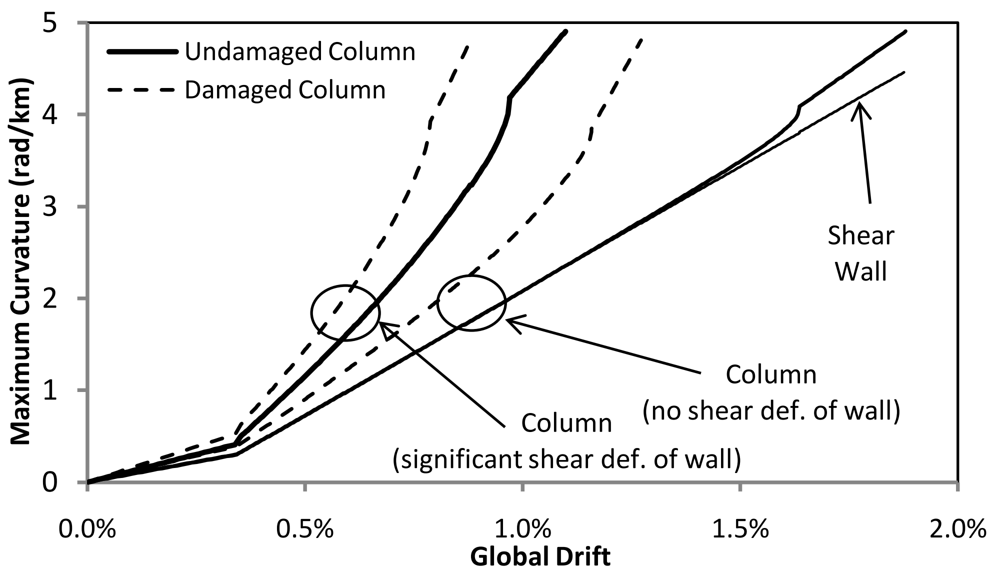


Figure 7. Comparison of pushover analysis results for damaged and undamaged columns, reduction in column drift capacity due to damage of the column.

Fig. 2 also shows the moment-curvature response of the damaged column. While the damage has reduced column stiffness and moment-strength by almost 25%, the damaged column "yields" and fails at nearly the same curvatures as the undamaged column.

Damage at the base of the column generally reduces column drift capacity which is due to localized accumulation of curvature in the softer section at the base of the column. Fig. 8 gives further insight into this phenomenon. Shown in Fig. 8 (a) is column curvature distribution at failure when no wall shear deformation was included. In this case, shear force in the first storey was reversed causing the maximum bending moment to occur at the top of the first floor. In this case, the column "yielded" at the top of the damaged zone. When significant wall shear deformation was included, shear force in the first storey was positive which caused maximum bending moment to occur at the base of the column (see Fig. 8 (b)). The relatively small height of the damaged zone caused the column to fail soon after "yielding".

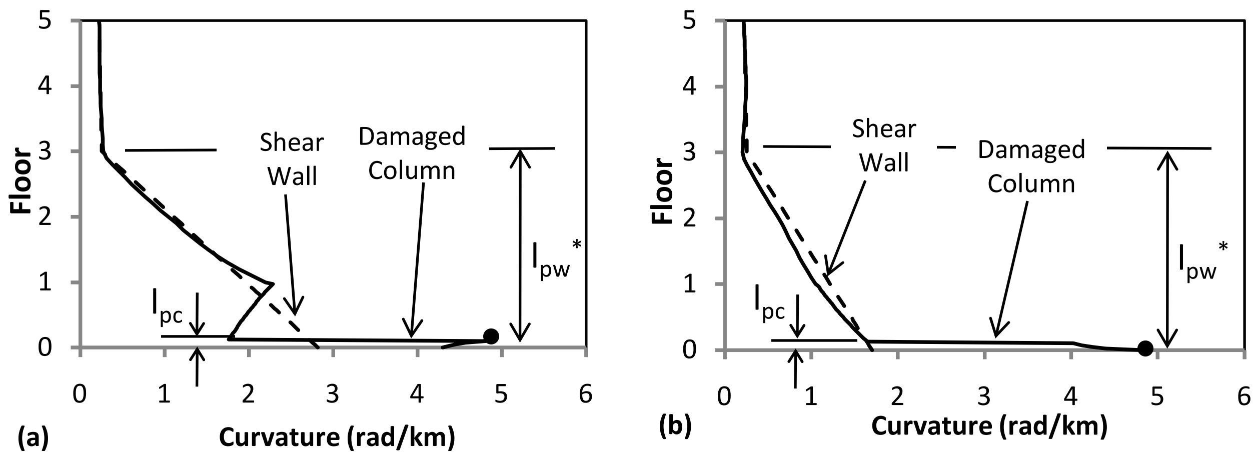


Figure 8. Curvature distribution of damaged column in wall plastic hinge region at column failure: (a) no wall shear deformation, (b) significant wall shear deformation.

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