



SIMPLIFIED SEISMIC ANALYSIS AND EXPERIMENTAL STUDIES ON BUILDINGS WITH MID-STORY ISOLATION SYSTEM

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ABSTRACT

The mid-story isolation design method is recently gaining popularity for the seismic protective design of buildings particularly located at highly populated areas. In a mid-story isolated building, the isolation system is incorporated into the mid-story rather than the base of the building. In this paper, the dynamic characteristics and seismic responses of mid-story isolated buildings are investigated using a simplified three-lumped-mass structural model for which equivalent linear properties are formulated. It is found that the nominal frequencies of the superstructure and the substructure respectively above and below the isolation system have significant influences on the isolation frequency and equivalent damping ratio of a mid-story isolated building. The mass and stiffness of the substructure are of greater significance than the superstructure in affecting the dynamic characteristics. The higher mode responses may contribute significantly to the story shear force of the substructure. Moreover, the adverse effect arising from the coupling of higher modes on the acceleration responses of the superstructure is presented numerically and experimentally. A simple method to guarantee the mid-story isolation design against the modal coupling due to the improper design of the substructure and superstructure is proposed.

Introduction

The excellent performance of seismically isolated structures in past earthquakes, in addition to numerical and experimental validation, has encouraged the seismic isolation design for structural protection (Fujita 1998, Nagarajaiah et al. 2000, Zhou et al. 2007). The application of seismic isolation design in Taiwan has also been extensive after the 1999 Chi-Chi earthquake (Chang et al. 2007). The mid-story isolation design, in which the isolation system is typically installed on the top of the first story of a building, is recently attracting intense attention because it can satisfy the architectural concerns of aesthetics and functionality as well as enhance the construction feasibility at highly populated areas. The effectiveness of mid-story isolation design in reducing seismic demand on the superstructure has been numerically proved (Ogura et al.

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1999). A simplified two-lumped-mass structural model has been proposed to numerically investigate the seismic responses of mid-story isolated structures (Koh et al. 2000).

The existing seismic isolation design guidelines are tailored for base-isolated buildings rather than for mid-story isolated buildings (International Building Code 2006, Seismic Design Code of Buildings 2006). The current design of mid-story isolated buildings usually follows the equivalent lateral force procedure provided for base-isolated buildings, assuming that the substructure is sufficiently stiff or rigid. However, the dynamic behaviors of a mid-story isolated structure and a base-isolated structure may not be identical since the seismic responses of a mid-story isolated building may be significantly affected by the flexibility of the substructure. Furthermore, the interaction between the superstructure and substructure may not be negligible.

The mid-story isolation design should consider the following issues: (1) the seismic performance of the superstructure of a mid-story isolated building is warranted to be as good as that of a base-isolated building; (2) the fundamental period and effective damping ratio of the isolated building should be accurately captured if the equivalent lateral force procedure is to be applied; (3) possible damages to the substructure should be prevented; and (4) special attention has to be paid such that the unexpected seismic responses due to the coupling of higher modes can be avoided (Chang et al., 2009, Koh et al. 2004). Therefore, this study aims to investigate the dynamic characteristics and seismic responses of mid-story isolated buildings represented by a simplified three-lumped-mass structural model. The corresponding equivalent linear properties of the simplified model are formulated and the effects of various parameters on the dynamic characteristics of mid-story isolated structures are examined. The response spectrum analysis is conducted to investigate the significances of all vibration modes on the seismic responses of mid-story isolated buildings. Furthermore, the adverse effect arising from the coupling of higher modes on the seismic responses of the isolated structure is numerically and experimentally presented. A simple method to guarantee the mid-story isolation design against the modal coupling due to the improper design of the substructure and superstructure is proposed.

Equivalent Linear Characteristics

Referring to the simplified two-lumped-mass structural model for base-isolated buildings proposed by Kelly (Kelly, 1990) as given in Fig. 1(a), a mid-story isolated building may be represented by a simplified three-lumped-mass structural model, composed of the superstructure, isolation system and substructure, as shown in Fig. 1(b). All structural elements, except the isolation system, are assumed to remain elastic under earthquakes. The hysteretic behavior of the seismic isolation system is represented by an equivalent linear model composed of effective stiffness and equivalent damping ratio. The equation of motion is written in terms of story drifts

$$\begin{aligned}
& \begin{bmatrix} m_{sub} & 0 & 0 \\ m_{iso} & m_{iso} & 0 \\ m_{sup} & m_{sup} & m_{sup} \end{bmatrix} \begin{Bmatrix} \ddot{v}_{sub} \\ \ddot{v}_{iso} \\ \ddot{v}_{sup} \end{Bmatrix} + \begin{bmatrix} c_{sub} & -c_{iso} & 0 \\ 0 & c_{iso} & -c_{sup} \\ 0 & 0 & c_{sup} \end{bmatrix} \begin{Bmatrix} \dot{v}_{sub} \\ \dot{v}_{iso} \\ \dot{v}_{sup} \end{Bmatrix} \\
& + \begin{bmatrix} k_{sub} & -k_{iso} & 0 \\ 0 & k_{iso} & -k_{sup} \\ 0 & 0 & k_{sup} \end{bmatrix} \begin{Bmatrix} v_{sub} \\ v_{iso} \\ v_{sup} \end{Bmatrix} = - \begin{bmatrix} m_{sub} & 0 & 0 \\ 0 & m_{iso} & 0 \\ 0 & 0 & m_{sup} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \ddot{u}_g
\end{aligned} \tag{1}$$

where v_{sub} , v_{iso} and v_{sup} = the story drifts of the substructure, isolation layer and superstructure, respectively; \ddot{u}_g = the ground acceleration; m_{sub} , m_{iso} and m_{sup} = the seismic reactive masses of the substructure (or sub-floor), isolation layer (or super-floor) and superstructure, respectively; k_{sub} and k_{sup} = respectively the elastic lateral stiffnesses of the substructure and superstructure; k_{iso} = the effective lateral stiffness of the isolation system; c_{sub} and c_{sup} = respectively the viscous damping coefficients of the substructure and superstructure; and c_{iso} = the equivalent viscous damping coefficient of the isolation system. The mass ratios of the substructure and superstructure to the super-floor are defined as $r_{sub} = m_{sub} / m_{iso}$ and $r_{sup} = m_{sup} / m_{iso}$, respectively. The ‘‘nominal frequencies’’ (Kelly, 1990) are defined as $\omega_{sub} = \sqrt{k_{sub} / m_{sub}}$, $\omega_{sub}^* = \sqrt{k_{sub} / (m_{sub} + m_{iso} + m_{sup})}$, $\omega_{iso} = \sqrt{k_{iso} / (m_{iso} + m_{sup})}$ and $\omega_{sup} = \sqrt{k_{sup} / m_{sup}}$. The component damping ratios of the substructure, isolation system and superstructure are respectively expressed as $\xi_{sub} = c_{sub} / 2\omega_{sub}^* (m_{sub} + m_{iso} + m_{sup})$, $\xi_{iso} = c_{iso} / 2\omega_{iso} (m_{iso} + m_{sup})$ and $\xi_{sup} = c_{sup} / 2\omega_{sup} m_{sup}$. The characteristic equation can be obtained by solving the eigenvalue problem of Eq. 1

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0 \tag{2}$$

where

$$a = - \left(\omega_{sub}^2 + \frac{(1+r_{sub})(1+r_{sup})}{r_{sub}} \omega_{iso}^2 + (1+r_{sup}) \omega_{sup}^2 \right) \tag{3}$$

$$b = (1+r_{sup}) \omega_{sub}^2 \omega_{iso}^2 + (1+r_{sup}) \omega_{sub}^2 \omega_{sup}^2 + \frac{(1+r_{sub}+r_{sup})(1+r_{sup})}{r_{sub}} \omega_{iso}^2 \omega_{sup}^2 \tag{4}$$

$$c = -(1+r_{sup}) \omega_{sub}^2 \omega_{iso}^2 \omega_{sup}^2 \tag{5}$$

From Eq. 2 to 5, the n^{th} modal natural frequency ω_n and the n^{th} mode shape of story drifts $\{(\phi_{sub})_n, (\phi_{iso})_n, (\phi_{sup})_n\}^T$ can be calculated in which $n=1 \sim 3$. The fundamental modal natural frequency ω_1 may be very close to the isolated frequency ω_{iso} and is well separated from the residual modal natural frequencies if the elastic lateral stiffnesses of the substructure and superstructure are much greater than the effective lateral stiffness of the isolation system.

Assuming $(\omega_{iso} / \omega_{sub})^2$ and $(\omega_{iso} / \omega_{sup})^2$ are of an order less than 10^{-1} and neglecting the high order terms of $\omega_{iso} / \omega_{sub}$ and $\omega_{iso} / \omega_{sup}$, the first modal damping ratio ξ_1 can be approximated by

$$\xi_1 = \frac{\xi_{iso}}{\left(1 + \frac{2(1+r_{sup})}{r_{sub}} \left(\frac{\omega_{iso}}{\omega_{sub}}\right)^2 + \frac{2r_{sup}}{1+r_{sup}} \left(\frac{\omega_{iso}}{\omega_{sup}}\right)^2\right)} \quad (6)$$

If the elastic lateral stiffnesses of the substructure and superstructure are much greater than the effective lateral stiffness of the isolation system (i.e. $\omega_{iso} / \omega_{sub}$ and $\omega_{iso} / \omega_{sup}$ are sufficiently small), it is reasonable to assume that the first mode is the isolation mode and the effective damping ratio is equal to the first modal damping ratio. To investigate the condition that the coupling of higher modes occurs in the simplified three-lumped-mass structural model (i.e. $\omega_2 \approx \omega_3$), the characteristic equation as given in Eq. 2 can be factorized into

$$(\lambda - \omega_{iso}^2)(\lambda^2 + (a + \omega_{iso}^2)\lambda + b + \omega_{iso}^2(a + \omega_{iso}^2)) \quad (7)$$

Two roots for the quadratic equation in terms of λ in Eq. 7 are obtained as

$$\lambda_{2,3} = \frac{-(a + \omega_{iso}^2) \pm \sqrt{(a + \omega_{iso}^2)^2 - 4(b + \omega_{iso}^2(a + \omega_{iso}^2))}}{2} \quad (8)$$

Since $\omega_{iso}^2 \ll |a|$, the component in the square root in Eq. 8 can then be approximated by

$$f(a, b) = a^2 - 4(b + a\omega_{iso}^2) \quad (9)$$

It is obvious that the coupling of higher modes occurs when $\lambda_2 \approx \lambda_3$ in Eq. 8, which indicates that there exists a minimum positive value for $f(a, b)$ as given in Eq. 9. Multiplying $1/\omega_{iso}^4$ into Eq. 9 in which a and b are respectively substituted by Eq. 3 and 4, the first partial differential equations with respect to $(\omega_{sub} / \omega_{iso})^2$ and $(\omega_{sup} / \omega_{iso})^2$ are set to zero to determine the frequency ratio condition that the coupling of higher modes occurs. The differential equations can be rewritten in the standard forms of hyperbolas with the identical asymptotes as follows

$$(\omega_{sub} / \omega_{iso}) = (\omega_{sup} / \omega_{iso}) \sqrt{1 + r_{sup}} \quad \text{or} \quad \omega_{sub} = \omega_{sup} \sqrt{1 + r_{sup}} \quad (10)$$

The adoption of Eq. 10 can facilitate engineers to prevent the mid-story isolation design from the modal coupling. It also provides some useful and meaningful information as follows

1. The asymptote is capable of accurately predicting the condition that the coupling of higher modes occurs with less difficulty because it is linear function of ω_{sub} , ω_{sup} and r_{sup} .

2. The condition of the modal coupling is independent of ω_{iso} . That is, the mid-story isolation design with the modal coupling is independent of various types of seismic isolation bearings, diverse mechanical properties of seismic isolation bearings and different deformation extent that the isolation system undergoes under frequent, moderate or major earthquakes.

Parametric Study

A series of parametric studies are conducted with two cases of mass ratios $(r_{sub}, r_{sup}) = (1, 5)$ and $(r_{sub}, r_{sup}) = (2, 5)$. The frequency ratios $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ are assumed to vary from 3 to 40. ξ_{sub} and ξ_{sup} are assumed to be 5% while ξ_{iso} is presumed to be 20%. The effective period of the isolation system is assumed to be 2.0 seconds. The comparison between ω_1 and ω_{iso} with respect to various $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ is shown in Fig. 2. Furthermore, ξ_1 varying with respect to $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ is illustrated in Fig. 3. It is found that the important system characteristics of a mid-story isolated building, including ω_1 and ξ_1 , are more significantly affected by both the stiffness and mass of the substructure than by those of the superstructure. The assumptions that $\omega_1 = \omega_{iso}$ and $\xi_1 = \xi_{iso}$ are rational or acceptable when $\omega_{sub} / \omega_{iso}$ is sufficiently large.

The dynamic characteristics of the higher modes varying corresponding to $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ are summarized in Figs. 4 to 5. From Fig. 4, it is seen that the third modal participation mass ratio is nearly zero in the frequency ratio region where the second modal participation mass ratio is effective and vice versa. In between the two frequency ratio regions where either the second or the third mode is effective, there exists a frequency ratio bandwidth in which the coupling of higher modes occurs. In summary, the dynamic characteristics of higher modes can be categorized in Fig. 5 in which the domain formed corresponding to various frequency ratios is divided into three regions: (1) the second mode effective region; (2) the third mode effective region; and (3) the bandwidth of modal coupling.

Response Spectrum Analysis

The 5% damped design spectrum with $S_{DS} = 0.8$ and $S_{D1} = 0.4$ is selected. The design spectrum is modified by damping adjustment factors (International Building Code 2006, Seismic Design Code of Buildings 2006) corresponding to the modal damping ratios. The Complete Quadratic Combination (CQC) method is used to estimate the maximum responses from each modal peak spectral value (Wilson et al. 2007). The second modal, third modal and the maximum inertia forces at the substructure, $(F_{sub})_2$, $(F_{sub})_3$ and F_{sub} , are shown in Figs. 6(a), 6(b) and 6(c) respectively. It is found that the maximum inertia force exerting at the substructure of Fig. 6(c) is almost equal to the maximum modal inertia force of either the second or the third mode of Fig. 6(a) or 6(b). Therefore, it is concluded that the maximum inertia force exerting at the substructure is primarily attributed to the higher modal responses rather than the fundamental modal response. Besides, a larger r_{sub} generally results in a larger maximum inertia force F_{sub} .

acting at the sub-floor (or substructure) as shown in Fig. 6(c). Therefore, the calculation of design force of the substructure (or the base shear force) should carefully consider the contribution of higher mode responses. The maximum acceleration responses (or maximum inertia forces) at the super-floor and superstructure are mainly attributed to the first modal inertial forces. However, it is noted that significant response amplifications are seen in Fig. 7, at the super-floor in particular, when $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ fall within the frequency ratio bandwidth where the coupling of higher modes occurs. The response amplification due to the modal coupling is particularly obvious when $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ are relatively small.

The analytical study discloses that the effect of the substructure on the seismic responses of mid-story isolated buildings is of particular importance when the substructure possesses a relatively small nominal frequency ω_{sub} . Designing the substructure and superstructure with higher nominal frequencies, ω_{sub} and ω_{sup} , will produce a better seismic performance of the isolated structure. Furthermore, the participation of higher mode responses should be taken into account when designing the substructure. When the substructure possesses a larger mass, the modal participation mass ratio of higher modes becomes larger. Therefore, the effects of higher modes should be appropriately considered when designing the substructure using the equivalent lateral force procedure for mid-story isolated buildings. More importantly, the improper design of the substructure and superstructure resulting in the coupling of higher modes should be avoided.

Experimental Study

Two test structures are 1/4 scaled down three-story steel frames and the mid-story isolation system above the first story is composed of four lead-rubber bearings (LRB). The seismic reactive masses assigned in the first floor (or substructure), second floor (or super-floor) and third floor (or superstructure) are equal to $5 \text{ kN-sec}^2 / \text{m}$, $3 \text{ kN-sec}^2 / \text{m}$ and $14 \text{ kN-sec}^2 / \text{m}$, respectively. Thus, r_{sub} and r_{sup} are respectively equal to 1.7 and 4.7 for both test structures. Besides, the effective period of the isolation system corresponding to the design displacement of 55 mm is designed to be 1.0 seconds or $\omega_{iso} = 2\pi$. The first test structure, denoted as Specimen A and illustrated in Fig. 8(a), is to simulate a building isolated at the top of the first story without the modal coupling. The heights of the substructure and superstructure are respectively 0.8 m and 2.5 m so that $(\omega_{sub} / \omega_{iso}, \omega_{sup} / \omega_{iso}) = (17.7, 3.3)$. The second test structure, denoted as Specimen B and illustrated in Fig. 8(b), is to simulate a mid-story isolated building with the modal coupling. All the design parameters of Specimen B are identical to those of Specimen A except the height of the substructure is 2 m so that $(\omega_{sub} / \omega_{iso}, \omega_{sup} / \omega_{iso}) = (7.8, 3.3)$. The coupling of higher modes is expected to occur in Specimen B based on Eq. 6 (i.e. $(\omega_{sub} / \omega_{iso}) = 3.3\sqrt{(1+4.7)} \approx 7.8$).

Two ground motions recorded at TAP097 and TAPBAF stations respectively during the 2002 Hualien earthquake and the 1999 Chi-Chi earthquake (respectively denoted as 331TAP097 and 921NCREE) are normalized to the peak ground acceleration (PGA) of $0.32g$ for the uniaxial shaking table tests. The story drift and acceleration response histories at the substructure, super-

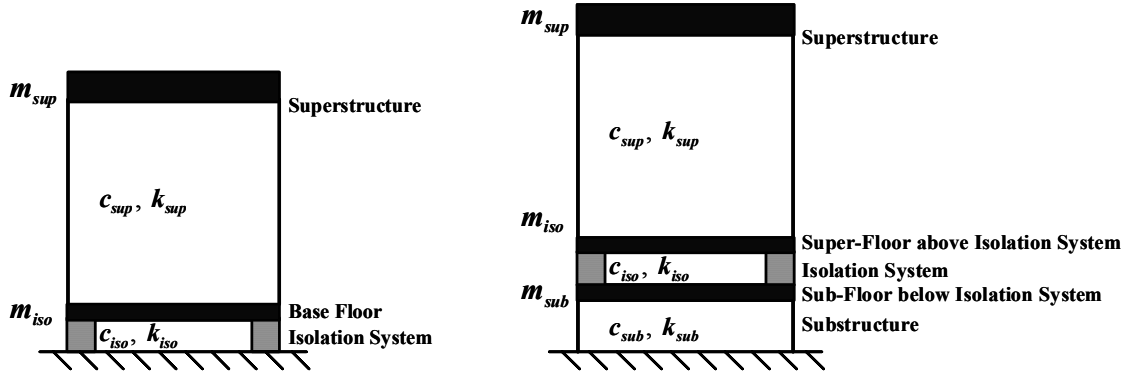
floor and superstructure of the test structures subjected to 331TAP097 are shown in Fig. 9. It is seen that the mid-story isolation design reveals the satisfactory seismic performance since the acceleration response transmitted to the superstructure can be reduced effectively compared with the ground excitation. However, the story drift and acceleration responses at the substructure of Specimen B are much severer than those of Specimen A. It is because that the substructure of Specimen B is designed to exhibit the more flexible behavior compared with that of Specimen A. In addition, the higher mode participation is more significant and the acceleration response at the super-floor is severer in Specimen B than those in Specimen A due to modal coupling effect. The seismic response at the superstructure is almost not affected by the flexibility of the substructure and the modal coupling. In summary, the flexibility of the substructure and the coupling of higher modes lead to the enlarged seismic responses at the substructure and super-floor, respectively.

Conclusions

The equivalent linear characteristics of mid-story isolated buildings represented by a three-lumped-mass model are formulated. It is found that the two frequency ratios, $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$, have significant influences on the first modal frequency and first modal damping ratio. Since the effect of $\omega_{sub} / \omega_{iso}$ is more significant, the mass and stiffness of the substructure are more important than those of the superstructure in affecting the dynamic characteristics of a mid-story isolated building. Observed from the response spectrum analysis, the maximum seismic responses at the super-floor and superstructure are mainly attributed to the first modal responses. However, it is noted that significant response amplifications of the acceleration responses may occur when $\omega_{sub} / \omega_{iso}$ and $\omega_{sup} / \omega_{iso}$ fall within the frequency ratio bandwidth of the coupling of higher modes. Therefore, a simple method to guarantee the mid-story isolation design against the modal coupling is presented. Besides, it is noted that the contribution of higher mode responses to the inertia force at the substructure is significant such that the design of substructure should carefully consider the higher mode contribution. Consequently, the lateral force distribution often used in the equivalent lateral force procedure for base-isolated buildings may not be applicable to mid-story isolated buildings without appropriately considering the effect of higher modes.

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(a) Two-lumped-mass model
 (b) Three-lumped-mass model
 Figure 1. Simplified models for base isolated and mid-story isolated buildings.

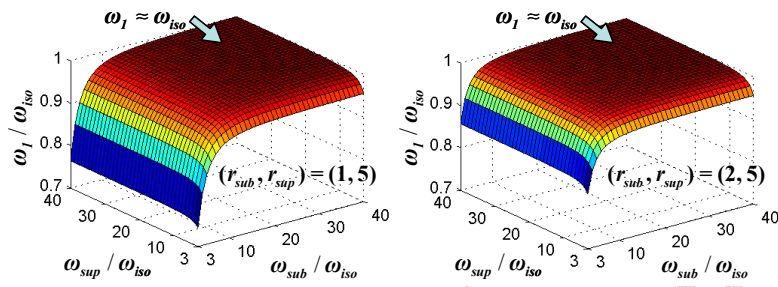


Figure 2. Comparison between ω_1 and ω_{iso} .

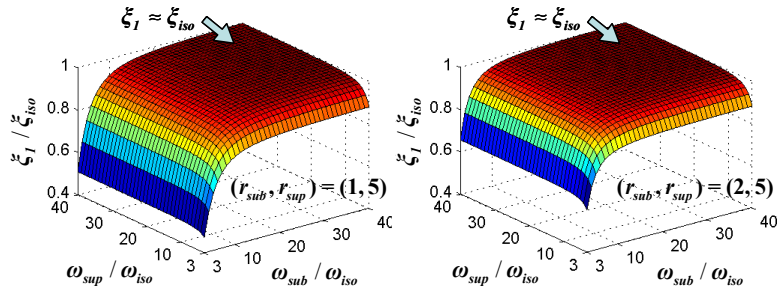
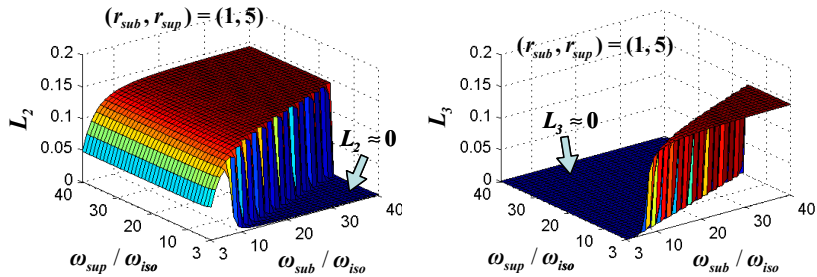


Figure 3. Comparison between ξ_1 and ξ_{iso} .



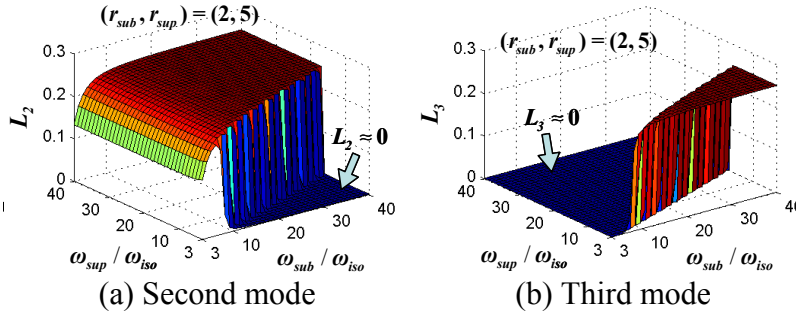


Figure 4. Higher modal participation mass ratios.

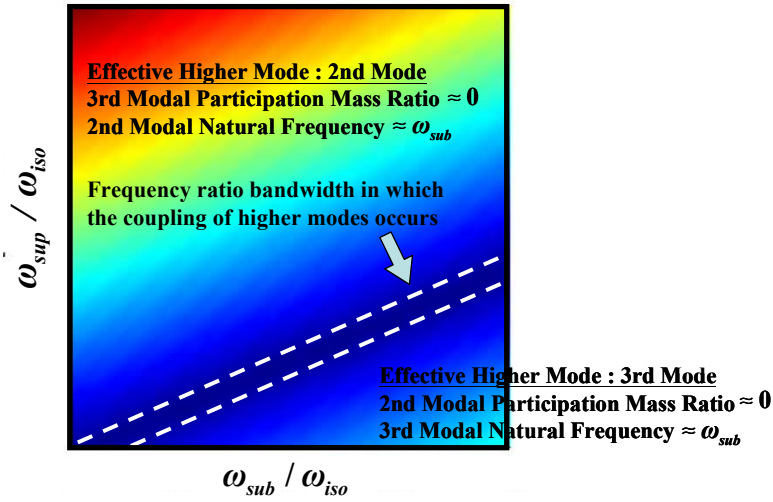


Figure 5. Summarized dynamic characteristics of higher modes.

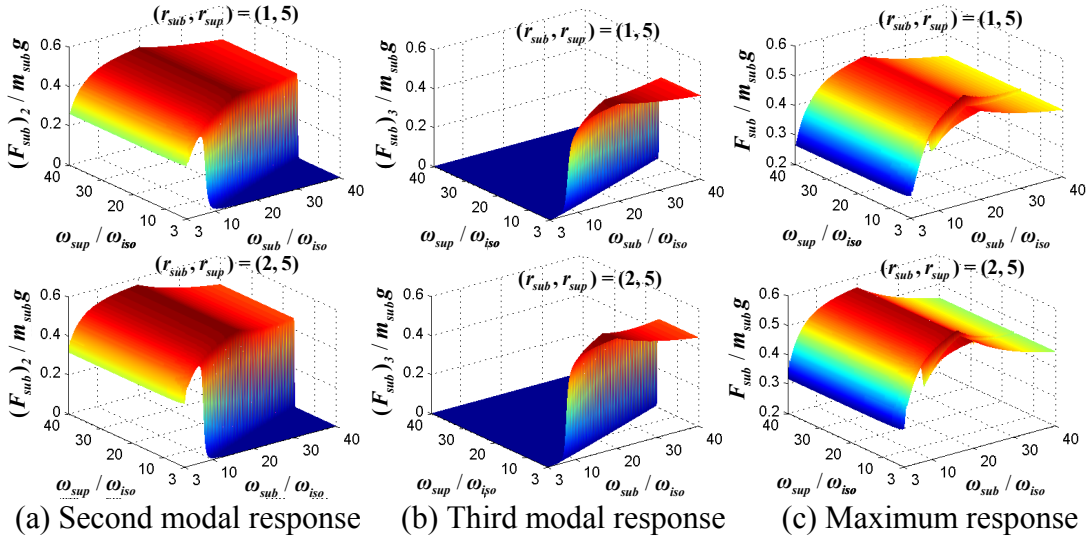


Figure 6. Inertia force responses at substructure.

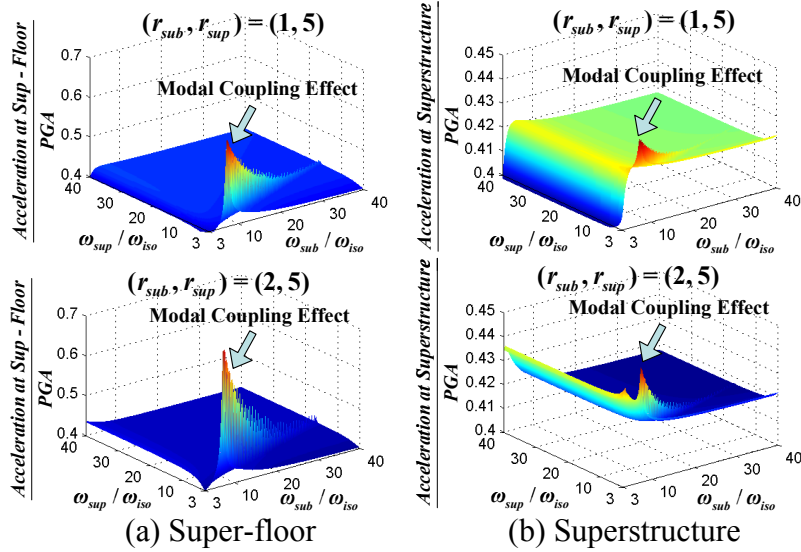


Figure 7. Ratios of maximum accelerations at super-floor and superstructure to PGA values



(a) Specimen A



(b) Specimen B

Figure 8. Installation photos of test structures.

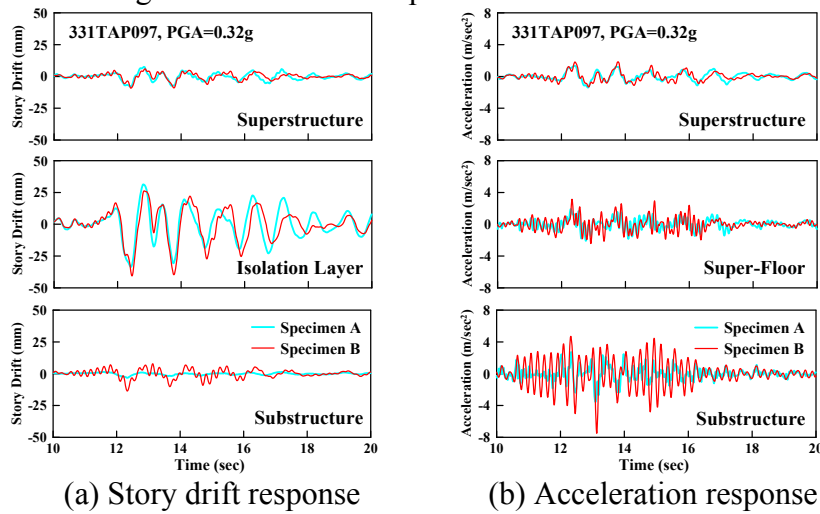


Figure 9. Comparisons of seismic response histories of test structures subjected to 331TAP097 with PGA of 0.32g.

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