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SYSTEM IDENTIFICATION FOR SEISMIC RESPONSE PREDICTION OF TORSIONALLY COUPLED BUILDING

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ABSTRACT

There is considerable research interest in the area of system identification of buildings using in-situ vibration measurements. The identification of system properties of the existing structures is important to evaluate structural damage and to ascertaining its present condition. The identified system parameters can be used to evaluate the expected structural responses to future earthquake excitations. The design and modification of structural control systems also needs knowledge of the system parameters of the structure. In this study, the modal model of the existing structure is extracted from the measured earthquake responses where the excitation measurement is not available. After determining structural dominant modal frequencies and damping ratios, the mode shapes are obtained at the instrumented floor level. Since the mode shapes are extracted only at instrumented floor level, a new mode shape estimation technique based on interpolated responses has been proposed by the authors to estimate the mode shape at other floor levels. Numerical investigation of an 8-story torsionally coupled building shows that the proposed system identification technique is able to identify dominant structural modal parameters even with the high-level noise content in the measurements. The numerical verification of the proposed system identification technique is conducted on a 6-story model building subjected to base excitations. Since the dominant frequencies, damping ratios, and the mode shapes are estimated accurately, the unmeasured floor responses can be determined with low error. The Finite Element model can be updated using the modal parameters extracted from the vibration records taken on the building. The updated model can be used to predict the response to future base excitations.

Introduction

Vibration monitoring is a well-known technique for damage identification and condition assessment of civil engineering structures. The modal parameters of a structure can be determined from the vibration data using system identification methods. The data recorded during seismic excitation can also be used for system identification and model updating methods. For structures damaged due to earthquakes, the change in strength due to retrofitting can be ascertained by comparing its dynamic properties identified before and after retrofitting. Due to economical considerations, the responses are commonly recorded at only few locations on the structure. This paper describes a method that uses interpolation techniques to increase the

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number of available responses for system identification for shear buildings subjected to base excitation. The responses at non-instrumented floor levels of building are reconstructed by interpolating using spline shape function. The recorded and reconstructed responses are used in system identification algorithm to extract the modal properties of the building (natural frequencies, damping ratios and mode shapes). A rigorous analysis procedure that extends the spline shape function method to shear building for identifying effective locations of sensors is used. The error in the reconstructed responses are defined by global minimax error and used in identifying locations of sensors on the building.

Traditional system identification techniques require measurement of input excitations and its corresponding responses (Kozin 1986). However, a real structure usually possesses a large number of degrees of freedom making it very expensive if not impossible to acquire full measurements of all significant degrees of freedom because of limited number of sensors. Thus, system identification based on response measurements of a few degrees of freedom becomes very useful from practical considerations. There are several different approaches to extract modal parameters from limited response measurements using general input (earthquake base excitation) and output (floor response to earthquake) method. The Eigen Realization Algorithm (ERA) and Observer/Kalman filter Identification (OKID) approach has been used to identify the modal parameter from earthquake induced time histories of the structural response (Lus et al. 1999). Comparative studies have highlighted the difference and similarities in current modal identification algorithms, viz. Least-Squares Complex Exponential (LSCE), the poly reference time domain (PTD), Ibrahim time domain (ITD), and Eigen-system Realization Algorithm (ERA), Rational Fraction Polynomial (RFP), Poly Reference Frequency Domain (PFD), and Complex Mode Indication Function (CMIF) methods (Allemang et al. 1994). In many of these studies the modal parameters are extracted by modeling building as a linear system with only one or two translational DOF per floor.

The accuracy of proposed system identification method depends on location of sensors on the structure. To come up with accurate modal parameters, one often needs to collect response data from instruments located at various positions within the structure. For given types of instruments, which are to be used, one often wants to locate them such that data collected from those locations yield the best estimates of the modeled structural parameters. There are several different techniques proposed for optimal sensor locations for recording structural vibrations (Udwadia 1994, Heredia and Esteva 1998). For example, a spline shape function error criterion has been proposed for the choice of optimal location of a limited number of recording sensors for reconstruction of seismic responses of multi-storey frames (Limongelli 2003). Reconstruction of unknown responses in this method is performed by modeling the evolution of relative acceleration along the height of the building through a cubic-spline shape function. Many of the methods discussed above are found suitable for parameter identification of mechanical structural components and are not been adequately explored for civil engineering structures. In this paper, a method has been devised and used to determine modal parameters of torsionally-coupled building from reconstructed earthquake responses. The proposed system identification method considers the linear behavior of the structures subjected base excitations.

Torsionally-Coupled Shear Building

In this study, the building has been idealized as consisting of rigid floors supported on mass less axially inextensible columns and walls. The general torsionally-coupled multi-story shear building considered in this study as shown in Fig 1 has the following general features: (1) The principal axes of resistance for all of the stories are identically oriented along the x-and y-axes shown in the Fig 1, (2) The centers of the mass of the floors do not lie on a same vertical axis, (3) The centers of resistance of the stories do not lie on a same vertical axis, i.e., the static eccentricity at each story are not equal, (4) Floors may have different radii of gyration, *r*, about the vertical axis through the centre of mass, (5) The ratio of three stiffness quantities, the translational stiffness in *x* and *y*-directions, K_x, K_y, and K_θ torsional stiffness for any story may be different. For the above general torsionally-coupled *N*-storey shear building, each floor has three degrees-of-freedom (DOF), *x*- and *y*-displacements, relative to the base and rotation about a vertical axis. For floor *i*, the DOFs are denoted by subscripts x, y and θ, respectively.

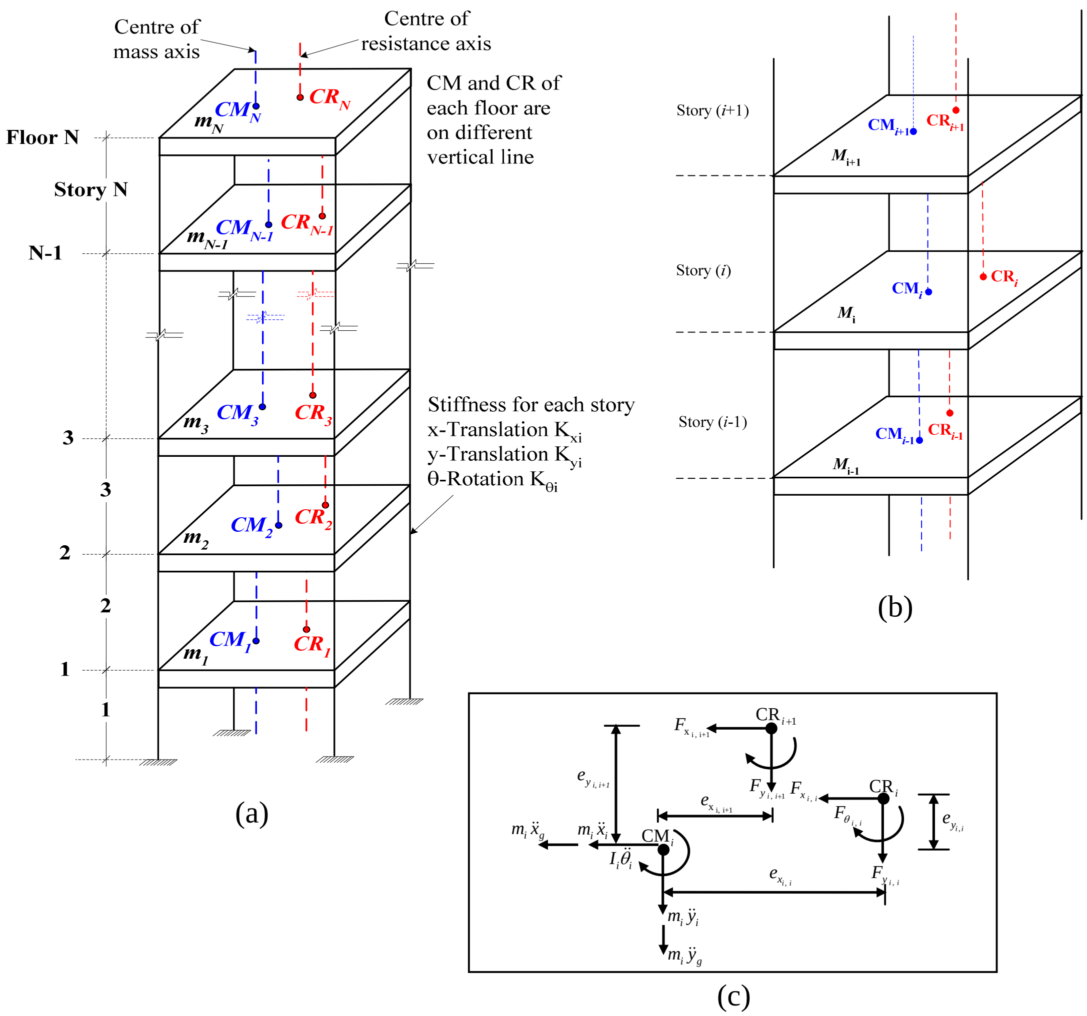

Figure 1: Torsionally coupled multi-storey building.

For an N-storey shear building as shown in Fig 1(c), x_i, y_i and θ_i are displacements in x, y and θ-direction of the centre of mass of the i-th floor. The stiffness of i-th story in x, y and θ-direction are denoted as $K_{x(i)}$, $K_{y(i)}$ and $K_{\theta(i)}$, respectively. In this figure, $F_{x(i,i)}$, $F_{y(i,i)}$, and $F_{\theta(i,i)}$ are forces acting on i-th floor at Centre of Resistance (CR_i) due to the eccentricity of i-th story and $F_{x(i,\,i+1)}$, $F_{y(i,\,i+1)}$ and $F_{\theta(i,\,i+1)}$ are the forces acting on the i-th floor at CR_{i+1} due to eccentricity of $(i+1)$-th story. The static eccentricity of the floor are denoted by $e_{x(i,j)}$ and $e_{y(i,j)}$, where $j=i$ for the story considered and $j=i+1$ for the eccentricity due to story above. The base excitations in x- and y-directions, respectively, are given by $\ddot{x}_g$ and $\ddot{y}_g$. The forces acting on the floors can be evaluated considering the resistive force generated by story above and below each floor. The detailed force calculation and formulation of stiffness and mass matrix has been presented in literature (Uneg et al. 2000). Now, the undamped dynamic equations of motion for the building subjected to two horizontal ground accelerations $\ddot{x}_g$ and $\ddot{y}_g$ assumed to be the same at all points of the base can be expressed in the matrix form as

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}} + \boldsymbol{K}\,\boldsymbol{u} = -\boldsymbol{M}\,\boldsymbol{r}\,\ddot{\boldsymbol{u}}_{\boldsymbol{g}} \tag{1}$$

Where, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{u}$ is the displacement vector with respect to the base, $\boldsymbol{r}$ is the force-influence vector, $\boldsymbol{K}$ is the stiffness matrix, and

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 & \cdots & \boldsymbol{u}_N \end{bmatrix}^T,\ \ddot{\boldsymbol{u}}_g = \begin{Bmatrix} \ddot{x}_g & \ddot{y}_g & 0 \end{Bmatrix}^T \tag{2}$$

$$\boldsymbol{r} = \begin{bmatrix} \boldsymbol{r}_1 & \boldsymbol{r}_2 & \cdots & \boldsymbol{r}_N \end{bmatrix}^T,\ \boldsymbol{M} = \mathrm{diag}\begin{Bmatrix} \boldsymbol{M_1} & \boldsymbol{M_2} & \cdots & \boldsymbol{M_N} \end{Bmatrix} \tag{3}$$

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{11} & \boldsymbol{K}_{12} & & & & & \\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22} & \boldsymbol{K}_{23} & & & & \\ & \boldsymbol{K}_{32} & \boldsymbol{K}_{33} & \boldsymbol{K}_{34} & & & \\ & & \vdots & \vdots & \vdots & & \\ & & & \vdots & \vdots & \vdots & \\ & & & & \vdots & \vdots & \\ & & & & \boldsymbol{K}_{N-1\,N-2} & \boldsymbol{K}_{N-1\;N-1} & \boldsymbol{K}_{N-1\;N} \\ & & & & & \boldsymbol{K}_{N\;N-1} & \boldsymbol{K}_{NN} \end{bmatrix} \tag{4}$$

Further, the submatrices are given as follows

$$\boldsymbol{r}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T,\ \boldsymbol{u}_i = \begin{Bmatrix} x_i & y_i & \theta_i \end{Bmatrix}^T,\ \text{and}\ \boldsymbol{M_i} = \mathrm{diag}\begin{Bmatrix} m_i & m_i & I_i \end{Bmatrix} \tag{5}$$

where $\boldsymbol{r_i}$ is the force influence coefficient matrix with elements equal to 0 and 1, m_i is the lumped mass of i-th floor, and I_i is the mass moment of inertia of the i-th floor. The formulations of the stiffness sub-matrices ($\boldsymbol{K}_{11}, \boldsymbol{K}_{12}$, …, $\boldsymbol{K}_{NN}$) follow the procedure given by Uneg et al. (2000). After formulating the mass and stiffness matrices, the natural frequencies and mode shapes of the building are obtained by solving the corresponding eigenvalue problem.

The damping in the building has been represented by its Raleigh damping matrix, with defined damping ratio for each natural mode of vibration. In most cases of structural engineering interest, modal damping ratios are used in the computer model to approximate nonlinear energy dissipation within the structure. In this paper, Raleigh damping has been considered to include the influence of all sources of energy dissipation in the building that behaves linearly, thereby avoiding the need to formulate the damping matrix based on the physical properties of the real structure (Hart and Wong 1999).

System Identification Using Reconstructed Seismic Responses

The technique to reconstruct unmeasured floor level responses without using the mode shape coefficients has been presented by (Limongelli 2003). In this approach, a spline based reconstruction is applied to plane multi-storey shear frames having similar system properties at each story level. The frame was modeled as lumped mass system with one DOF per floor. The technique was applied to reconstruct the seismic response of real building frames. The application of the said technique to other types of structures different from multi-storey shear plane frames has not been investigated in the published literature. In the present study, this approach is extended to a general multi-storey shear building (different mass and stiffness for each story with varying eccentricity for each floor) modeled as lumped mass shear system with three DOF per floor. The assumptions made in the investigations by (Limongelli 2003) are also applicable in this study. The assumptions are: (1) The responses in terms of absolute acceleration available in a limited number of locations along the building height and the input ground acceleration is known, (2) The evolution of relative displacement x of the building along its height z can be modeled by means of a function of position and time $x(z, t)$, and (3) The second derivative of relative displacement with respect to time gives the evolution of the relative acceleration along height of the building.

By knowing the input base acceleration, $\ddot{u}_g(t)$, the time history of absolute acceleration $\ddot{u}(z,t)$ at a given location on a building can be evaluated by solving Eq. 1. However in practice sensors are used to record the accelerations at only a few DOFs of the building. The locations where responses are recorded by sensors are assumed as knots of the spline function and, for each time instant, the unknown coefficients in the spline function are determined from continuity, interpolation and boundary conditions. The advantage of spline functions as interpolating functions is that among all twice differentiable functions approximating a given set of data, the spline functions are the ones corresponding to the minimum value of the overall curvature. A detailed discussion on the use of cubic spline interpolation function can be found in published literature (for example, Limongelli 2003). The advantage of this method is that the responses at non-instrumented floor levels are obtained by interpolating the recorded responses at instrumented floor level, without using the mode shapes of the building.

The modal parameter of general torsionally-coupled building can be extracted from the recorded vibration signatures using different techniques. To extract the first few natural frequencies, it is sufficient to have vibration record at the top floor level (Mau and Aruna 1994). In this case the complete mode shape is not identified. To identify complete mode shapes one need to have simultaneous vibration records at all floor levels or use mode shape interpolation techniques or modal expansion techniques. It has been shown that vibration records collected at top and first floor level are sufficient to get the complete mode shapes of a symmetric torsionally-coupled shear building (Hegde and Sinha 2008). The mode shape interpolation techniques can be applied if the building is symmetric and follows uniform shear criterion (Mau and Aruna 1994). In case of general torsionally-coupled multi-story building one cannot expect to have uniform shear at all floor levels. Modal expansion techniques fetch good result only if experimentally identified mode shapes coefficients are available at sufficient degrees of freedom. Most of the expansion techniques use finite element model as a mechanism to obtain mode shape coefficients at unmeasured degrees of freedom from the experimental modal model (Leandro et al. 2006). Hence in this paper a different technique has been proposed to obtain the complete mode shapes, using vibration records at limited number of locations along the height of the building.

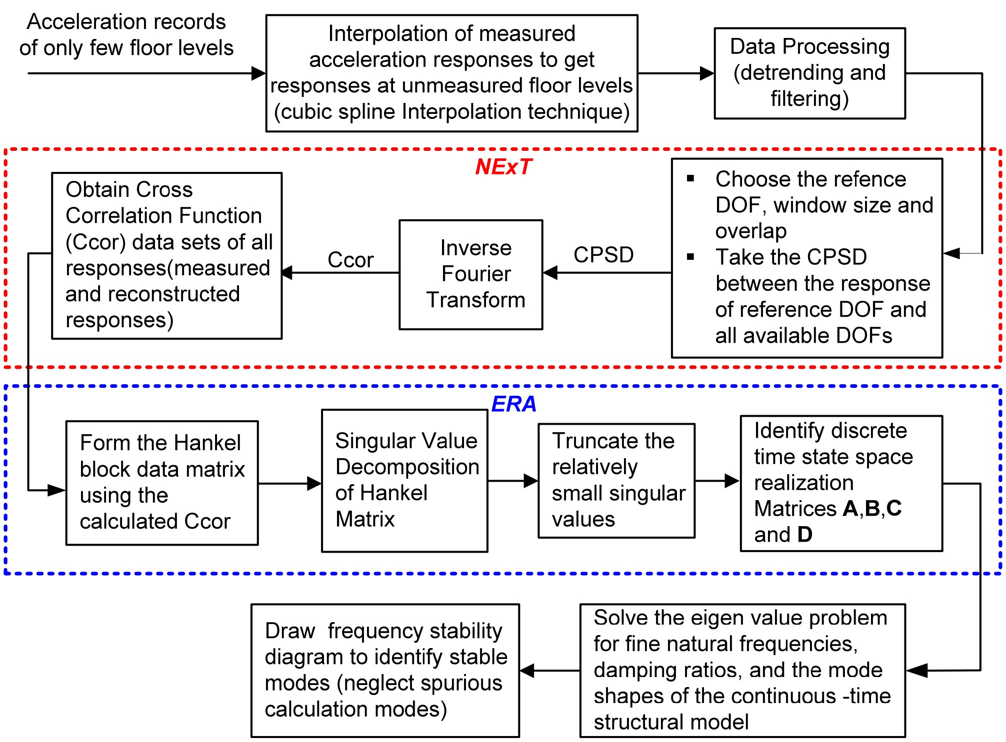

Figure 2. Flow chart for system identification of general torsionally-coupled building.

In typical problems of forced vibration of buildings, it is difficult or impossible to measure the frequency response functions or Transfer Function (FRFs or TFs) derived from recorded excitation and structural responses, and hence these quantities are not available for modal property extraction. . In this study, modal identification was performed using only the response measurements, hence the parameter identification procedure based on Natural Excitation Technique (NExT) (Farrar and James, 1997), which utilizes the cross-correlation between measured responses, has been implemented for general torsionally-coupled building without using recorded excitations. The time domain curve fitting algorithms such as polyreference method, complex exponential method, or eigensystem realization algorithm (ERA) (Juang and Pappa 1985), Ibrahim Time Domain method (Ibrahim and Mikulcik 1997), which is developed to analyze the impulse response functions, can be applied to cross-correlation functions to obtain resonant frequencies and modal damping exhibited by the structure (Caicedo et al. 2004).

In the present study modal parameters are extracted using the principles of NExT and ERA. The detail procedure is presented in the flow chart shown in Fig 2. The unmeasured floor responses are obtained by interpolating the available measured responses using cubic spline method explained earlier (Limongelli 2003). Thus the complete set of all floor response are made available for parameter identification. The cross-correlation between each floor responses with respect to referenced measurement is used in Eigen Realization Algorithm for identifying first few resonant frequencies, damping ratios and their complete mode shapes (Juang and Pappa 1985). In case of typical torsionally-coupled buildings, the first few modes have the highest contribution to the effective mass participation (Chopra 2004). It is also well known that the mode shapes are linearly independent of each other. When a building is subjected to base excitations, the lower modes contribute more to the vibration response than the higher modes. Hence it is important to accurately capture the first few modes of vibrations.

Numerical Verification

A 6-storey torsionally-coupled shear building has been considered for numerical evaluation of the procedure. First a simulation is carried out to verify the effectiveness of the spline function in interpolating the acceleration responses along the height of the building. The physical properties of the example building are given in the Table 1.

Table 1. System properties of the example building

Floor (i)	**Mass** m_i **(kg)**	**Moment of Inertia, I (kg.m^4)**	**C.M.** (x, y) **(m)**	**Story (i)**	K_x **(N/m)**	K_y **(N/m)**	K_θ **(N/m)**	**C.R.** (x, y) **(m)**
1	200000	1.28×10^7	(1.0,1.0)	1	9.0×10^8	8.5×10^8	5.0×10^{10}	(4.0, 4.0)
2	200000	1.28×10^7	(1.0,1.0)	2	9.0×10^8	8.5×10^8	5.0×10^{10}	(4.0, 4.0)
3	200000	1.28×10^7	(1.0,1.0)	3	9.0×10^8	8.5×10^8	5.0×10^{10}	(4.0, 4.0)
4	190000	1.07×10^7	(0.8, 0.9)	4	8.5×10^8	7.5×10^8	4.5×10^{10}	(3.2, 3.0)
5	190000	1.07×10^7	(0.8, 0.9)	5	8.5×10^8	7.5×10^8	4.5×10^{10}	(3.2, 3.0)
6	190000	1.07×10^7	(0.8, 0.9)	6	8.5×10^8	7.5×10^8	4.5×10^{10}	(3.2, 3.0)

In this example, it is assumed that the sensors are located at first floor, third floor and fifth floor level (based on rigorous analysis results) and the acceleration responses are recorded only at these floor levels (Hegde and Sinha 2007). The numerical model is subjected to unidirectional earthquake at the base (1940 El-Centro North-South component) and responses at all floor levels are obtained in the three directions. The system damping is introduced as the viscous damping ratios in each vibration modes. The calculated earthquake base excitation responses at floor levels are added with noise up to 20% (RMS) to simulate actual field measiurements. A cubic spline interpolation method described in Limongelli (2003) has been used to calculate the responses at unmeasured floor levels. The acceleration time responses at 2-

nd, 4-th and 6-th floors are reconstructed by interpolating the recorded acceleration time histories at 1-st, 3-rd and 5-th floor levels. The spline interpolations are carried out separately for each direction. Fig 3 shows the comparisons of some of the reconstructed responses at unmeasured floor levels.

It is seen that the reconstructed responses are a good approximation of their actual values, and this simple example demonstrates the accuracy of cubic spline function to reconstruct unmeasured responses. Using the reconstruction technique, the responses at all floor levels become available for modal parameter identification. The Cross Power Spectral Density (CPSD) functions are calculated for all floor responses taking the first floor responses (*x, y* and rotational responses) as the reference measurement. The cross-correlation functions are obtained by inverse Fourier Transform of CPSD. Thus a total of six cross-correlation functions for each direction for each reference measurement, totaling 54 CPSDs are used in Eigen Realization Algorithm (ERA) for modal parameter identification. A Hankel matrix $\mathbf{H}$ of size (100×100) is constructed using the CPSD functions. The Singular Value Decomposition of $\mathbf{H}$ is carried out to determine the state space matrices A, B and C. These matrices are further modified by eliminating the rows and columns corresponding to the smaller singular values produced by computational modes (Caicedo et al. 2004). The modified matrices $\mathbf{A}_{(22\times22)}$, $\mathbf{B}_{(22\times3)}$ and $\mathbf{C}_{(18\times22)}$ are used to extract the modal properties of the example building. The natural frequencies are obtained by Eigen analysis of system matrix $\mathbf{A}_{(22\times22)}$. The mode shapes corresponding to all 18-DOFs are obtained by multiplying eigenvector of $\mathbf{A}_{(22\times22)}$ with the output matrix $\mathbf{C}_{(18\times22)}$.

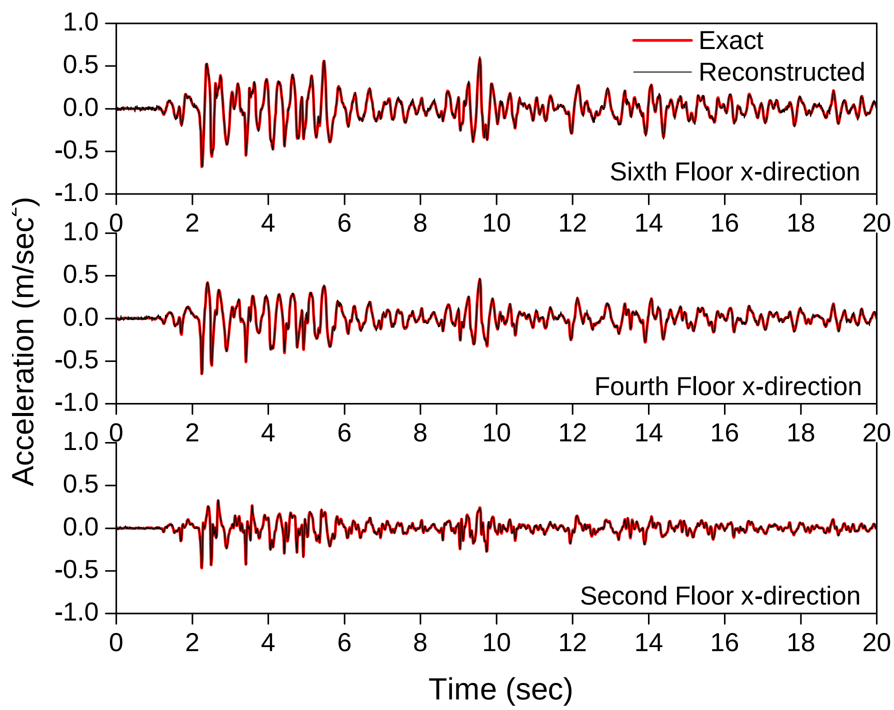

Figure 3. Floor responses of example building.

The frequency stability diagram is used to identify the stable frequencies among the ERA extracted modes. The identified frequencies and damping ratios are given in the Table 2, and are found to be good agreement with their actual values.

Table 2. Identified System properties of the example building

Mode	Frequency (Hz)		Damping (%)	
	Actual	Identified	Actual	Identified
1	1.61	1.59	5.00	9.69
2	2.56	2.59	4.18	3.92
3	3.23	3.20	4.12	3.08
4	5.61	5.93	4.86	3.55
5	7.30	7.49	5.73	4.65

The mode shapes extracted from the set of reconstructed responses have compared with their actual values to illustrate the accuracy of mode shape determination procedure. Fig. 4 shows the first mode shape corresponding to 1.61 Hz, when compared with the mode shapes obtained by proposed method using ERA. It is seen that the extracted mode shapes are in excellent agreement with their actual values in terms of shape; however there is some error in its magnitude. The error is found to be more in the rotational direction when compared to two translation directions. This illustrates that the method accurately determines the complete mode shapes of the structure and is comparable to the other methods proposed in literature (Ibrahim and Mikulcik 1997).

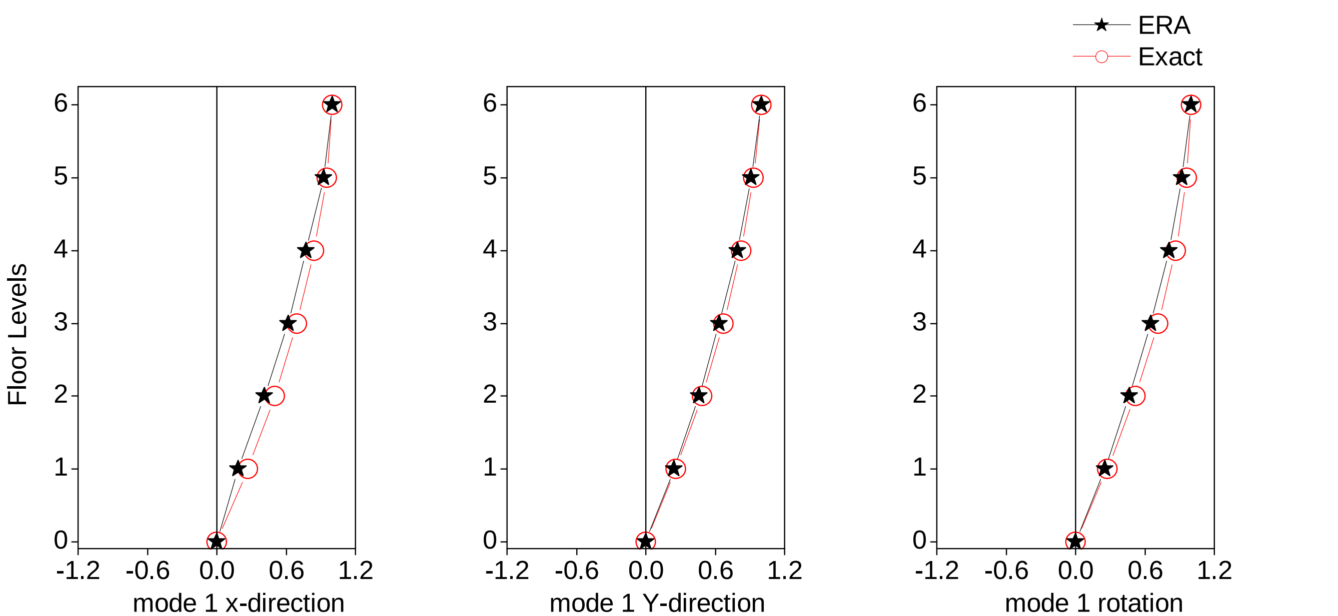

Figure 4. Mode shapes extracted from reconstructed responses.

Conclusions

This paper presents a method for determining structural modal properties when responses of a torsionally-coupled shear building subjected to base excitations are recorded at only a few floors. The investigations show that the cubic spline shape functions can be used for reconstruction of unmeasured floor responses. The reconstruction of responses of torsionally-coupled shear building has been performed by extending the spline shape function method. The investigation also show that the accuracy of reconstructed responses increase with number of sensors used for recording. The parameter extraction from the set of reconstructed and actual responses are carried out and found to be effective in identifying first few fundamental frequencies and corresponding complete mode shapes. The effective location of the senor has been determined based on minimum error criteria in the reconstructed responses. The identified modal properties have been compared to their actual values using correlation techniques. The correlation results show good accuracy in the identified frequencies and mode shapes for first few fundamental modes. The Finite Element model can be updated using the modal parameters extracted from the vibration records taken on the building. The updated FE model can be used to predict the response to future base excitations. The procedure can be effectively applied for condition assessment and structural health monitoring problems since vibration records are required at only a few floor levels, and the fundamental frequencies and mode shapes are identified with required accuracy.

References

Allemang, R. J., Brown, D. J., and W. Fladung, 1994, Modal parameter estimation: A unified matrix polynomial approach. *Proceedings of International Modal Analysis Conference*, 501-514.

Caicedo, J. M., Dyke, S. J., and E. A. Johnson, 2004. NExT and ERA for Phase-I of the IASCE-ASCE benchmark problem: simulated data. *ASCE Journal of Engineering Mechanics*. 130.

Chopra, A. K., 2004. *Dynamics of Structures, Theory and Application to Earthquake Engineering*, 2nd ed. Prentice-Hall of India Pvt. Ltd, New Delhi.

Farrar, C. R. and G. H. James III, 1997. System identification from ambient vibration measurements on a bridge, *Journal of Sound and Vibration*. 205, 1-18.

Hart, G. C., and K . Wong, 1999. *Structural Dynamics for Structural Engineers*. John Wiley & Sons, New York, USA.

Hegde, G., and R. Sinha, 2007. A technique for locating sensors on buildings for complete modal identification from limited number of response measurements. *III ECCOMAS Thematic Conference on Smart Structures and Materials*, Gdansk, Poland.

Hegde G. and R. Sinha, 2008. Parameter identification of torsionally coupled buildings from earthquake response records. *Earthquake Engineering and Structural Dynamics*, 37, 1313-1331.

Heredia, E. Z., and L. Esteva, 1998. Optimal instrumentation of uncertain structural systems subject to earthquake ground motions. *Earthquake Engineering and Structural Dynamics*, 27, 343-362.

Ibrahim, S. R, and E. C. Mikulcik, 1997. The experimental determination of vibration parameters from time responses. *The Shock & Vibration Bulletin*, 46,187-196.

Juang, J. N., and R. S. Pappa, 1985. An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of Guidance and Control*, 8, 620-627.

Kozin, F., and H. G. Natke, 1986. System identification techniques. *Structural Safety*, 3:269-316.

Leandro, F., Fadel, M., Ruy, C. R., de Menezesa, and Fadel, L. F. M. (2006). “Mode shape expansion erom data-based system identification procedures.” *Mecánica Computacional*, 25, 1593-1602

Limongelli, M. P., 2003. Optimal location of sensors for reconstruction of seismic responses through spline function interpolation. *Earthquake Engineering and Structural Dynamics*. 32, 1055-1074.

Lus, H., Betti, R., and R.W. Longman, 1999. Identification of linear structural systems using earthquake-induced vibration data. *Earthquake Engineering and Structural Dynamics*, 28,1449-1467.

Mau, S. T., and V. Aruna, 1994. Story-drift, shear, and OTM estimation from building seismic records. *ASCE Journal of Structural Engineering*, 120, 11, 44-51.

Udwadia, F. E., 1994. Methodology for optimum sensor locations for parameter identification in dynamic systems. *ASCE Journal of Engineering Mechanics*, 120, 368-390.

Uneg, J. M., Lin, C. C., and P. L. Lin, 2000. System identification of torsionally coupled buildings. Computer & Structures. 74, 667-686.