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AN OVERVIEW ON THE NUMERICAL/ANALYTICAL METHODS OF SITE RESPONSE ANALYSIS FOR THE CITY OF OTTAWA

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ABSTRACT

City of Ottawa is located in the Western Quebec Seismic zone and the majority of the surficial geology of the city consists of soft soil sediments with low shear wave velocity (~150 m/sec) underlain by hard bedrock with very large shear wave velocity (>2000 m/sec). This high contrast shear wave velocity between the soft soil and the underlying bedrock can cause multiple reverberations of seismic waves inside the soil layers leading to the wave trapping and large seismic amplification values. This issue was the motivation for carrying out an extensive site response analysis as a part of the seismic microzonation studies for the city of Ottawa. First, we implemented the generalized method of the reflection/transmission (R/T) to simulate the probable internal reflections of seismic waves and we calibrated this method with the well-known equivalent linear method. Second, we used the one-dimensional finite element method (FEM) for the site response analysis of some deep sites located in the eastern part of the city. Then, using the analysis results, we investigated the variation of the peak amplification ratio with the contrast ratios of the examined sites and the shaking intensities of the input motions. Finally, we proposed a mathematical model that correlates the expected peak amplification ratio to two parameters of contrast ratio and peak ground acceleration.

Introduction

The surficial geology of the near-surface sediments in the Ottawa region, Canada consists of very loose and very low shear wave velocity (~150 m/sec) post-glacial deposits (Holocene age sediments) underlain by very high shear wave velocity bedrock (>2000 m/sec) (see Motazedian and Hunter, 2008 and Pugin et.al. 2007). According to the definition of contrast ratio (z):

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$$z = (\rho_b V_b)/(\rho_s V_s) \quad (1)$$

Where ρ_b, V_b , ρ_s and V_s denote the bedrock density, bedrock shear wave velocity, soil density and the soil shear wave velocity, respectively. Taking ($V_b = 2500$ m/sec, $V_s = 150$ m/sec) and assuming ($\rho_b = 2500$ kN/m^3 and $\rho_s = 1700$ kN/m^3), Eq. 1 gives the contrast ratio of 24.5 for a typical soil-bedrock formation in the region. This unusual contrast can cause multiple internal reflections of seismic waves inside the soil layers, leading to the subsequent wave trapping and large seismic amplification ratio (AR). On the other hand, sparse earthquakes recorded in the area show large seismic amplification values compared to the amplification values used in geotechnical practice (Motazedian et.al. 2008). These observations including the mentioned soil-bedrock formation highlight the significance of the site response analysis methods for the area. In this research, we will have an overview on two site response analysis methods applied on some sites in the Ottawa region.

First, generalized method of reflection/transmission (R/T) will be described that benefits from a shaking intensity-dependent equation for the soil damping. This means that a damping equation as a function of shaking intensity will be introduced and used in R/T method. The main advantage of R/T method is a scheme that considers all the internal reverberations of seismic waves inside the soil layers. Following the mentioned damping implementation in R/T method, for a deep site located in the eastern part of the city, the results of the calibration of R/T method with ProShake software (EduPro Civil Systems 2001) will be presented. ProShake software is a site response analysis program that is based on the well-known equivalent linear method (see Schnabel et.al. 1972) for the vertical propagation of shear waves.

Second, combined effect of two parameters, shaking intensity and contrast ratio, on the seismic amplification will be studied using the one-dimensional finite element method. To investigate the effect of these parameters, seismic amplification analysis of three sites will be performed for a variety of input motions. Finally, according to the variation pattern of seismic amplification with each of these parameters, a model will be proposed that correlates the amplification ratio to the contrast ratio and the shaking intensity.

Generalized reflection/transmission (R/T) method for the site response analysis

R/T method is based on the older matrix method of Thompson-Haskell (Thompson 1950 and Haskell 1953). The R/T method formulation was developed using two concepts of scattering matrices at the interfaces of soil layers (see Aki and Richard 1980) and the propagator matrix (Gilbert and Backus 1966). The method's formulation was described by Kennett 1974. In this formulation, reflection and transmission matrices are obtained at the boundaries of soil layers for the upward and downward wave propagation. For example, the reflection matrix for the upward propagation is obtained through the following equation:

$$R^{13}_U = R^{23}_U + T^{23}_D R^{12}_U T^{23}_U + T^{23}_D R^{12}_U R^{23}_D R^{12}_U T^{23}_U + \dots \quad (2)$$

R and T stand for the reflection and transmission matrices, respectively. U and D subscripts denote the up-going and down-going waves, respectively. Superscripts of 1,2 and 3 indicate the levels of soil layers shown in Fig.1 . This Figure illustrates the Kennett's scheme for internal reflections of waves. According to this Figure, the combinations of reflection and transmission matrices can be calculated at the interfaces of soil layers until the upward propagating seismic waves reach the ground surface.

In order to implement the R/T method for the city of Ottawa, damping of soil layers should be estimated. For the eastern Ottawa region, Khaheshi Banab et. al. 2009 introduced a damping equation based on the target peak ground acceleration (PGA) in the area. The equation is:

$$D = 14.33\ PGA^{0.2537} \tag{3}$$

In this equation, a damping ratio (D) is attributed to the expected shaking intensity represented by the PGA (expressed in the ratio of gravitational acceleration). To incorporate damping in the algorithm of R/T method, the shear wave velocity is considered as a complex quantity and the damping is included in the imaginary part of the velocity function. This definition of shear wave velocity in complex form can be found in Aki and Richard, 1980. Examples of the application of this damping-modified R/T algorithm are shown in Fig.2 and Fig.3. These Figures demonstrate the application of R/T method for two shaking intensity levels at the Heritage Park located in eastern part of the city. Furthermore, these Figures exhibit the comparison of the R/T method with the well-know equivalent linear method (using ProShake software 2001). As seen in these Figures, the agreement between two methods is fairly good and the first peak amplification values are adequately close together. Fig. 4 and Table 1 give the location and the mechanical parameters of soil-bedrock formation at Heritage Park respectively. The input motion data of the analysis were chosen from the artificial time history acceleration records for Eastern Canadian Earthquakes (Atkinson and Beresnev 1998).

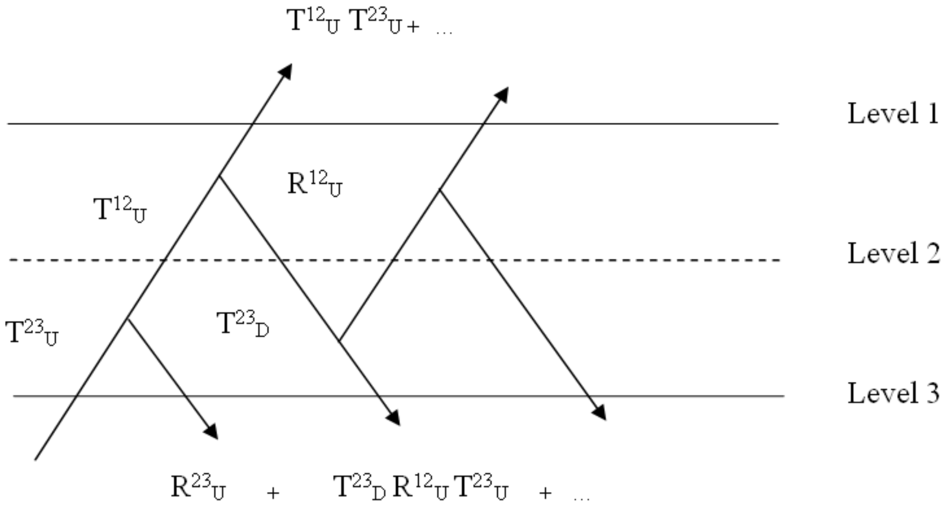

Figure 1. R/T method scheme showing the reverberations inside the soil layers for upward wave propagation.

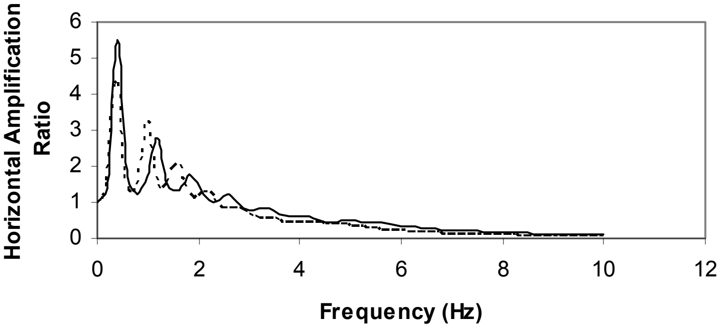

Figure 2. Comparison of the amplification ratios at Heritage Park site obtained from R/T method (solid line) and the equivalent linear method (dashed line) for the target PGA of 0.2g.

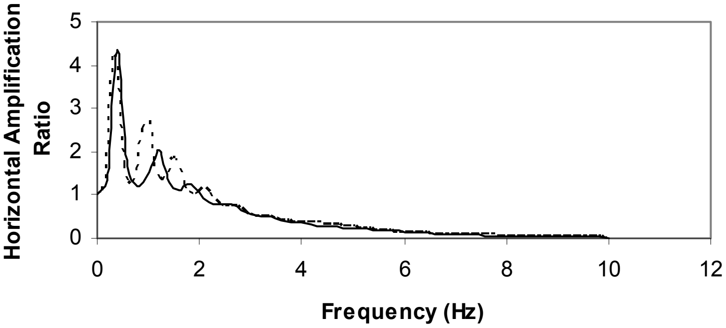

Figure 3. Comparison of the amplification ratios at Heritage Park site obtained from R/T method (solid line) and the equivalent linear method (dashed line) for the target PGA of 0.4g.

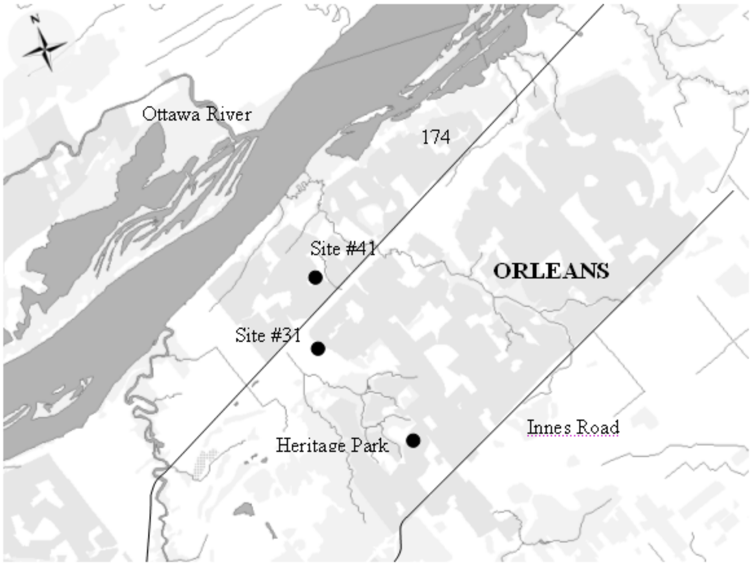

Figure 4. Orleans area in the eastern part of the city of Ottawa and the locations of three investigated sites.

Table 1. Mechanical parameters of soil-bedrock formation for three sites located in the eastern part of Ottawa

Site # / Name	Heritage	31	41
Depth (d)-dependent /average velocity function	200 (m/sec)	1.37d+147.2	1.51 d+91.9
Average bedrock velocity (m/sec)	3000	3000	3000
Average soil density (kN/m^3)	1700	1700	1700
Average bedrock density (kN/m^3)	2500	2500	2500
Site depth (m)	81	60	39
Average contrast ratio	22.05	23.42	36.35

Finite element method for the site response analysis

In the finite element method (FEM), (see Reddy 2005 and Bathe 1996) the soil layer is modeled using number of elements connected at their nodes (See Figure 5 that shows the finite element mesh for a soil deposit). At the base of the finite element mesh, viscous boundary conditions leading to perfect wave absorption (Lysmer and Kuhlemeyer, 1969) are applied. The following equation of motion is exploited in this method:

$$[M]\ddot{u} + [C]\dot{u} + [K]u = R \qquad (4)$$

Where [M], [C] and [K] are the total mass, damping and stiffness matrices, respectively, obtained from assembling the element matrices. R denotes the load vector (input time history acceleration at bedrock). u , $\dot{u}$ and $\ddot{u}$ represent the displacement, velocity and the acceleration, respectively. Based on the obtained level of shear strain from equation (4), the shear modulus and damping values are updated implying that the hysteretic model is considered for the solution of the equation of motion. Finally, Solving Eq. 4, horizontal response at the ground surface is calculated. Having the ground response, amplification curve is obtained if the Fourier spectrum of the ground response is divided by the Fourier spectrum of the bedrock motion.

To apply FEM for the seismic amplification analysis, three sites in the eastern part of the city were chosen (see Fig.4). These sites include the Heritage Park site, GSC (Geological Survey of Canada) site # 31 and GSC site #41. Mechanical parameters of all the sites are given in Table 1. These sites were subjected to seven time histories selected from the artificial time histories of Eastern Canada (Atkinson and Beresnev 1998). Peak ground acceleration (PGA) of these records varies from 23 gal (cm/sec^2) to 349 gal.

To cover a broad range of contrast ratios, in addition to the real V_s profiles of the mentioned sites, three more shear wave velocity profiles were generated for each site. The goal was to reach the contrast ratios of 4, 8 and 12 using the Eq. 1. Taking the constant shear wave velocity for the underlying bedrock, 3000 m/sec, having ρ_s equal to 1700 kN/m^3 and ρ_b equal to 2500 kN/m^3 ,shear wave velocity (V_s) of the target models can be calculated for each contrast ratio, z. Thus, using seven time histories and four contrast ratios (real and three generated contrast ratios) 28 seismic amplification curves are produced for each site. Totally, 84 amplification curves will be obtained for three investigated sites. Finally For each pair of (PGA, z), peak amplification ratios are extracted from the obtained curves. As one example among three examined sites, Fig.6 shows the variation of peak amplification ratio with two parameters of PGA and contrast ratio, z at Heritage Park.

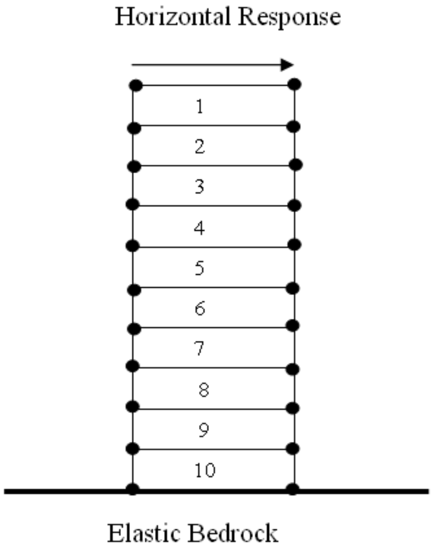

Figure 5. Scheme of the finite element mesh for site response analysis. The seismic waves propagate from the elastic bedrock. Viscous boundary conditions are applied at the base of model.

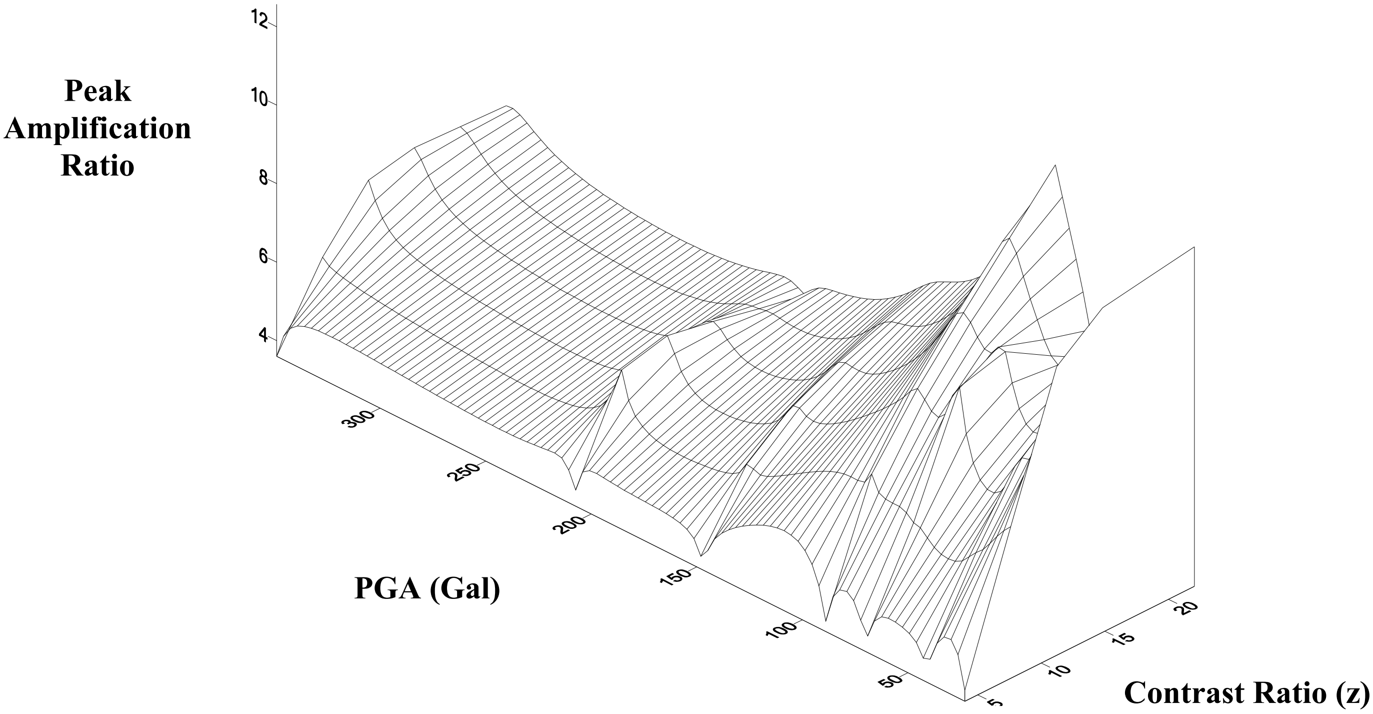

Figure 6. Peak seismic amplification ratio (from FEM) variation with two parameters of contrast ratio and the peak ground acceleration at Heritage Park. This Figure is comparable with previous results from R/T method. For example, at the shaking intensity of 0.2g, FEM and R/T give the peak AR values of 6 and 5.6 respectively.

Variation of seismic amplification ratio with contrast ratio

As mentioned before, the main geological units of concern in Ottawa area, consist of a very loose post- glacial soil (Vs <150 m/s) and very firm bedrock (Vs> 2000 m/s). This large shear wave velocity contrast exhibits the high potential of the soil profiles to trap the incident seismic waves. This wave-trapping potential can be clearly confirmed if the reflected energy coefficient (E_R) is considered (Reynolds 1997). Here, the Reynolds‘s equation is expressed in terms of the contrast ratio, z:

$$E_R = (z-1)^2 / (z+1)^2 \qquad (5)$$

Plugging a range of high contrast ratio values from 15 to 25 (which is the range for most of the available profiles throughout the city) in the Eq. 5, E_R values from 0.76 to 0.85 are obtained. This means that 76% to 85% of the energy of the propagated seismic wave will be reflected and trapped in the soil layer that might lead to large ground surface motions.

Considering this contrast ratio effect, mathematical models were sought that correlate the amplification ratio (AR) to the contrast ratio (z) for our examined sites. To achieve this model, the variation of AR values with z values was monitored for each shaking intensity level (PGA). For example, Fig. 6 shows that, for the most of the PGAs, the AR values get saturated when the contrast ratio exceeds the contrast ratio of 12. Due to this saturation behavior, the logarithmic function Log (z) is suggested as the mathematical model for contrast ratio. Logarithmic function gets partially saturated and by examining three analyzed sites, we have concluded that Log $_{1.35}$ (z) gives a better saturation behavior. For instance, if the contrast ratio increases from 12 to 20, the function (Log $_{1.35}$ z) will vary from 8.28 to 9.9. This small variation of AR value is satisfactory enough according to the observed saturation behavior.

Variation of seismic amplification ratio with shaking intensity

Many researchers have investigated effect of shaking intensity on the nonlinear behavior of soils during earthquakes. For instance, Jarpe et.al. 1989 studied the seismic amplification of some sites in San Francisco Bay resulting from the weak and strong motions of Loma Prieta earthquake, 1989. Their study on this earthquake and the following aftershocks exhibited the significant reduction of the amplification ratios with the increased level of shaking. As another worthwhile study, Beresnev et.al. 1994 carried out a research on the nonlinear behavior of soils during large earthquakes. They analyzed the recorded acceleration data from two strong motion arrays in Taiwan. This study gave significant evidences on the remarkable role of the shaking intensity on the seismic response.

Our analysis results of three examined sites confirm the significance of shaking intensities on the seismic response. For instance, Fig. 6 clearly shows that AR tends to decrease as the PGA level of input motions increases. To simulate this decreasing effect of increased shaking intensity (PGA) on the AR values, different functions were tested and finally, according the obtained results, the exponential function, $e^{-PGA/g}$, was proposed.

For the current analysis, the lowest PGA of the input records is 23 gal (0.023g) and the highest PGA is 349gal (0.35g); therefore, the proposed exponential function gives the coefficients of 0.977 and 0.704 for the mentioned PGAs. These coefficients approximately exhibit the peak amplification decrease of 3% and 30 % for the PGAs of 23gal and 349gal respectively, in a constant contrast ratio. This means that the expected AR values, at a constant contrast ratio, should be multiplied by factors ranging from 0.977 to 0.704 as the shaking intensity varies from 23 gal to 349 gal.

Proposed model for the combined effect of contrast ratio and the shaking intensity

Following the discussion above, the combined function of [$\log_{1.35} (z)\, e^{-PGA/g}$] is introduced as the indicator of the combined effect of z and PGA on the variation of peak AR values. To simplify the logarithmic-exponential function, we can express the logarithmic part of this indicator in base 10 (Log $_{10}$) and therefore, the new indicator is obtained [$7.67 \log_{10} (z)\, e^{-PGA/g}$]. Then, using the obtained AR values of three examined sites, the Best-fit curve (shown in Fig. 7) between this indicator and AR values is determined. The obtained equation is:

$$AR = 8.16 \left[\log_{10} (z)\, e^{-PGA/g}\right]^{0.9} \qquad (R^2 = 0.8) \qquad (6)$$

Eq. 6 can be used for the determination of the expected peak amplification ratio for each pair of (PGA, z) in the region.

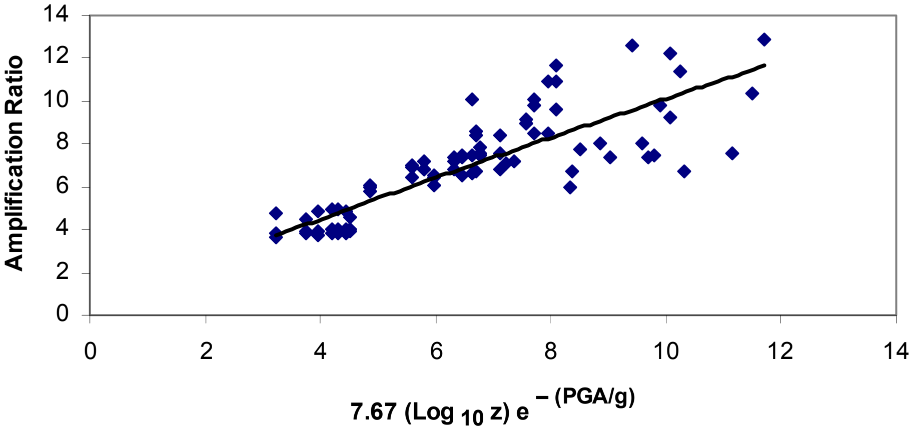

Figure 7. Graph showing the combined effect of PGA and contrast ratio (z) on the expected peak amplification ratio. [$7.67\mathrm{Log}_{10}(z)\, e^{-PGA/g}$] as the indicator of the mentioned combined effect is calculated for each pair of (PGA, z) and then, the best fir-curve is determined using the corresponding peak amplification ratio.

Conclusions

In this research, the results of two site response analysis methods were presented for the city of Ottawa using the artificial time history acceleration records of Eastern Canada earthquakes. First, the reflection/transmission (R/T) method with a shaking intensity-dependent soil damping was applied for a deep site located in the eastern part of the city. The peak amplification values obtained from R/T method showed good agreement with those obtained from the well-known equivalent linear method. Second, site response analysis of three sites located in the eastern part of the city was performed using the one-dimensional FEM and the variation of the peak amplification values was investigated for different combinations of the contrast ratio and peak ground acceleration. Finally, a combined Logarithmic-exponential function was proposed that correlates the peak amplification ratio to the pair of peak ground acceleration of the input motions and contrast ratio of soil-bedrock profiles.

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