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FRAGILITY CURVES FOR RC BUILDINGS USING HIGH DIMENSIONAL MODEL REPRESENTATION

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ABSTRACT

Fragility curves represent the conditional probability that a structure's response may exceed the performance limit for a given ground motion intensity. Conventional methods for computing building fragilities are either based on statistical extrapolation of detailed analyses on one or two specific buildings or make use of Monte Carlo simulation with these models. However, the Monte Carlo technique usually requires a relatively large number of simulations in order to obtain a sufficiently reliable estimate of the fragilities, and it is computationally expensive and time consuming to simulate the required thousand of nonlinear seismic analyses. In this paper, High Dimensional Model Representation (HDMR) based response surface method together with the Monte Carlo Simulation is used to develop the fragility curve, which is then compared with that obtained by using Latin Hypercube sampling. It is used to replace the algorithmic performance-function with an explicit functional relationship, fitting a functional approximation, thereby reducing the number of expensive numerical analyses. After the functional approximation has been made, Monte Carlo simulation is used to obtain the fragility curve of the system

Introduction

Fragility analysis was originally employed to evaluate the seismic safety of nuclear power plants and was later accepted as a reliable method for the evaluation of the seismic performance of civil infrastructures like bridges and buildings (Cimellaro et al. 2006). Fragility curve represents the conditional probability that the response of a structure may exceed the performance limit for a given ground motion intensity. They are useful tools for the estimation of probability of structural damage due to earthquakes as a function of ground motion indices or other design parameters.

Fragility curves can be derived using empirical or analytical methods, based on the source of data and the type of analysis (Basöz and Kiremidjian 1999; Shinozuka et al. 2000a; Singhal and Kiremidjian 1998; Yamazaki 1999). The former is based on the interpretation of test data and engineering judgments, while the latter uses the response of a structure to a set of ground motions. The response can be either from a linear or nonlinear time history analysis, elastic spectral analysis or nonlinear static analysis. Analytical fragility curves are used when the actual earthquake damage data is limited and there is a lack of sufficient statistical information. The

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seismic response data may be obtained from the computational analysis of the structure using simulated ground motions.

Conventional means of generating fragility curve involves the use of Direct Monte Carlo Simulation (MCS) and other sampling techniques. The Monte Carlo Simulation technique usually requires relatively large number of simulations in order to obtain a sufficiently reliable estimate of the fragilities, making it computationally expensive and time consuming (Towashiraporn et al. 2004). This approach was later replaced by metamodels, which are essentially a statistical approximation of the complex and implicit phenomena, formulated using response surface methods. Response is estimated in a closed-form function of input variables thus requiring less computational effort (Buratti et al.; Schotanus and Pinto 2002; Towashiraporn et al. 2004).

The present study adopts an efficient means for reducing the computational cost for developing fragility curves and also overcoming the inherent difficulties associated with the response surface methodology (like Design of Experiments etc.) by using High Dimensional Model Representation (HDMR) based response surface method. The basic idea of the method is to replace the complex computationally burdensome mechanistic model by an approximate yet accurate analytical relationship between the response of the structure (output variables) and the basic random variables (input variables) involved in the analysis. Monte Carlo simulation is then performed over the simpler analytical model instead of a large number of complex dynamic analyses, to obtain the fragility curve for the structure, giving the conditional probability of exceedance of particular limit state for different values of the seismic intensities

High Dimensional Model Representation

High Dimensional Model Representation (HDMR) is a tool developed to express input-output relations of complex, computationally burdensome models in terms of hierarchical correlated function expansions (Alis and Rabitz 2001; Li et al. 2001a; Rabitz and Alis 1999; Rajib et al. 2008). Application of HDMR methodology to complex nonlinear model provides an efficient means to obtain an accurate reduced model of the original system. The uncertainty analysis of the outputs of the computationally burdensome model can then be well approximated by MC simulation of the corresponding reduced model outputs, which is thus performed at a much lower computational cost without compromising accuracy. It is a general set of quantitative model assessment and analysis tool for capturing the high dimensional relationships between sets of input and output model variables, where the order of input variables may go up to order of 10^2 ~ 10^3 or more.

As an example, let an N-dimensional vector $x = \left\{ x_1, x_2, x_3 .... x_n \right\}$ be the input variable and $f(x)$ be the output variable. HDMR expresses the output variable as a hierarchical correlated function expanded in terms of the input variables as given in Eq. 1

$$f(x) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1 \le i \le j \le N} f_{ij}(x_i, x_j) + \sum_{1 \le i \le j \le k \le N} f_{ij}(x_i, x_j, x_k) + ... + f_{1,2....N}(x_1, x_2, ..., x_N) \quad (1)$$

where, f_0 denotes the mean response or response of the system evaluated at a reference point. The function $f_i\,(x_i)$ is a first-order term expressing the effect of variable x_i acting alone, although generally nonlinear, on the output $f(x)$. The function $f_{i_1 i_2}(x_{i_1}, x_{i_2})$ is a second-order term which describes the cooperative effects of the variables x_{i_1} and x_{i_2} on $f(x)$. The higher order terms gives the cooperative effects of increasing numbers of input variables acting together to influence the output $f(x)$. The last term $f_{12....N}(x_1, x_2, ..., x_N)$ contains any residual dependence of all the input variables locked together in a cooperative way to influence the output $f(x)$. Once all the relevant component functions in Eq. (1) are determined and suitably represented, the component functions constitute HDMR, thereby replacing the original computationally expensive method of calculating $f(x)$ by the computationally efficient model.

Depending on the method adopted to determine the component functions in Eq. 1. HDMR expansions are classified into: ANOVA-HDMR and cut-HDMR (Rajib et al. 2008). ANOVA-HDMR is useful for measuring the contributions of the variance of individual component functions to the overall variance of the output. On the other hand, cut- HDMR expansion is an exact representation of the output *f(x)* in the hyper plane passing through a reference point in the variable space.

In the cut-HDMR method, first a reference point $c = \{c_1, c_2, ... c_N\}$ is defined in the variable space. In the convergence limit, cut-HDMR is invariant to the choice of reference point *c*. In practice, *c* is chosen within the neighborhood of interest in the input space. The expansion functions are determined by evaluating the input-output responses of the system relative to the defined reference point *c* along the associated lines, surfaces, sub volumes, etc. (i.e. cuts) in the input variable space. This process reduces to the following relationship for the component functions in the Eq.. 1

$$f_0 = f(c) \tag{2}$$

$$f_i(x_i) = f(x_i, c^i) - f_0 \tag{3}$$

$$f(x_i, c^i) = f(c_1, c_2......c_{i-1}, x_i, c_{i+1}, ...c_N) \tag{4}$$

In Eq. 4 all the input variables are at their reference point values except x_i, thus rendering a one dimensional form.. The f_0 term is the output response of the system evaluated at the reference point c. Each first-order term is evaluated along its input variable space through the reference point. In this study first-order cut-HDMR is used for generating the response surface.

General Procedure for Calculating HDMR Based Fragility Curves

The first step in calculating the seismic fragility curves utilizing the HDMR metamodel concept is to define the input and output (response) variables. A response measure that best describes damage from seismic loadings should be selected as the output variable (Towashiraporn et al. 2004). Damage limit states or performance limit states corresponding to the selected damage measure or output variable must then be identified. Parameters such as base shear, maximum roof displacement, peak inter-storey drift, damage indices, ductility ratio, and energy dissipation capacity can be used to identify the damage states depending on the type of structure being investigated. Input variables include aleatory uncertainties caused due to the randomness in construction material properties are defined together with their statistical parameters.

Seismic intensity parameter is also defined and used as an input variable. Typical seismic intensity measures include the peak ground acceleration (PGA) and the spectral acceleration (S_a) at a fundamental period of the structure. Selection of an appropriate intensity measure is based primarily on its correlation with the damage potential (Buratti et al.). In the present study, PGA is chosen as the seismic intensity measure. The ground motion records in the suite are scaled to have the same level of seismic intensity measure. The scaling of the ground motion records ensures that the damage probabilities calculated based on a suite of ground motions is conditioned on a specific seismic intensity level.

Various combinations of input variables are generated corresponding to the HDMR sampling scheme and computational seismic analyses are then performed on these structural models and the maximum response is obtained. A metamodel for the maximum response is formulated by applying HDMR technique as in Eq. 1. Monte Carlo technique with a large number of simulations is carried out on this metamodel using probability density functions for the input variables. Consequently, the probability of the chosen response exceeding certain damage limit states can be extracted from the simulation outcomes. This probability value is conditioned on a specific earthquake intensity level and represents one point in a fragility curve. Repetition of the process over different levels of earthquake intensity provides exceedance probability values at other intensity levels, and the fragility curves can be constructed

Application -1: Three Bay Four Storey RC Building

In this case study, a three-bay, four storeyed reinforced concrete building designed according to IS 456-2000 using M20 concrete and Fe415 steel is subjected to nonlinear static analyses to estimate the fragility curve using HDMR.

Description of the Structure

The details of the building elevation and reinforcement details of typical beam and column are shown in Fig. 1.

The height of each storey is 3.6m and the width of the bay is 5m and the supports are assumed to be fixed. In addition to beam and column self-weights, the dead load of 35 kN/m is prescribed for all beams. The live load intensity is 15 kN/m. All the above parameters have been defined on the basis of engineering judgement with reference to the construction practice in India, for this kind of building. Beams and columns are modelled as 3D frame elements. All the frame elements are modelled with nonlinear hinges at the possible yield locations. Non-linear constitutive relations have been defined for concrete and steel. Modified Mander’s (Mander et al. 1988) model have been used for concrete constitutive relation, while for steel the constitutive relation for reinforcing steel given in IS 456:2000 is well accepted and hence considered for the present study. The pushover analyses of the structural models are carried out using the commercially available SAP2000 software.

Modelling of Uncertainties

Uncertainties in modeling occur due to the materials, structural response, and also in the determination of the magnitude and nature of the seismic loads. An important source of uncertainty in the analysis of structures is due to the material variability. In the present study, ultimate strength, and modulus of elasticity of steel are modeled as deterministic factors. The random variables involved are the concrete characteristic strength *(f_{ck})*, the Young's modulus of concrete *(E_c)*, and the steel yield strength *(f_y)* and their distribution characteristics and the values used in this study are given in Table 1 (Ranganathan 1990).

Table 1 Statistics of Random Variables (Ranganathan 1990)

	Variable	Mean (MPa)	COV (%)	Distribution
Concrete	f_{ck} (MPa)	19.54	21.0	Normal
	E_c (MPa)	34100	20.6	Normal
Steel	f_y (MPa)	469	10	Normal

Failure Criteria and Performance Limit States

The present case study uses the maximum inter-story drift ratios at the performance point of the capacity spectrum as the response parameter due in part to its simplicity, but largely because the drift is well-correlated with seismic damage. The limit states (immediate occupancy, life safety, and collapse prevention) associated with various performance levels of reinforced concrete frames is given in Table 2. (Ghobarah 2004).

Table 2 Limits associated with various structural performance levels

Structural Damage States	Permissible Inter storey Drift
Light Damage	0.2%
Medium Damage	0.5%
Severe Damage	0.8%

Formulation of Metamodel

Different combinations of input variables are generated corresponding to the HDMR sampling scheme as shown in Table 3.

Table 3 Range of input variables used for 3 point sampling in HDMR

Variables	μ-2σ	μ	μ+2σ
f_{ck}	11.33	19.54	27.75
f_y	375.2	469	562.8
E_c	20050.8	34100	48149.2
PGA	0.1	0.25	0.4

The sampling is carried out such that only one random variable is estimated at a given time, while all the other random variables are kept at their reference points (μ). Pushover analysis is carried out on these combinations and the maximum response (interstory drift) at the performance point is calculated. The metamodel for the response is then formulated by applying HDMR technique as in Eq.1.

Monte Carlo simulation is performed subsequently on the metamodel by randomly generating values for the input variables based on their probability distributions and the response is calculated from the metamodel. Fragility curves as shown in Fig. 2 are generated for different damage levels by comparing the response from the above simulations with the limit states associated with those performance levels.

In order to validate the proposed method for fragility analysis, fragility curves using Latin Hypercube Sampling (LHS) is formulated and compared with that obtained using HDMR. 100 samples for each of the random variables are created from their probabilistic characteristics. Pushover Analyses are carried out on these 100 building samples for different levels of PGA. The fragility curves are then compared with those obtained by using HDMR with 3 point sampling as shown in Fig. 3. It is seen that the fragility curve generated using HDMR closely matches with those obtained using LHS.

Conclusion

In this work, it is seen that High Dimensional Model Representation (HDMR) response surface methodology simplifies the process of fragility computation. It is used to replace the algorithmic g-function with an explicit functional relationship, fitting a second order polynomial, thereby reducing the number of expensive numerical analyses. A comparison of the computational effort involved in the development of fragility curves using HDMR and a conventional method like LHS is also presented. It is evident from the case studies that the computational effort required for computing fragility curves using HDMR is very low compared to that using conventional method without any compromise in accuracy. Application of MC simulation directly on the structural model would require about 100 pushover analyses, and the use of MC simulation on the other hand the proposed method require only 13 pushover analyses to generate the curve.

FIGURES

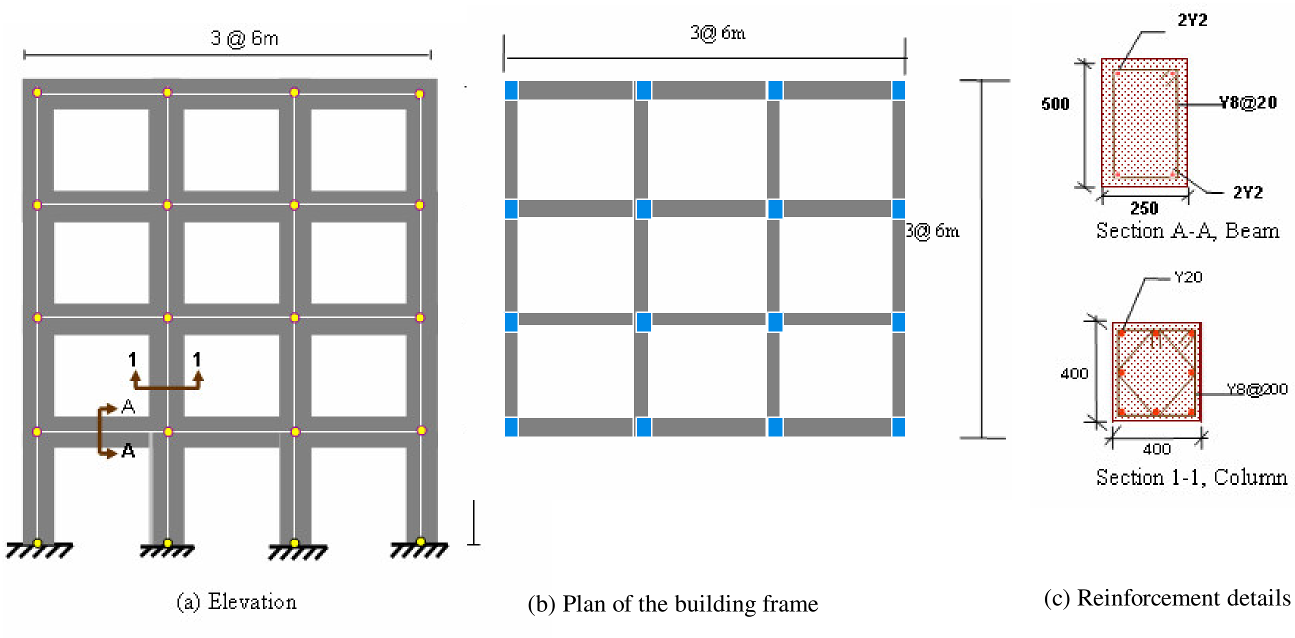

Figure 1 (a) Elevation, (b) Plan of the Building frame (C) reinforcement details of Columns and beams

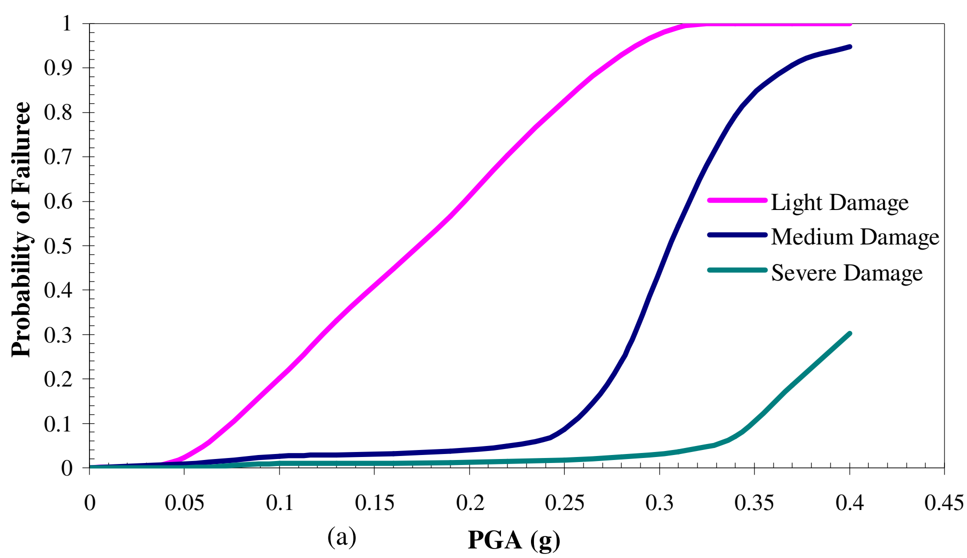

Figure 2 Fragility Curves for different Performance Limits using 3 sample points

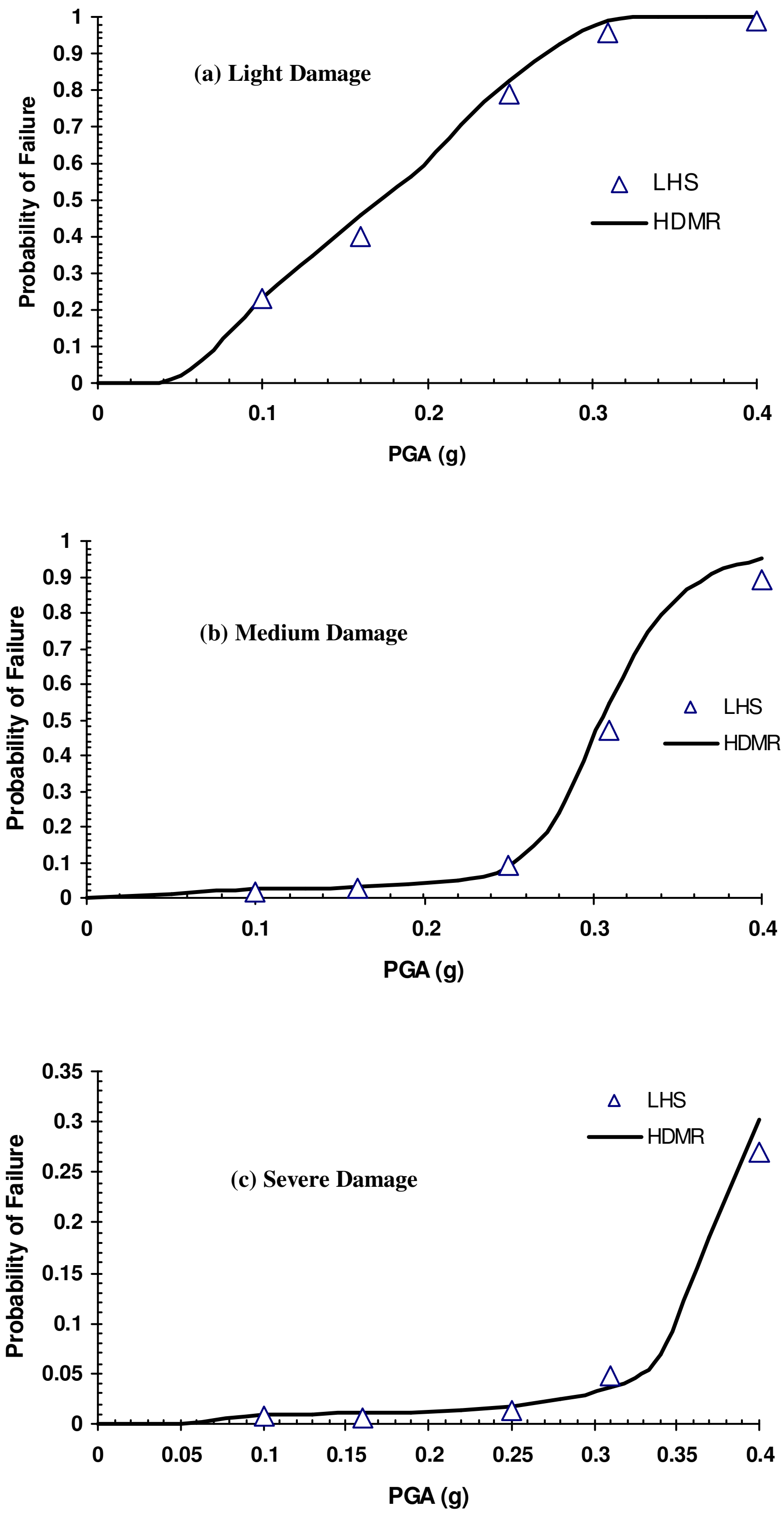

Fig. 3 Comparison of Fragility Curves obtained using HDMR & LHS for different damage states (a) Light damage (b) Medium damage (c) Severe damage

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