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ESTIMATION OF COLLAPSE CAPACITY AND COLLAPSE FRAGILITY USING MODAL PUSHOVER ANALYSIS

Sang Whan Han[1], Ki-Hoon Moon[2], Anil K. Chopra[3]

ABSTRACT

Structural collapse during earthquakes causes massive loss of economy and human lives. For protecting structures from collapse, accurate estimation of collapse intensity and collapse fragility is very important. To estimate the collapse intensity of a structure, incremental dynamic analysis (IDA) can be used. The IDA of practical structures, however, is computationally extremely demanding since the IDA requires repeated nonlinear response history analyses (RHA) of a structure for an ensemble of ground motions. This study develops a simple and accurate procedure for estimating the collapse intensity and collapse fragility of a structure using modal pushover analyses (MPA) and an empirical equation of collapse strength ratios.

Introduction

Prediction of collapse potential of buildings is an important issue in earthquake engineering. Incremental Dynamic Analysis (IDA) (Vamvatsikos, 2002) can be used to estimate collapse potential of the buildings. Since the IDA requires nonlinear response history analysis (RHA) of the structure for an ensemble of ground motions, each scaled to many intensity levels, selected to cover a wide range of structural response all the way from elastic behavior to global dynamic instability, the IDA is computationally extremely demanding for practical structures.

This paper develops a Modal Pushover Analysis (MPA) based approximate procedure to quantify the collapse potential of buildings. The MPA (Chopra and Goel, 2002), an approximate analysis procedure rooted in structural dynamics theory (Chopra, 2007), has been utilized to estimate seismic demands instead of nonlinear RHA. The accuracy of MPA in estimating IDA curves was demonstrated in Han and Chopra (2006). This paper explores the potential of MPA in investigating the collapse of buildings. Collapse here is synonymous with dynamic sideway instability in one or several stories of the structural system. For verifying the accuracy of the proposed procedure using MPA, collapse intensity and collapse fragility curves for two steel moment frames are estimated using exact IDA and the proposed approximate method.

[1] (corresponding author)Professor of Architectural Engineering, Hanyang University, Seoul 133-791, Korea.
e-mail: swhan82@hotmail.com Fax: +82-2-2291-1716
[2] Graduate student of Architectural Engineering, Hanyang University, Seoul 133-791, Korea.
[3] Horace, Dorothy, and Katherine Johnson Chair in Engineering, University of California, Berkeley, Calif. 94720-1710 USA.

Collapse intensity and collapse fragility curves

The ground motion intensity that causes collapse of a building is defined as collapse intensity (A_c). IDA can be used to estimate A_c . From the IDA, one IDA curve is obtained under one ground motion, which is a plot of ground motion intensity against a seismic demand parameter. In this study, the ground motion intensity is characterized by $A(T_1, \zeta_1)$, the spectral pseudo-acceleration corresponding to the period(T_1) and damping ratio (ζ_1)of the first-mode of elastic vibration, and the demand parameter is represented by maximum over all stories of the peak inter-story drift ratio ($\theta_{\max}$), defined as the story drift divided by the story height.

Figure 1(a) shows the IDA curves for the SAC-Los Angeles 9-story building (Gupta and Krawinkler, 1999) for an ensemble of 20 ground motions (Vamvatsikos and Cornell, 2004). Calculated from these data, the 16, 50, and 84% fractile values of the intensity measure (IM) for a given $\theta_{\max}$ are presented in Figure 1(a). Global dynamic instability is identified by the vertical segment of the IDA curve, where the seismic demand increases greatly with the slightest increase in ground motion intensity. Adopting this terminology, the collapse intensity value, A_c, of the IM is identified by solid circles for individual ground motions in Figure 1(a) and for the fractile values for the ground motion ensemble in Figure 1(a).

Following Zareian and Krawinkler (2007), the "collapse fragility curve" is defined as the cumulative distribution function, assuming a lognormal distribution, of the A_c values for the ground motions in the ensemble considered. Figure 1 (b) shows collapse fragilities using 20 A_c marked in Fig. 1(a) , together with their lognormal distribution calculated from the A_c data.

IDA curves of model buildings

Two model buildings with rectangular plan are considered and used as model buildings: 9- and 20-story buildings (Gupta and Krawinkler, 1999). Here, the lateral resistance for these buildings is provided by special steel moment-resisting frames (SMRF) along the plan perimeter; the N-S exterior frames of the two buildings are used as examples. The frame is idealized by the M1 model, a basic centerline model in which size, stiffness, and strength of the panel zone are not included (Gupta and Krawinkler, 1999). $P-\Delta$ effect due to gravity loads are included in the analysis, but the strength and stiffness deterioration of structural members are not considered.

The first three natural periods of the structures vibrating within the elastic range are: 2.34, 0.88, and 0.50 sec for the 9-story building; and 3.98, 1.36, and 0.79 sec for the 20-story building. The damping matrix is constructed as $\mathbf{c} = a_o\mathbf{k} + a_I\mathbf{m}$, where $\mathbf{k}$ and $\mathbf{m}$ are the initial elastic stiffness matrix and mass matrix, respectively, and the constants a_0 and a_1 are determined from specified damping ratios (ζ) at two periods. For the 9 story building, damping ratios of 2% are specified at the first- and fifth mode periods, whereas for the 20-story building, damping ratios of 2% are specified at the first-mode period and at 0.2 sec.

Figure 2 shows the 1st mode pushover curve for each of the two buildings. It is idealized by the strength limited bilinear model (Ibarra et al., 2005), shown to be appropriate to represent the behavior of moment resisting steel frame buildings (Han and Chopra, 2006). Table 1summarizes the properties of the hysteretic model for the 1st mode pushover curve for the two buildings. The details of strength limited bilinear model can be found in Ibarra et al. (2005).

An ensemble of 20 ground motions was selected. Listed in Vamvatsikos and Cornell, (2004), these motions were recorded on firm soil, during three earthquakes of M 6.5-6.9 (Loma Prieta, 1989; Superstition Hills, 1987; and Imperial Valley, 1979) at distances ranging from 15 to 32 km.

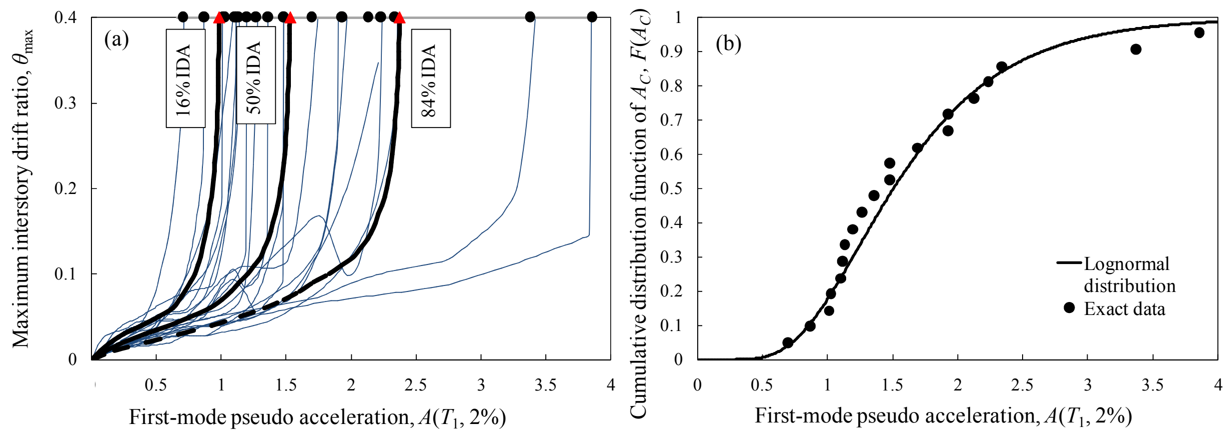

Figure 1 (a) individual IDA curves and 16%, 50%, and 84% fractile IDA curves (collapse intens ity is denoted by solid circles); (b) Collapse fragility curve for SAC LA 9-story building:

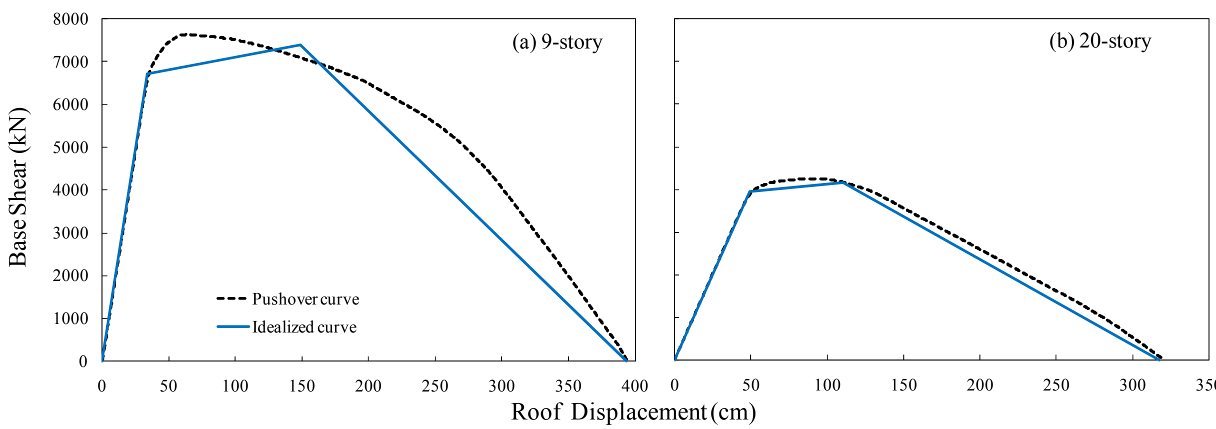

Figure 2. First-mode static pushover curves and their idealization for two buildings: (a) 9-story building; and (b) 20-story building.

The dynamic response of each frame to each of the 20 ground motions scaled to the selected intensity A (T_1, 2%) was determined by two procedures: nonlinear RHA and MPA. The 16%, 50%, and 84% fractile values of A (T_1, 2%) for a given θ_{max} were computed, and plotted against θ_{max}. The MPA procedure provides a computationally efficient, although approximate, alternative to nonlinear RHA. The MPA procedure is available in a convenient step-by step form (Chopra and Goel, 2002). In applying MPA to obtain IDA curves for all fractiles, an nth-mode pushover analysis of the structure is implemented only once. The resulting database provides all the response information needed to estimate seismic demands due to any ground motion scaled to any intensity level. The "modal" response is extracted from this database at the roof displacement u_{rn} due to the selected ground motion at the selected intensity level.

Table 1. Properties of first-mode SDF system

Property parameters	9-story	20-story
T_1	2.34 sec	3.98 sec
α_s	0.03	0.04
μ_c	4.40	2.25
α_c	-0.15	-0.25
A_y	0.18 g	0.09 g

Accuracy of MPA-Estimate of Collapse Fragility Curve

Figure 3 compares the MPA-based approximate IDA curves, including contributions of a variable number of modes, and the exact IDA curves for 9- and 20-story buildings. Figure 4 shows the ratio of approximate collapse intensity, A_c obtained from the MPA-IDA to A_c obtained from exact IDA using nonlinear RHA of the MDF systems. This plot permits three observations: (1) the higher mode contributions are significant in determining the IM corresponding to smaller values of demand; (2) the first mode alone is sufficient in the MPA-based approximate procedure to determine the IM corresponding to the larger values of demand; and (3) the MPA-based approximate value of the IM becomes increasingly accurate at larger values of demand. The last observation implies that the collapse intensity, A_c, and hence the collapse fragility curve, can be determined accurately by the MPA-based approximate IDA by including only the first vibration mode. Higher modes of vibration have essentially no influence on the approximate value of A_c, and the one-mode value is within 8% of the exact value.

Figure 5 presents the cumulative distribution function (CDF), $F(A_c)$, determined by fitting a lognormal distribution to the data of 20 values of Ac, determined by two methods: nonlinear RHA based exact analysis, and MPA based approximate analysis including only the first mode. The influence of higher vibration modes on the collapse fragility curve is seen to be negligible, and one mode alone is adequate to estimate the collapse fragility curve.

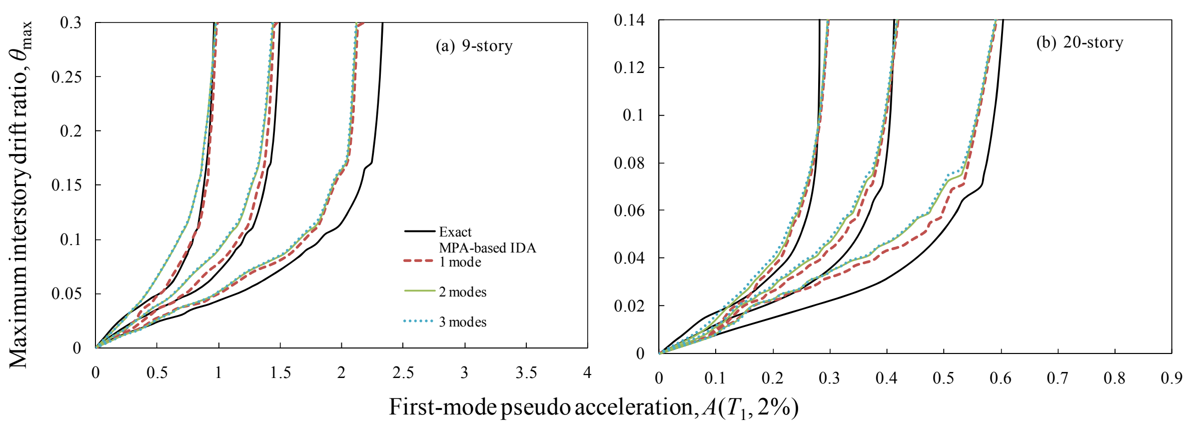

Figure 3. 16%, 50%, and 84% fractile IDA curves from MPA based IDA, versus exact IDA

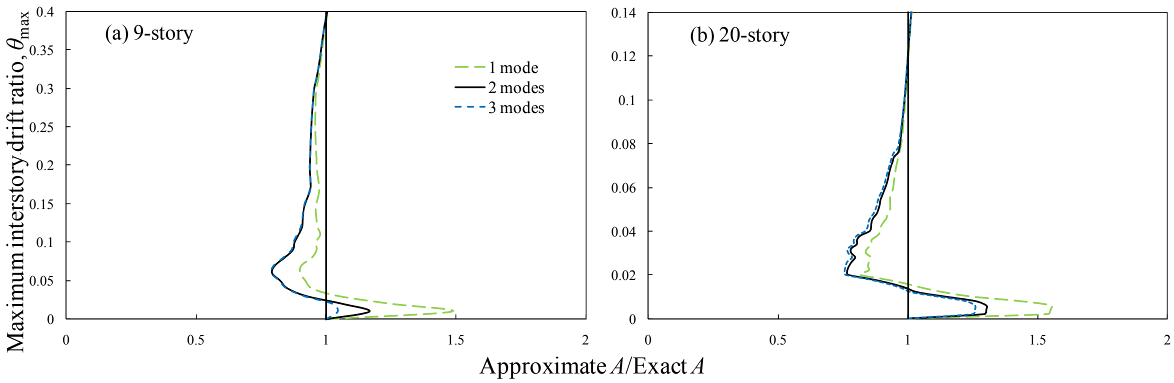

Figure 4. Ratio of approximate and exact values of IM, $A(T_1, 2\%)$

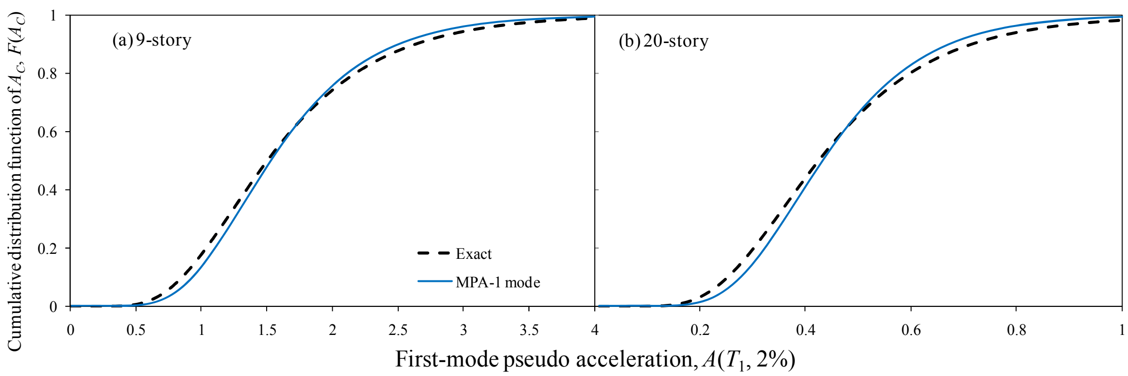

Figure 5. Collapse fragility curves determined by MPA-based IDA versus exact IDA

Collapse fragility curves for SDF systems

To encompass the full range of response from elastic to collapse to determine the collapse intensity of the SDF system, this MPA-based approximate IDA would require a series of nonlinear RHA of the first-mode inelastic SDF system subjected to each of the ground motions scaled to several levels of intensity. For practical application, we avoid such nonlinear RHA by developing empirical equations to estimate the desired intensity for strength-limited bilinear systems.

Collapse Intensity

The ground motion intensity required to cause collapse of SDF systems can also be determined by IDA. As mentioned earlier, an IDA curve is a plot of the ground motion intensity, characterized by, say, $A(T_n, \zeta)$, versus a demand parameter, say, the peak deformation of the system. Such an IDA curve is presented in Figure 6(a) for three strength-limited bilinear SDF systems. Dynamic instability is identified by the vertical segment of the IDA curves, where the demand increases greatly with the slightest increase in ground motion intensity. The collapse intensity, A_c, is identified by a solid circle (Figure 6); also shown for reference is the peak deformation u_o of the corresponding linear system and the yield strength A_y of the nonlinear system.

Figure 6(b) shows the IDA curves of Figure 6(a) re-plotted using normalized scales: the vertical axis is now the inelastic deformation ratio, defined as the ratio of peak deformation u_m to peak deformation u_0 of the corresponding linear system; the horizontal axis plots the strength ratio $R = f_o / f_y$, where f_y is the yield strength of the system and $f_o = mA(T_1, 5\%)$ is the minimum strength required for the system to remain elastic. The strength ratio, R_c, at which results are presented for three different strength-limited bilinear systems, all with $T_n = 0.5$ sec and the same slopes α_s and α_c of the force-deformation relation but different values of the ductility coefficient μ_c, is referred to as the *collapse strength ratio*: $R_c = m\, A_c / f_y$.

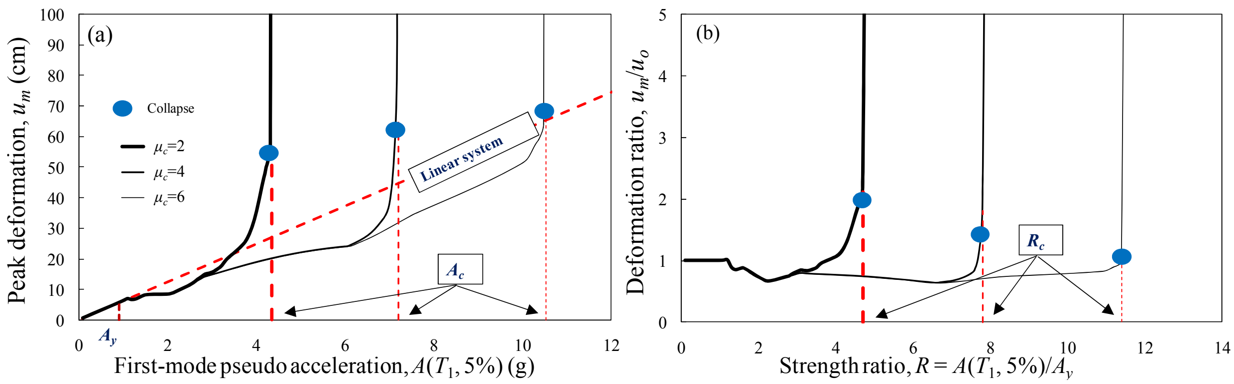

Figure 6. Collapse intensity of excitation: (a) $A_c(T_1, 5\%)$; and (b) collapse strength ratio R_c.

Empirical Equations for Median and Dispersion of R_c

To avoid the large number of nonlinear RHA of SDF systems for each ground motion required to determine the collapse strength ratio, R_c, an empirical equation for R_c is available for bilinear systems (Miranda and Akkar, 2003). Developed herein for strength-limited bilinear system with 5% damping, the empirical equation was determined by regression analysis of the database of R_c values developed by IDA of 1200 $(= 20 \times 5 \times 3 \times 4 \times 1 \times 1)$ SDF systems, covering the range of parameters listed in Table 2, implemented for 240 ground motions; thus the database included 288,000 $(= 1200 \times 240)$ values of R_c. The basis of an earlier study (Ruiz and Miranda, 2005), included are 80 ground motions recorded on three NEHRP site classes B, C, and D, leading to a total of 240 ground motions recorded during earthquakes with magnitudes ranging from 5.8 to 7.7. Because R_C depends on five parameters (T_n, ζ, α_S, α_C, and μ_C) and their influence on R_C is inter-related with the other parameters, it was difficult to choose a functional form of R_C. It was selected by trial and error and the equation parameters were determined by regression analysis of the exact data from nonlinear RHA, resulting in the following equation for the median value of R_C. For systems with 5% damping:

$$\left[(R_c)_{\zeta = 5\%} \right]_{50\%} = 1 + \mu_c \left(1 - e^{-2.5 T_n}\right) \left(-0.38 \mu_c^a \alpha_c^{-1}\right)^{\mu_c^{-b} + 0.03 T_n - 0.03 \ln T_n} \qquad (1)$$

wherein coefficients a and b are presented in Table 3 for selected values of α_s; these parameter values may be determined by interpolation for other values of α_s.

For any other value of damping ratio ζ, R_c is estimated from its value for 5%-damped systems as follows:

$$(R_c)_\zeta = C_\zeta (R_c)_{\zeta=5\%} \tag{2}$$

Regression analysis of the database of its exact values determined by IDA of 4800 $(= 20\times5\times3\times4\times4\times1)$ SDF systems (Table 2) and 240 ground motions, resulted in the empirical equation:

$$C_\zeta = \left\{1-\left[\frac{0.07\ln\zeta+0.20}{T_n^{0.38}(-\alpha_c)^{-0.26}\,\mu_c^{-0.44}}\right]\right\} \tag{3}$$

Regression analysis of the database of 1,152,000 values of R_c led to an empirical equation for the dispersion measure:

$$\sigma_{\ln R_c} = \frac{0.22}{\zeta^{0.05}}\left[(-\alpha_c)^{0.03\mu_c-0.29}+\frac{0.04\mu_c^{1.12}-0.14}{\zeta^{0.28}\cdot T_n^{0.36}}\right] \tag{4}$$

An overall test of the accuracy of the proposed empirical equations is presented in Figure 7, wherein estimated and exact values of 16, 50, and 84% fractiles of R_c are presented. The estimated values are determined from empirical Eqs. (1)-(4); and the exact values come from exact IDAs. This comparison indicates that the empirical equations provide a satisfactory estimate of R_c. The agreement is better with the 16% and 50% fractile data and deteriorates for the 84% data.

Table 2. System parameter values selected for developing empirical equations for R_c.

Parameters	**Range**	**Number of Values**
T_n	0.2 ~ 4.0 (ΔT_n = 0.2)	20
α_s	0.00, 0.03, 0.05, 0.10, 0.20	5
α_c	-0.1, -0.3, -0.5	3
μ_c	1, 2, 4, 6	4
ζ	2, 5, 10, 20 %	4
f_r	0	1

Table 3. Numerical values for coefficients a and b in Eq. (1) for different values of α_s

Coefficients	**Strain-hardening stiffness ratio, α_s**				
	0.00	**0.03**	**0.05**	**0.1**	**0.2**
a	0.00	0.30	0.38	0.65	1.57
b	1.09	0.85	0.61	0.51	1.07

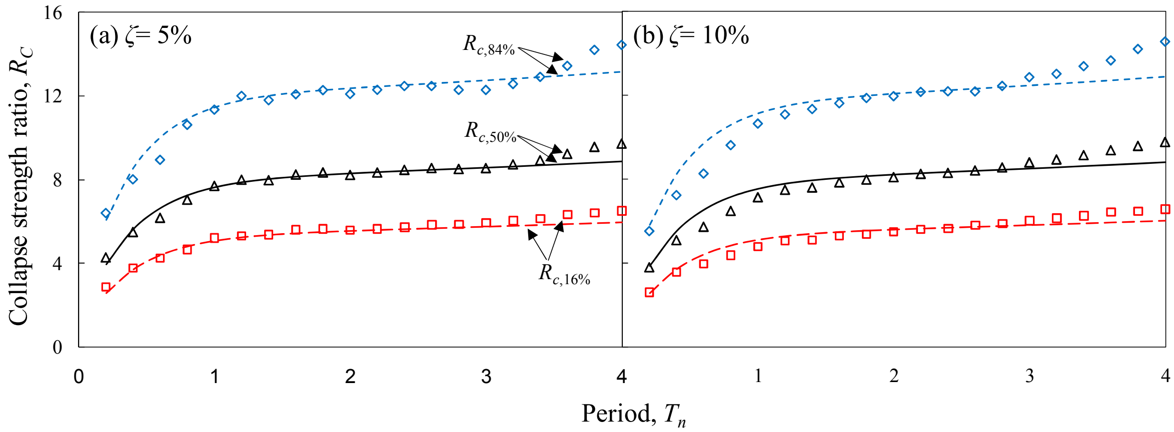

Figure 7. Comparison of empirically determined 16%, 50%, and 84% fractile values of R_c with data from exact IDA for systems ($\alpha_s = 0.03$, , and $\mu_c = 4$)

Estimating collapse fragility curves FOR MDF systems

In an earlier section, the collapse fragility curve for a building was determined by IDA of the structure for 20 ground motions. Against this exact curve, we compared the approximate results determined by MPA-based approximate IDA, considering only the first mode of vibration. The first-mode pushover curve is converted to the force-deformation curve for SDF systems, which is idealized as a strength-limited bilinear system by established equations (Chopra, 2007; Chapter 19). The pseudo-acceleration, A_y, associated with the yield-strength, f_y, of the SDF system and the system parameters are presented in Table 1. Corresponding to these properties, the median value of collapse strength ratio R_c is determined from Eqs. (1) and (2), and the value of $\sigma_{\ln A_c}$ from Eq. (4); because A_y is assumed to be a deterministic quantity, not a random variable, $\sigma_{\ln A_c} = \sigma_{\ln R_c}$.

The collapse intensity A_c for the first-mode SDF system is determined from $A_c = R_c A_y$. Using this median value (50%) of A_c and $\sigma_{\ln A_c}$ and assuming a lognormal distribution, approximate values of the 16 and 84% fractile A_c are computed as median A_c times $e^{\mp\delta}$ where dispersion δ is $\sigma_{\ln A_c}$; these are presented in Table 4 where they are compared with the data determined by IDA of the structure.

Figure 8 shows approximate and exact fragility curves assuming lognormal distributions with approximate median A_c and $\sigma_{\ln A_c}$ calculated by using empirical Eqs. (1)-(4), and with exact median A_c and $\sigma_{\ln A_c}$ determined by exact IDA for 20 ground motions, respectively. Clearly, the approximate results are quite accurate. The accuracy of the MPA-based approximate method is excellent in estimating the collapse fragility curves and the 16, 50, and 84% fractile values of A_c for a range of building heights; the accuracy deteriorates slightly for the 20-story building, presumably because higher modes contribute significantly to its response.

The MPA based approximate procedure, which provides good estimates of ground motion intensity necessary to cause collapse of a building, requires a small fraction of the computational effort compared to that required in the "exact" procedure that uses nonlinear RHA to compute demands. If a Pentium 4 processor with 3.0 GHz CPU and DDR 2GB RAM is used, the computer time for analysis of one frame of the SAC Los Angeles 9 story building is reduced from 3 hours for the exact result to only 2 minutes for the approximate procedure. Thus, a fast estimate of the collapse fragility curves for multistory buildings is achieved at only a small loss in accuracy.

Table 4. Exact and approximate values of 16, 50, and 84% fractile values of A_c (units of g).

Building	16%		50%		84%	
	Exact	Approximate	Exact	Approximate	Exact	Approximate
9-story	0.970	1.018	1.507	1.522	2.341	2.273
20-story	0.283	0.303	0.425	0.429	0.638	0.606

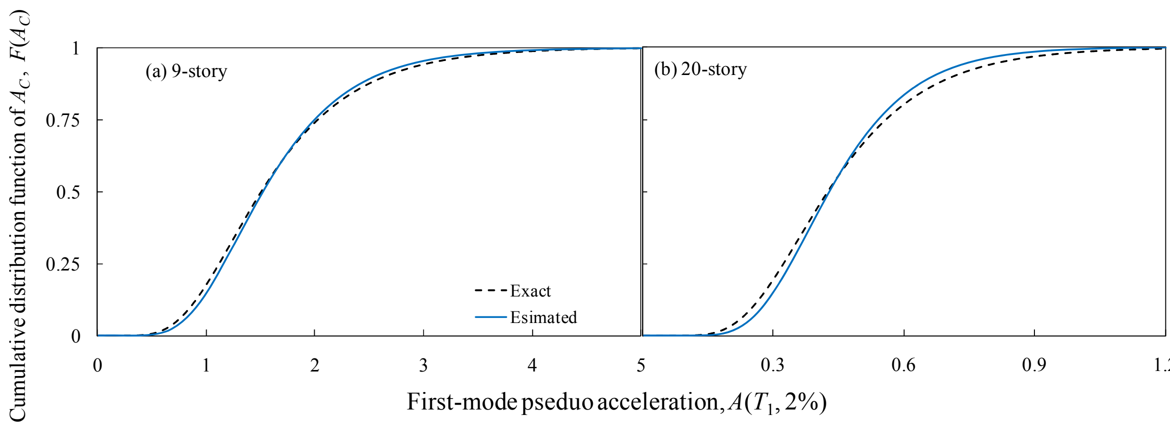

Figure 8. Collapse fragility curves determined by two methods: (1) one-mode MPA-based IDA and empirical equations for R_c for SDF systems, and (2) nonlinear RHA-based IDA for two buildings.

CONCLUSIONS

This study proposes an approximate method for estimating collapse intensity and collapse fragility of MDF systems. This study shows that only the first mode of vibration of a structure needs to be considered in an MPA-based approximate method to determine the ground motion intensity, A_c, necessary to cause collapse of a building. Thus the collapse fragility curve can be determined by IDA of the first-mode inelastic SDF system derived by a first-mode pushover analysis of the building. In the proposed method, empirical equation of R_c is used which is proposed to avoid the series of nonlinear RHA of this SDF system subjected to each ground motion scaled to several intensity levels, empirical equations have been developed to determine the collapse strength ratio, R_c. These equations were developed for the strength limited bilinear model. Results for 6-, 9-, and 20-story steel SMRF buildings demonstrated that a one-mode MPA-based approximate method combined with the empirical equations for R_c for SDF systems estimates collapse fragility curves that are highly accurate. Estimating collapse fragility of the structure by MPA leads to a highly efficient procedure. In one of the examples considered, the MPA-based approximate method required a small fraction (roughly 1%) of the computational effort required in the “exact” procedure.

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