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AN APPROXIMATE METHOD FOR DYNAMIC ANALYSIS OF HIGHWAY SKEWED BRIDGES WITH CONTINUOUS DECK

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ABSTRACT

In this paper using a 3DOFs analytical model an approximate method is presented for dynamic analysis of highway skewed bridges with continuous deck. In modeling of the superstructure it is assumed that the deck is rigid in-plane. the stiffness of substructure and shear stiffness of the elastomeric bearings on the two abutments are modeled with linear springs. To calculate the natural circular frequencies and internal forces of skewed bridges, simplified formulae are presented. The capability and accuracy of the proposed method is compared with a finite element model. Results indicate that this method is capable for dynamic analysis of the highway skewed bridges with continuous deck in preliminary stage of design process. Moreover, the preliminary values of this method can help to identify the unknown errors occur during FE modeling of the structure.

Introduction

Dynamic behavior of highway bridges during earthquakes is complex and depends on the dynamic characteristics of the bridge. Therefore, understanding the effects of these characteristics is important for seismic analysis of the highway bridges. Skewed bridges are commonly used as overcrossings in highway intersections and interchanges, especially in crowded urban areas where lack of space necessitates the use of skew geometries. This type of bridge shows unusual behavior under dynamic load and was found to be quite susceptible to severe damage during the 1971 San Fernando and 1994 Northridge earthquakes (Wakefield et al 1991, Meng and Lui 2000). Hence, the necessity of understanding this complex behavior and effective important characteristics is felt. The dynamic characteristics of structures, such as circular frequencies and mode shapes, are usually determined by Finite Element Model (FEM) in final stage of the design process. Some simplified approaches for seismic calculation of tall buildings subjected to earthquake have been presented (Meftah et al 2007). However for highway bridges it has not been explicitly discussed. In an early work, the rigid body motions of skewed bridges using a one-dimensional rigid bar was truly analysed (Maragakis 1984). But the way of determination of natural frequencies and mode shapes was not explicitly discussed. An analytical solution for free vibration problem of single-span skewed bridge with symmetric slab-beam, was derived (Maleki 2000). However, the effect of stiffness of the skewed substructure was not

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considered. In this paper an approximate hand-method is presented for dynamic analysis of highway skewed bridges with continuous deck. The purpose of this paper is to develop a simple approximate expression for the calculation of the natural frequencies and seismic loads of the highway skewed bridge. It is assumed that the deck is rigid thereupon rigid body motion are the predominate motion of the bridge. The stiffness of substructure as well as the shear stiffness of elastomeric bearings are modeled with linear springs. The equation of motion and frequency equation are derived. Accuracy of the proposed method is surveyed using a FEM of a highway skewed bridge with continuous deck. It is shown that the presented equations are capable of studying the natural circular frequencies and mode shapes of a skewed bridge. The proposed method is simple and accurate enough to be used at the concept design stage in particular. It can be useful to verify the results of the FEM where the time-consuming operations and handling all the data can always be a source of error. Also, in next step of the research plan, authors of this paper are going to apply the presented model to control the undesirable seismic behavior of skewed bridges by using semi-active methods.

Dynamic model and equations of motion

Dynamic model

It is assumed that the deck of the bridge is rigid in-plane and the material remains linear during earthquake excitation. Consider Fig. 1 which shows an analytical model of a typical asymmetric highway skewed bridge with continuous deck. In the model the point O(0,0) is the coordinate origin. X and Y are longitudinal and transverse coordinate axes, respectively. N and T coordinate axes were obtained by counter-clockwise rotating X and Y axis through an angle β about the origin, as shown in Fig. 1. The angle β is the skew angle of the bridge. The point $C_m(X_m,Y_m)$ is the mass center of the bridge, U_X, U_Y and U_θ are the degrees of freedom of the model that are put at the mass center. The points $C_{ss}(e_{sx},e_{sy})$ and $C_{sb}(e_{bx},e_{by})$ are the stiffness centers of the substructure and elastomeric bearings with respect to the mass center, respectively. The coordinate of the stiffness center of substructure in the NT coordinate system is also shown with $C_{ss}(e_n,e_t)$. The stiffness of substructure is modeled with two linear translational springs K_N and K_T and a linear torsional spring K_θ. In addition, the shear stiffness of elastomeric bearings on the abutments is modeled with two linear translational springs K_{bX} and K_{bY} where $K_{bX}=K_{bY}=n_b k_b$. To compare with a realistic model, the elevation of a highway skewed bridge is shown in Fig. 2.

Equations of motion

The governing equation of the model in Fig. 1, subjected to strong ground motion can be written as follows:

$$[M]\{\ddot{U}(t)\}+[C]\{\dot{U}(t)\}+[K]\{U(t)\}=-[M]\{\ddot{U}_g(t)\} \tag{1}$$

Where $\{\ddot{U}(t)\}$, $\{\dot{U}(t)\}$ and $\{U(t)\}$ are the relative acceleration, velocity and displacement vectors of the mass center of the model and $\{\ddot{U}_g(t)\}$ is ground acceleration vector. [M], [C] and [K] are

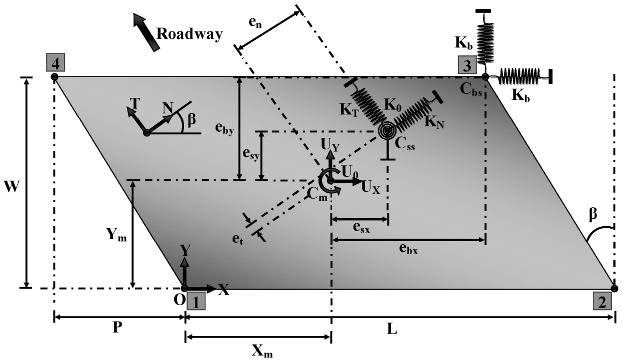


Figure 1. The 3DOFs dynamic model of a highway skewed bridge with continuous deck

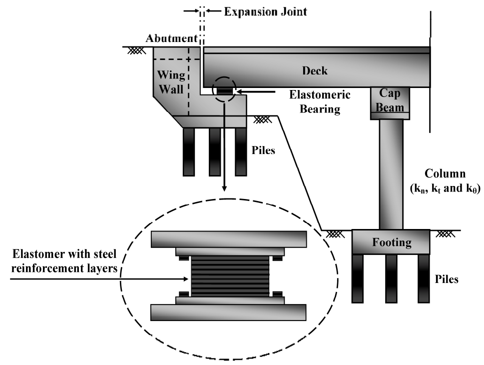


Figure 2. The elevation of a highway skewed bridge

mass, damping and stiffness matrices of the system. For the simplicity, the effect of rotational acceleration of ground on the model is neglected; hence, it is assumed that $\ddot{U}_{g\theta}(t)=0$.

The mass matrix and mass moment of inertia of the superstructure about the vertical axis passing through the center of mass of the bridge are defined as follows:

$$[M]=\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & I_{mz} \end{bmatrix} \quad \text{and} \quad \begin{aligned} I_{mz} &= \sum_{span} M_{di}\left(\frac{L_{di}^{2}+W^{2}+P^{2}}{12}+d_{zdi}^{2}\right) \\ &+ \sum_{capbeam} M_{ci}\left(\frac{L_{ci}^{2}+B_{ci}^{2}}{12}+d_{zci}^{2}\right) = M_{SupStr}\, r_z^{2} \end{aligned} \tag{2}$$

Where M is the summation of the masses of the substructure and superstructure, M_{SupStr} is the mass of the superstructure which is equal to $\Sigma M_{di}+\Sigma M_{ci}$. r_z is radius of gyration of the superstructure. In the first term of I_{mz}, M_{di} is mass of the deck in the *i*-th spans. L_{di} and W are the length and width of the deck in the *i*-th span respectively. P is equal to W×tan(β). Also d_{zdi} is the horizontal distance between the mass center of the deck of the *i*-th span and the mass center of the bridge. For the second term, M_{ci} is mass of the *i*-th capbeam. L_{ci} and W_{ci} are the length and width of the cross section of the *i*-th capbeam, respectively. Also d_{zci} is the horizontal distance between the mass center of the *i*-th capbeam and the mass center of bridge. Note that, due to the assumption of a direct connection between the capbeams and deck, the rotational inertia of them is considered in the formulations. However, the rotational inertia of the substructure is neglected.

Stiffness matrix of the model in XY coordinate system is defined as $[K]=[K_s]+[K_b]$. Where $[K_s]$ and $[K_b]$ are the stiffness matrices of the substructure and elastomer bearings, respectively. To develop the stiffness matrices of the substructure and elastomer bearings, the direct equilibrium method mentioned in text books on structural dynamics is employed (Chopra 1996, Clough and Penzien 2003). Hence, the matrix $[K_s]$ in NT coordinate system can be written as follows:

$$[K_s]_{NT}=\begin{bmatrix} K_N & 0 & -e_t K_N \\ 0 & K_T & +e_n K_T \\ -e_t K_N & +e_n K_T & e_t^{2} K_N + e_n^{2} K_T + K_\theta \end{bmatrix} \quad \text{and} \quad \begin{aligned} e_t &= -\sin\beta e_{sx} + \cos\beta e_{sy} \\ e_n &= +\cos\beta e_{sx} + \sin\beta e_{sy} \end{aligned} \tag{3}$$

And the matrix $[K_b]$ in XY coordinate system can be written as follows:

$$[K_b] = n_b k_b \begin{bmatrix} 1 & 0 & -e_{by} \\ 0 & 1 & +e_{bx} \\ -e_{by} & +e_{bx} & {e_{by}}^2 + {e_{bx}}^2 \end{bmatrix} \quad , \quad e_{bx} = \frac{\sum_{\text{bearing}} x_{bmi}}{n_b} \quad \text{and} \quad e_{by} = \frac{\sum_{\text{bearing}} y_{bmi}}{n_b} \tag{4}$$

Where k_b is the shear stiffness of an elastomer bearing, and n_b is the number of the elastomer bearings on two abutments. Also e_{bx} and e_{by} are stiffness eccentricities of elastomer bearings in X and Y direction with respect to the mass center, respectively. (x_{bmi}, y_{bmi}) is the coordinate of the *i*-th elastomer bearing on two abutments with respect to the mass center. To state the stiffness matrix of the substructure in XY coordinate system, $[K_s]$ can be written as $[K_s]_{XY} = [T]^T [K_s]_{XY} [T]$ where $[T]$ is the transformation matrix and is defined as follows:

$$[T] = \begin{bmatrix} +\cos\beta & +\sin\beta & 0 \\ -\sin\beta & +\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Finally, we have:

$$[K_s] = [K_s]_{XY} = \begin{bmatrix} K_{sXX} & K_{sXY} & e_{sx} K_{sXY} - e_{sy} K_{sXX} \\ K_{sXY} & K_{sYY} & e_{sx} K_{sYY} - e_{sy} K_{sXY} \\ e_{sx} K_{sXY} - e_{sy} K_{sXX} & e_{sx} K_{sYY} - e_{sy} K_{sXY} & {e_{sy}}^2 K_{sXX} - 2 e_{sx} e_{sy} K_{sXY} + {e_{sx}}^2 K_{sYY} + K_\theta \end{bmatrix} \tag{6}$$

Where:

$$K_{sXX} = K_N \cos^2\beta + K_T \sin^2\beta \quad , \quad K_{sYY} = K_N \sin^2\beta + K_T \cos^2\beta \quad , \quad K_{sXY} = \frac{1}{2}(K_N - K_T)\sin 2\beta$$

$$K_N = \sum_{\text{column}} k_{ni} \quad , \quad K_T = \sum_{\text{column}} k_{ti} \quad \text{and} \quad \begin{aligned} K_\theta = & \sum_{\text{column}} \left[(y_{smi} - e_{sy})\cos\beta - (x_{smi} - e_{sx})\sin\beta \right]^2 k_{ni} \\ & + \left[(x_{smi} - e_{sx})\cos\beta + (y_{smi} - e_{sy})\sin\beta \right]^2 k_{ti} + k_{\theta i} \end{aligned} \tag{7}$$

Here, k_{ni} and k_{ti} are flexural stiffness of the *i*-th column of substructure in N and T directions, respectively. $k_{\theta i}$ is the torsional stiffness of the *i*-th column of substructure about its vertical axis passing through the centroid of section. e_{sx} and e_{sy} are the stiffness eccentricities of substructure in the X and Y directions with respect to the mass center, respectively, and can be expressed as follows:

$$e_{sx}=\frac{\sum_{column}(k_{ni}\sin\beta+k_{ti}\cos\beta)x_{smi}}{\sum_{column}(k_{ni}\sin\beta+k_{ti}\cos\beta)}\quad\text{and}\quad e_{sy}=\frac{\sum_{column}(k_{ni}\cos\beta+k_{ti}\sin\beta)y_{smi}}{\sum_{column}(k_{ni}\cos\beta+k_{ti}\sin\beta)} \tag{8}$$

Where (x_{smi},y_{smi}) is the coordinate of the *i*-th column of substructure with respect to the mass center.

Natural circular frequencies and mode shapes

Natural circular frequencies of the model in Fig. 1 are obtained from the solution of free vibration equation of a similar undamped system by using classical damping assumption. The uncoupled circular frequencies of the system are defined as follows:

$$\omega_X=\sqrt{\frac{K_{sXX}}{M}},\omega_Y=\sqrt{\frac{K_{sYY}}{M}},\omega_{XY}=\sqrt{\frac{K_{sXY}}{M}},\omega_\theta=\sqrt{\frac{K_\theta}{I_{mz}}}\ \text{and}\ \omega_b=\sqrt{\frac{n_b k_b}{M}} \tag{9}$$

Where ω_{XY} is called the uncoupled cross circular frequency and is an important characteristic for a skewed bridge. It should be noted that for a straight bridge (β=0) ω_{XY}=0. Considering above definitions, the frequency equation can be written as follows:

$$\begin{vmatrix} \omega_X^{\ 2}-\omega^2 & \omega_{sXY}^{\ 2} & e_{sx}\omega_{sXY}^{\ 2}-e_{1sby}\omega_{sX}^{\ 2} \\ \omega_{sXY}^{\ 2} & \omega_Y^{\ 2}-\omega^2 & e_{1sbx}\omega_{sY}^{\ 2}-e_{sy}\omega_{sXY}^{\ 2} \\ e_{sx}\omega_{sXY}^{\ 2}-e_{1sby}\omega_{sX}^{\ 2} & e_{1sbx}\omega_{sY}^{\ 2}-e_{sy}\omega_{sXY}^{\ 2} & e_{2sby}^{\ 2}\omega_{sX}^{\ 2}-2e_{sx}e_{sy}\omega_{sXY}^{\ 2}+e_{2sbx}^{\ 2}\omega_{sY}^{\ 2}+r_z^{\ 2}\omega_\theta^{\ 2}-\alpha r_z^{\ 2}\omega^2 \end{vmatrix}=0 \tag{10}$$

Where $\alpha=M_{SupStr}/M$ and:

$$\begin{aligned} &\omega_X^{\ 2}=(1+\Psi_X)\omega_{sX}^{\ 2}\quad,\quad \Psi_X=n_b k_b/K_{sXX} \\ &\omega_Y^{\ 2}=(1+\Psi_Y)\omega_{sY}^{\ 2}\quad,\quad \Psi_Y=n_b k_b/K_{sYY} \\ &\omega_{sX}^{\ 2}=\omega_N^{\ 2}\cos^2\beta+\omega_T^{\ 2}\sin^2\beta \\ &\omega_{sY}^{\ 2}=\omega_N^{\ 2}\sin^2\beta+\omega_T^{\ 2}\cos^2\beta \\ &\omega_{sXY}^{\ 2}=\frac{1}{2}\left(\omega_N^{\ 2}-\omega_T^{\ 2}\right)\sin 2\beta \\ &\omega_N^{\ 2}=K_N/M\quad,\quad \omega_T^{\ 2}=K_T/M \end{aligned} \quad\text{and}\quad \begin{aligned} &e_{1sbx}=e_{sx}+e_{bx}\Psi_Y \\ &e_{2sbx}^{\ 2}=e_{sx}^{\ 2}+e_{bx}^{\ 2}\Psi_Y \\ &e_{1sby}=e_{sy}+e_{by}\Psi_X \\ &e_{2sby}^{\ 2}=e_{sy}^{\ 2}+e_{by}^{\ 2}\Psi_X \end{aligned} \tag{11}$$

Neglecting the shear stiffness of the elastomer bearings ($\Psi_X=\Psi_Y\approx 0$), the frequency equation of the model can be written as follows:

$$\begin{aligned} &\left(\omega_{sX}^{\ 2}-\omega^2\right)\left(\omega_{sY}^{\ 2}-\omega^2\right)\left(A^2+\omega_\theta^{\ 2}-\alpha\omega^2\right)+\left(B^2\omega_{sX}^{\ 2}+C^2\omega_{sY}^{\ 2}+D^2\omega_{sXY}^{\ 4}\right)\omega^2- \\ &\left(A^2\omega_{sX}^{\ 2}\omega_{sY}^{\ 2}+\omega_{sXY}^{\ 4}\omega_\theta^{\ 2}\right)=0 \end{aligned} \tag{12}$$

Where A, B, C, D and other parameters are defined as follows:

$$\begin{aligned} A^2 &= \mu_x{}^2 - 2\mu_{xy}{}^2 + \mu_y{}^2 \\ B^2 &= \mu_x{}^2 - 2\mu_{xy}{}^2 \\ C^2 &= \mu_y{}^2 - 2\mu_{xy}{}^2 \\ D^2 &= \lambda_x{}^2 + \lambda_y{}^2 + 1 \end{aligned} \quad \text{and} \quad \begin{aligned} \mu_x{}^2 &= \omega_{sX}{}^2\lambda_y{}^2 \quad , \quad \lambda_x = \frac{e_{sx}}{r} \\ \mu_y{}^2 &= \omega_{sY}{}^2\lambda_x{}^2 \quad , \quad \lambda_y = \frac{e_{sy}}{r} \\ \mu_{xy}{}^2 &= \omega_{sXY}{}^2\lambda_x\lambda_y \end{aligned} \qquad (13)$$

Note that neglecting the effect of elastomer bearings does not reduce the generality of the problem, because typical value for the shear stiffness of elastomer bearings is about 1000 to 2000 kN/m and in practice the order of parameters Ψ_X and Ψ_Y is approximately 10^{-3}~10^{-2} (Naeim and Kelly 1999).

In the analysis stage the RSA method is used for dynamic analysis of the model and the Complete Quadratic Combination (CQC) with damping ratio ξ=5% for the three modes is employed to combine the modal maximum responses. Finally, the maximum responses of the presented model subjected to the longitudinal and transverse translational acceleration are obtained using the square root of the sum of the squares (SRSS) (Chopra 1996, Clough and Penzien 2003).

Numerical example

The high torsional strength of the concrete box girder makes it particularly suitable for skewed piers and abutments, superelevation, and transitions such as interchange ramp structures (Lyang et al 2000). On the other hand, in this type of bridge capbeams are embedded in the deck and there is a direct connection between the deck and capbeams. In order to verify the accuracy of the proposed method in evaluation of the dynamic characteristics of the skewed bridges, a familiar highway skewed bridge with continuous deck in the literature is analysed (Wakefield et al 1991, Meng and Lui 2000, Jennings et al 1971). In Fig. 3, the plan, elevation, section of intermediate pier and cross section of the columns of southeastern bridge of Foothill Boulevard Undercrossing is shown. As can be seen in Fig. 3, this skewed bridge is a four-span continuous reinforced concrete box girder bridge with a skew angle of about 60˚. During the San Fernando earthquake on February 9th, 1971, this bridge suffered heavy damage in columns of the intermediate pier (Jennings et al 1971). It is assumed that the vertical translation and rotation about the longitudinal axis of the deck are restrained at the abutments. Also, the base of the columns in Bents 2 and 4 are modeled as pinned but those of the columns in Bent 3 are modeled as fixed. The retaining walls are not explicitly modeled in the analysis because the stiffness of the bents is not significantly affected by retaining walls (see Fig. 3). The capbeams are modeled as rigid beams to prevent excessive deflection. The modulus of elasticity is E=21689 MPa, Poisson's ratio is ν=0.2 and shear modulus is G=9037 MPa. The yield strength of the steel reinforcement is 270 MPa and compressive strength of concrete is 21 MPa. It is assumed that the mass density of reinforced concrete is about 2400 kg/m^3 (Meng and Lui 2000). The FEM of this bridge in SAP 2000 finite element program (SAP 2000 V.9) is shown in Fig. 4. Frame elements are used to model the columns and capbeams and four-node quadrilateral shell elements are used for modeling the rigid deck. Rigidity of the deck is achieved by assigning large values of

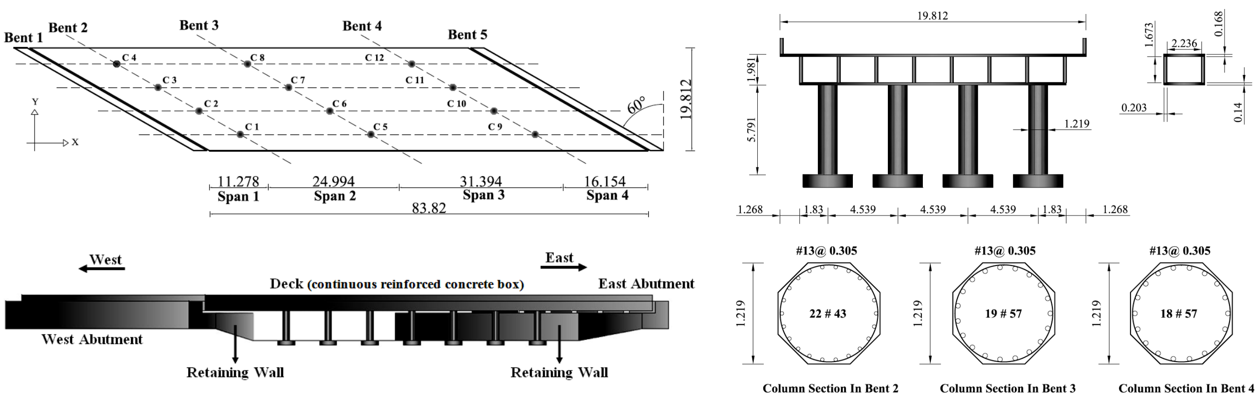


Figure 3. The pan, elevation, section of intermediate pier and cross section of columns of southeastern bridge of Foothill Boulevard Undercrossing (Unit: m).

modulus of elasticity to the shell elements. To model the elastomer bearings linear springs are used. The two components of the horizontal pseudo spectral acceleration of San Fernando earthquake on February 9th, 1971 with magnitude 6.6 M recorded in Pacoima Dam station (PEER Strong Motion Database) are used to analyze the FEM. Then, the dynamic characteristics and the response of the bridge to the earthquake will be calculated by the introduced hand-method and will be compared with the results of the FEM.

Analytical model of the Foothill Boulevard Undercrossing is shown in Fig. 5. It is seen that this model is an asymmetric bridge with an eccentricity in the X direction. The total mass of the bridge is M=2951140.8 kg and the mass moment inertia of the superstructure is $I_{mz}=2.377\times10^9$ kg.m^2. The mass center is C_m(25.898m,8.638m) with respect to the point O. The cross section of the columns is equated with a circular cross section with a diameter of D=1.269 m. Columns in Bent 2 and Bent 4 are fixed at top end and pinned at the base and the translational stiffnesses are $k_n=k_t=41044$kN/m and torsional stiffness is $k_\theta=0$. Columns in Bent 3 are fixed at two ends, the translational stiffnesses are $k_n=k_t=164175.9$kN/m and the torsional stiffness is $k_\theta=391642.9$kN.m. A typical shear stiffness for elastomer bearings has a value of $k_b=1500$kN/m and also, $n_b=16$. The stiffness eccentricities of the elastomer bearings using Eq.(4) are $e_{bx}=1.051$m and $e_{by}=0$ and the stiffness eccentricities of the substructure using Eq.(8) are $e_{sx}=-3.520$m and $e_{sy}=0$. Total translational and torsional stiffness of the substructure using Eq.(8) are $K_N=K_T=985055.2$ kN/m and $K_\theta=3.663\times10^8$ kN.m and total shear stiffness of the elastomer bearings is $n_bk_b=24000$kN/m. Uncoupled circular frequencies using Eq.(9) are obtained as $\omega_{sX}=\omega_{sY}=18.270$ rad/sec, $\omega_{sXY}=0$, $\omega_\theta=12.414$ rad/sec and $\omega_b=2.852$ rad/sec. The Natural circular frequencies using Eq.(12) are $\omega_1=12.265$ rad/sec, $\omega_2=18.491$ rad/sec and $\omega_3=18.728$ rad/sec. To compare with FEM results, the natural frequencies of Foothill Boulevard Undercrossing with the rigid deck assumption are calculated and shown in Table 1. The comparison of results indicates that the presented method is capable to obtain the dynamic characteristics of the skewed bridge.

On the other hand, it is seen that for both analytical and FEM of Foothill Boulevard Undercrossing the second mode shape is pure longitudinal translation. To explain this response, with note to above values of the skewed bridge it is seen that $K_N=K_T$ and as a result $\omega_N=\omega_T$. Therefore considering Eq.(7) and Eq.(9), for this bridge the uncoupled cross circular frequency is equal to zero ($\omega_{XY}=0$). Hence, neglecting the shear stiffness of the elastomer bearings ($\Psi_X=\Psi_Y\approx0$), Eq.(12) and its solution can be written as follows:

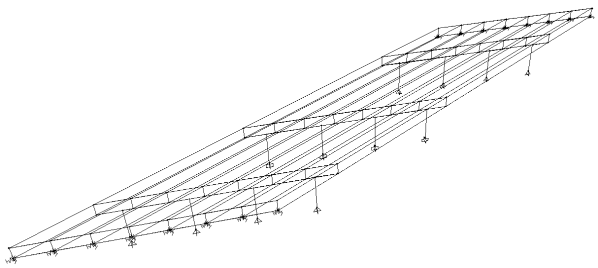


Figure 4. FEM of Foothill Boulevard Undercrossing - SAP 2000.

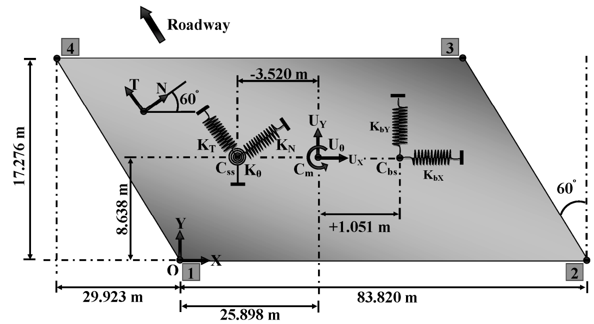


Figure 5. Analytical model of Foothill Boulevard Undercrossing

Table 1. The natural frequencies of the Analytical model and FEM

Mode	Analytical Model (rad/sec)	FEM (rad/sec)	Interpretation of the mode shapes
1 st	12.265	12.280	In-plan rotation coupled with translation parallel to Y direction
2 nd	18.491	17.834	Translation parallel to X direction
3 rd	18.728	18.061	Translation parallel to Y direction coupled with in-plan rotation

$$\left(\omega^2-\omega_N{}^2\right)\left(\omega^4-\left[\left(\lambda_x{}^2+\lambda_y{}^2+1\right)\omega_N{}^2+\omega_\theta{}^2\right]\omega^2+\omega_N{}^2\omega_\theta{}^2\right)=0$$
$$\omega=\omega_N \quad \text{and} \quad \omega=\omega_N\sqrt{\left(\frac{\Omega_N{}^2}{2}+\lambda^2\right)\pm\sqrt{\left(\frac{\Omega_N{}^2}{2}+\lambda^2\right)^2-\Omega_N{}^2}} \tag{14}$$

Where $\Omega_N=\omega_\theta/\omega_N$ and $\lambda^2=(\lambda_x{}^2+\lambda_y{}^2+1)/2$. The natural circular frequencies of Foothill Boulevard Undercrossing are generally stated by Eq.(14). It is seen that for this case the skewed bridge has a translational independent mode and two coupled modes. The ratio of the natural circular frequencies to ω_N (ω/ω_N) versus Ω_N for different values of λ^2 are simultaneously plotted in Fig. 6. Note that as in practice the value of λ^2 is limited such that $0.5\leq\lambda^2<1$. As shown the translational independent mode lies between the two coupled modes, hence, this mode always is the second mode. Also, it is observed that for the little values of Ω_N, the first mode approaches to the rotational mode (in the direction of the arrow) and for large values of Ω_N has a trend to the translational mode. Behavior of the third mode is reverse. For this bridge $\lambda^2=0.501$ and $\Omega_N=0.462$. Hence, the mode shapes of the second mode are obtained as follows:

$$\{\phi_N\}=c_N\left\{1.0 \quad \frac{e_{sy}}{e_{sx}} \quad 0\right\}^T=c_N\{1.0 \quad 0 \quad 0\}^T \tag{22}$$

As can be seen the second mode is a pure longitudinal translation mode. Therefore, it is obvious that the analytical model acceptably demonstrates the dynamic behavior of the skewed bridge.

Then, the seismic responses of the bridge include the torsional moment at the columns base, shear forces in the N and T direction and displacement of the corner nodes of the deck in the X and Y direction are calculated and shown in Fig. 7, Fig 8 and Fig 9, respectively.

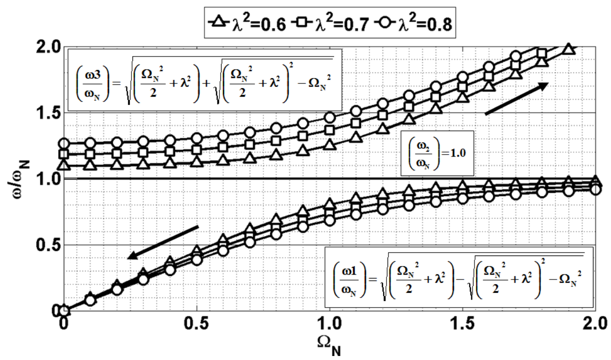


Figure 6. The natural circular frequencies of Foothill Boulevard Undercrossing

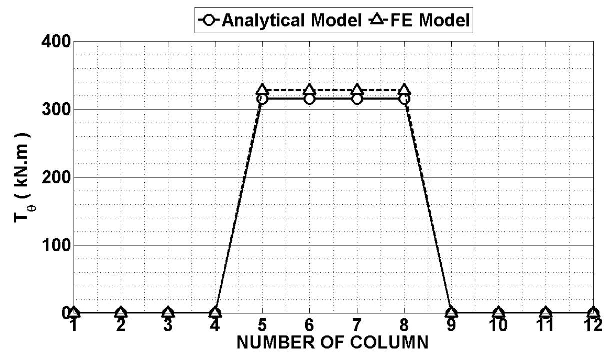
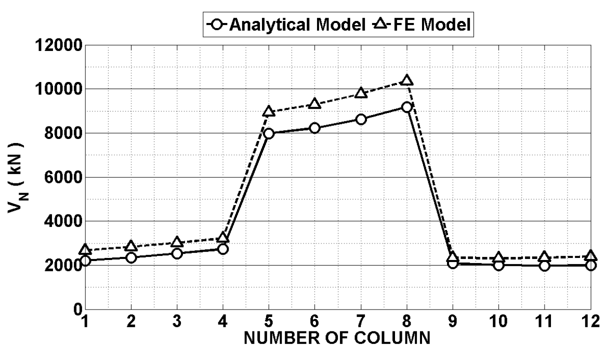
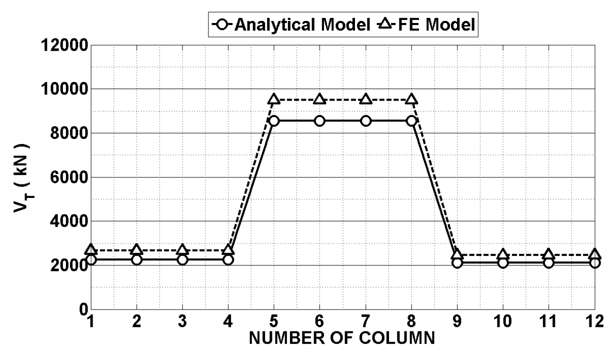


Figure 7. The torsional moments of the columns

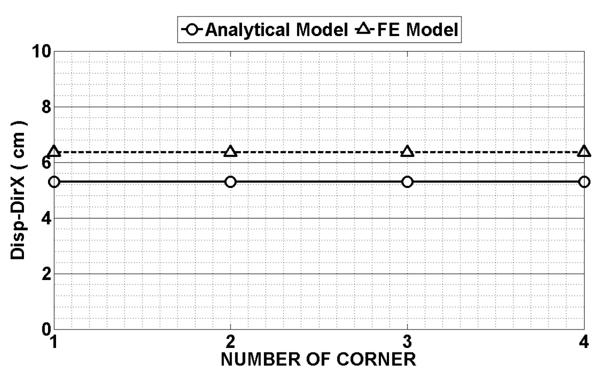


(a)

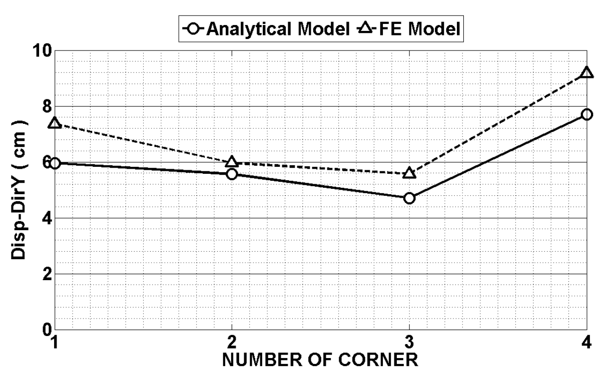


(b)

Figure 8. The shear forces of the columns, (a)N direction (b)T direction



(a)



(b)

Figure 8. The displacement of the corner nodes, (a)X direction (b)Y direction

The results show that the model is satisfactory to determine the general dynamic features of the skewed bridge with continuous rigid deck.

Conclusions

A generalized hand-method was presented for dynamic analysis of the highway skewed bridges with continuous deck. In the presented model it was assumed that the deck is rigid in-plane and the rotational effect of ground motion was neglected. The method was verified using a finite element model subjected to earthquake excitation. It was shown that the results from the proposed method as an approximate method in the preliminary analysis are in good agreement with the finite element model as a reliable method in the final stage of analysis. Since in practice, using the presented formulas is more available than a FE program, this approximate method is suggested to bridge engineers for seismic calculations of highway skewed bridges in preliminary phase of a seismic design process. In addition, the preliminary values of method can help identify the unknown errors occur during FE modeling of the structure in commercial package programs.

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