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OPTIMAL DESIGN OF FRICTION DAMPERS FOR MULTI-STOREY BUILDINGS

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ABSTRACT

This paper emphasises a design procedure with the aim to calculate the optimal number of Pall friction dampers incorporated in an existing building, to determine the slip-load and to estimate the optimal location of staggered braces equipped with dampers throughout structure in order to upgrade its seismic response. In this regard, the applied optimisation approach is based on minimising the difference between the total energy input and the energy dissipated by friction. In addition, the optimum location of braces equipped with dampers were found based on a second optimisation technique consisted of minimising torsional effect. The proposed design procedure was applied on an existing 10-storey building located in Montreal. It was found that the structural response reached the targeted objectives in terms of stiffness, strength, and lateral deformation under the selected ensemble of artificial earthquake-induced ground motions.

Introduction

The use of energy dissipation devices for seismic upgrade of existing buildings is an alternative retrofit approach. By integrating passive devices in existing structures, designed and erected prior to the introduction of seismic code and capacity design principle, the seismic response of buildings is substantially improved. These devices act as energy fuses and are able to move the input energy out of the existing frame system. To provide a cost-efficient retrofit design, the following parameters are investigated: the optimal number of friction dampers per floor; the location of braces with devices throughout structure; and dampers slip-load setting. However, maximising damper effectiveness while minimising the retrofit cost it is not a straightforward process.

In the recent past, many researchers have proposed optimisation techniques in order to find the optimal location of dampers, their number per floor, as well as the required amount of supplemental damping. Among them, Zhang and Soong (1992) addressed the damper location problem in a symmetrical building by extending the controllability index method. In this respect they proposed that the optimal location of dampers is where the interstorey drift of the undamped structure is the largest. Later on, Singh and Moreschi (2001) used a gradient-based approach to determine the optimal placement of dampers in a symmetric structure. They calculated the total

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amount of damping, as well as the damping distribution throught the structure based on controlling the performance index. This index is expressed as being the root-mean-square values of response quantities of interest such as the base shear, floor acceleration or interstorey drift. Further on (2002), they employed a genetic algorithm to optimise the magnitude of dampers slip-load and dampers location by minimising the interstorey drift of the damped versus undamped structure and by constraining the floor acceleration of the damped structure.

Studies related to the effect of supplemental damping on the seismic response of plan-asymmetric buildings were carried out by several researchers. Lin and Chopra (2001) concluded that the supplemental damping is more effective in reducing the seismic response of the building when dampers are distributed further from the centre of supplemental damping which is defined as being the location of the resultant of the damping forces when the system is subjected to a translational force. Llera et al (2005) studied the torsional balance of one-storey plan-asymmetric structure with friction dampers. It was found that friction dampers can control efficiently the lateral-torsional effect by placing the empirical center of balance (ECB) of the structure at equal distance from all edges of the building. The concept of ECB is general and its location depends on the inelastic response of damped structure. It was concluded that once the lateral-torsional effect is controlled, the plan-asymmetric buildings are transposed in plan-symmetric buildings.

This study is conducted with the aim to deliver an efficient retrofit design of existing multi-storey frame building equipped with friction dampers under different earthquake-induced ground motions. A design procedure that minimises the difference between the total energy input and energy dissipated by friction dampers while maintains constantly over the building height the ratio between the stiffness k_{br} of the braces and the storey stiffness $k_{f\&br}$ (structures with braces) is presented in this paper.

Retrofit Approach and Optimisation Technique

Existing bare frame buildings, designed and constructed prior to 1970s, thus before to the introduction of the seismic design provisions are vulnerable to earthquake induced forces in terms of both strength and deformability. The retrofit strategy which supplements the damping and stiffness of a given structure at a minimised retrofit cost is an appealing technique. In other words, the supplemental damping system is intended to dissipate a part of the seismic energy input E_I of the structure through minimising the cumulative strain energy E_s and kinetic energy E_k. Devices are usually distributed throughout the structure in order to absorb either E_k or E_s accumulated by the primary structural system from the seismic ground motions. The kinetic energy and the elastic strain energy constitute the elastic vibrational energy E_D which is related to the structural damage (Akiyama, 2000). The energy balance relation and the damage energy E_D are expressed in Eq. 1, respectively Eq. 2:

$$E_I = E_k + E_s + E_h + E_\zeta \tag{1}$$

$$E_D = E_I - (E_h + E_\zeta) = \frac{E_I}{(1+3\zeta+1.2\sqrt{\zeta})^2} \tag{2}$$

where E_h is the energy dissipated by the supplemental damping system (friction devices), E_ζ is the modal damping energy and ζ the damping constant of the system in which the 2-3% of critical damping is included. In a typical steel frame structure, the inherent damping is approximately 2-3% of critical. Based on Akyiama's research (2000), the variation of damage energy with the equivalent damping ratio, as defined by the following expression: $1/(1+3\zeta+1.2\zeta^{0.5})^2$ is shown in Fig. 1. However, for design purpose, the desirable seismic response of a structure equipped with friction devices is not necessarily associated with the amount of energy dissipated by dampers (E_h) or in other words, by maximising the energy dissipated by dampers, E_h does not necessarily conduct to minimise the damage energy value, since the amount of E_I may also increase (Christopoulos and Filiatrault, 2006). In this respect, Eq. 2 clearly emphasises the dependence of the damage energy with the total input energy. The energy that contribute to damage absorption due to damping, E_h depends on the magnitude of the base shear force in terms of seismic weight of the building and ground motion signature.

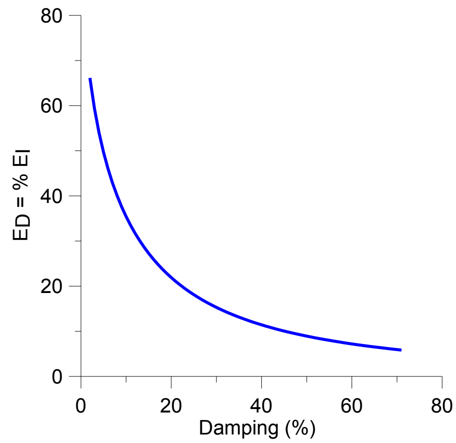

Figure 1. Structural damage energy versus equivalent damping ratio.

Generally, the optimisation process in designing passive devices consists of dampers slip-load setting and dampers location selection (Tirca, 2009). This design process is essentially made in two steps: i) the optimum load activation of friction dampers which is mainly dependent on the structure properties and ground motion frequency content; and ii) the optimum location of dampers which incorporates the building strength and its afferent displacement.

The first step related to the optimum activation load (slip-load) of each damper, $V_{s,j}$ consists of minimising the difference between the seismic input energy, E_I and the energy dissipated by the incorporated dampers, E_h. In the seismic retrofit approach, the purpose of adding dampers to a bare frame system is to protect the existing frame members by reducing the damage energy through activating the slip-load of all dampers. Herein, dampers are designed to slip when the shear deflection, $\Delta_{s,j}$ is reached. By transposing the shear deflection equation as well as the energy dissipated by friction devices equation for one-storey building (Baktash and Marsh,1987) to a multi-story building, the expression of the aforementioned parameters defined for the j-storey are given in Eq. 3 and Eq. 4:

$$\Delta_{s,j} = \frac{V_{f\&br,j} - V_{br,j}}{k_{u,j}} \tag{3}$$

where $V_{f\&br,j}$ is the shear force exerted by the frame and bracing system at storey j, $V_{br,j}$ is the shear force exerted by the bracing system alone at level j and $k_{u,j}$ is the lateral stiffness of the unbraced frame (bare frame). By considering that the shear deformation of the frame after slipping is equal to the slip drive distance, the energy dissipated by friction devices at storey-j is:

$$E_{f,j} = V_{br,j}\Delta_{s,j} = V_{br,j}\frac{V_{f\&br,j} - V_{br,j}}{k_{u,j}} \tag{4}$$

The value of seismic input energy developed at storey-j is:

$$E_{I,j} = V_{f\&br,j}\Delta_{f\&br,j} = (V_{f,j} + V_{br,j})\Delta_{f\&br,j} \tag{5}$$

where $\Delta_{f\&br,j}$ is the total shear deflection at floor j that contains the participation of frame and braces and $V_{f,j}$ is the lateral shear force at level j exerted by the unbraced frame alone. The minimised difference between the seismic energy input and the energy dissipated by friction is obtained by differentiating Eq. 6 with respect to $V_{br,j}$ as is shown in Eq. 7.

$$E_{I,j} - E_{f,j} = (V_{f,j} + V_{br,j})\Delta_{f\&br,j} - V_{br,j}\frac{V_{f\&br,j} - V_{br,j}}{k_{u,j}} \tag{6}$$

$$\frac{\partial E_{I,j}}{\partial V_{br,j}} - \frac{\partial E_{f,j}}{\partial V_{br,j}} = \Delta_{f\&br,j} - \frac{V_{f\&br,j}}{k_{u,j}} + \frac{2V_{br,j}}{k_{u,j}} = 0 \tag{7}$$

By expressing $\Delta_{f\&br,j} = V_{f\&br,j} / K_{f\&br,j}$,the solution of Eq. 7 is equal to:

$$V_{br,j} = \frac{V_{f\&br,j}k_{br,j}}{2k_{f\&br,j}} \tag{8}$$

where $k_{br,j}$ is the lateral stiffness of braces alone and $k_{f\&br,j}$ is the lateral stiffness of existing frame structure equipped with braces.

Once the shear force exerted by braces alone at the j-storey $V_{br,j}$ and the braces stiffness were found, slip-load calculation became straightforward. By employing FEMA 356 proceeding, the summation of the horizontal component of slip-load per floor is equal to the ratio $V_{br,j}/1.3$ or in other words braces should be verified to behave elastically in axial compression and tension under a force equal to 130% design slip-load. The optimum slip-load distribution over the building height was determined based on numerical analyses, where the characteristics of ground motions were considered beside the dynamic properties of the structure. In general, the optimum slip-load distribution depends on the lateral shear force distribution.

Therefore, the behavior of an existing structural frame system equipped with friction dampers in-line with single diagonal braces basically follows two stages (Levy et al, 2000): i) the stiffness of the retrofitted frame (bare frame with braces and dampers) is equal to the stiffness of the braced frame (bare frame with braces) as long as the slip-load is not activated in dampers and the velocity of the friction damper is equal to zero; ii) the stiffness of the retrofitted frame is equal to the bare frame stiffness when the friction force is equal with the slip-load in the brace and the velocity of friction dampers at each floor is equal to storey velocity. It is noted that any reverse horizontal movement applied to the structure returns the system to the first stage.

After the total number of dampers is calculated based on Eq. 8, their optimum location throughout the building may be assigned by controlling the torsional seismic response. In this regard, the optimum location of braces with dampers especially for plan-asymmetric buildings is proposed to be found by employing Kokil and Shrikhande (2007) objective function f(x) as given in Eq. 9, where V_u and Δ_u are the maximum base shear and maximum interstorey drift in the bare frame (undamped) structure.

$$f(x) = \frac{V_{f\&br}}{V_u} + \frac{\Delta_{f\&br}}{\Delta_u} \tag{9}$$

However, by choosing pairs of staggered tension-compression braces incorporated into an existing steel frame building, the processing data for optimal dampers location based on minimising Eq. 9 is reduced. Generally, there are four dampers located in X-direction and four dampers located in Y-direction. It is assumed that among the four dampers, two of them act in tension and the other two in compression. To minimise the torsional effect it is required that at least two dampers (one in each direction) to be located in one quarter of the building floor area.

Numerical Results

Building Description

The skeleton of the studied building, as shown in Fig. 2 is a non-ductile steel moment frame structure with concrete slab, completed in 1929 and designed to carry gravity and wind load only. The existing structure has one basement level and 10 stories above ground. Originally, the structure served as an industrial building for garment manufacturing. The floor plan is approximately 18m by 18m and the typical storey height is 3.4m, except for the ground floor which is 5.3m. The typical storey bay is 6.0m in both directions. The building has brick facades adorned with cast-iron prefabricated wall panels, which are typical for buildings of this generation. Columns and beams are W sections and the beam-column connections are riveted stiffened seat angles, which were widely used as rigid joints in steel frame design of that period. The floor structure consists of poured in place reinforced concrete joists that were monolithic with the floor slab and also the fire protecting concrete cover over the existing steel frame. This configuration was considered to be a rigid diaphragm. Because the steel beams supporting the floor joists were significantly stiffer than the beams in the opposite direction and those beams coincided with the orientation of the strong axis of the columns, the existing structure was stiffer in the Y-direction than in the X-direction.

Seismic Loads

The seismic retrofit design was performed according to the 2005 National Building Code of Canada (NBC 2005) and FEMA 356 recommendations. Based on the building code procedure, the minimum lateral seismic force, V is given by: $V = S(T_a)M_vI_EW/(R_dR_o)$, where S is the design spectral response acceleration obtained from the 2% in 50 years, T_a the fundamental period of the structure, W the seismic weight, M_v the higher mode factor, R_d and R_0 are the ductility-related respectively overstrength-related force modification factor. For the Montreal area, the response acceleration spectrum given in (T- S_a) pairs is defined as follows: (0.2s, 0.69g), (0.5s, 0.34g), (1.0s, 0.14g), and (2.0s, 0.048g). Firm ground conditions (Site Class C) with $F_a = F_v = 1$ were considered, where F_a, F_v are the acceleration-based and velocity-based site coefficients. For design purpose, the fundamental period of the existing steel moment frame system is taken as $1.5T_a$, where $T_a = 0.085(h_n)^{3/4}$ and $1.5T_a = 1.85$s (where the total building height, $h_n = 35.6$m).

However, the fundamental period of the building with no beam-to-column moment resisting connections, computed from free vibration analysis is much longer than that for similarly newly designed structures, and largely exceeds the code period (T = 1.85s). The seismic weight of the building which includes the 25% of the roof snow load is equal to 120360kN. For this non-ductile moment-frame system (R_dR_o=1.0), the lateral seismic force V is equal to 8060kN.

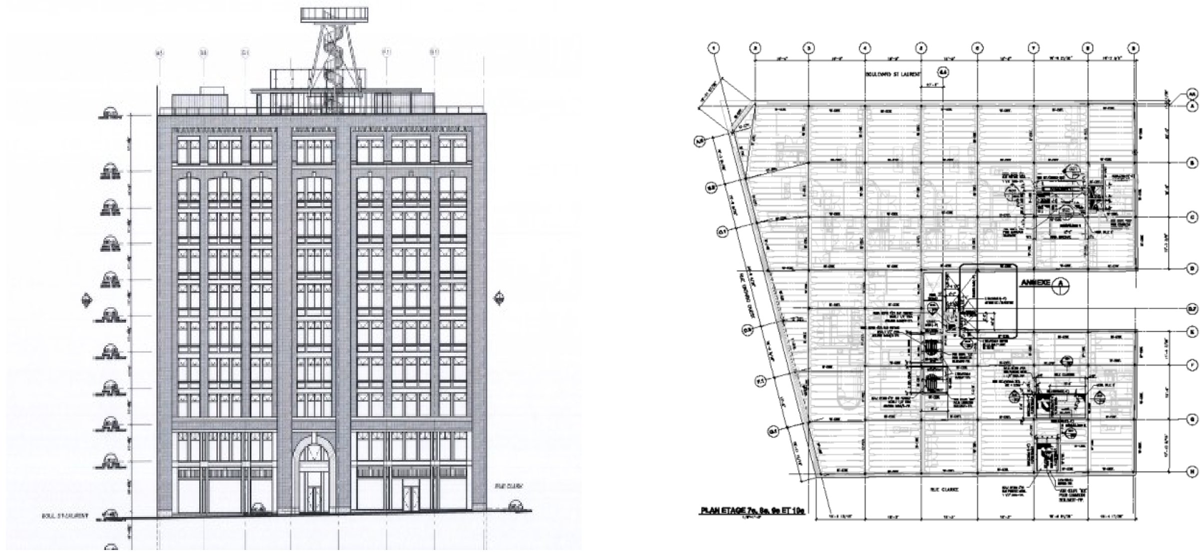

Figure 2. The studied building facade and its plan view.

Linear and non-linear dynamic time-history analyses were performed for this study and the structure was subjected to eight artificial ground motions simulated for the east-coast source zones matching the two dominant magnitude hypocentral distance scenarios for the Montreal area: M_w6 at 30km and M_w7 at 70km. The characteristics of the eight ground motions (peak ground acceleration, PGA_H and accelerogram duration) together with the scale factor applied to match the 2% in 50 year design spectrum for the Montreal area are shown in Table 1.

The computation of the scale factor for a given ground motion can be solved in several ways. The simple procedure (I) is to scale each selected accelerogram so that its spectral ordinate corresponding to the first vibration mode, $S(T_1)$ matches the ordinate of the design spectrum for the given site and the same fundamental period, T_1. In this case, the scale factor is the ratio between the design spectrum for the fundamental period of the building and $S(T_1)$. An alternative approach (II) is to scale each accelerogram so that the average response spectrum of the considered ground motions over the periods of interest (considering as being $0.2T_1$ to $1.5T_1$) to be equal to the design spectrum over the same periods (Baker, 2009). Herein, the scale factor is found by equating the area under the response acceleration spectrum of the selected ground motions over the periods of interest ($0.2T_1 - 1.5T_1$) and the area under the design acceleration spectrum over the same periods of interest. FEMA 356 proceeding requires practicing engineers to select either a minimum of three accelerograms and to consider the maximum response or to select a minimum of seven accelerograms and to consider the average response. When seven or more accelerograms are considered, the mean (Mean), the mean+one standard deviation (Mean+σ) and the mean-one standard deviation (Mean-σ) of the spectral acceleration responses over the periods of interest are calculated. Once this step was completed and all selected accelerograms were scaled, it is still possible that the ordinate of the response spectrum corresponding to the fundamental period $T_{1,}$ shows larger or smaller peak than Mean±σ. For the eight selected ground motions, the standard deviation σ was found as being equal to 1.3. In this case, a second adjustment is proposed such that all scaled ground motions have to match the design spectrum at $S(T_1)$ within ranges Mean±σ and all scaled ground motions have to fit approximately the design spectrum over the period of interest ($0.2T_1 - 1.5T_1$) between the Mean–σ and the Mean+σ as is shown in Fig. 3. Even if the scale factor consider in this paper is calculated based on the second procedure (II), Table 1 shows the value of scale factors as resulted from I and II procedure, as well as their ratio.

Table 1. Characteristics of selected ground motions for the Montreal area.

No.	Ground motions	PGA_H	Duration	Scale factor (I)	Scale factor (II)	(II) / (I)
R1	Simulated Trial #1 (M_w6-30km)	0.43g	6.3s	2.47	2.10	0.85
R2	Simulated Trial #2 (M_w6-30km)	0.52g	6.3s	1.51	1.70	1.13
R3	Simulated Trial #3 (M_w6-30km)	0.47g	6.3s	1.60	1.60	1.00
R4	Simulated Trial #4 (M_w6-30km)	0.44g	6.3s	1.55	2.05	1.32
R5	Simulated Trial #1 (M_w7-70km)	0.30g	20.1s	1.31	1.34	1.03
R6	Simulated Trial #2 (M_w7-70km)	0.29g	20.1s	1.31	1.40	1.07
R7	Simulated Trial #3 (M_w7-70km)	0.34g	20.1s	1.31	1.10	0.85
R8	Simulated Trial #4 (M_w7-70km)	0.29g	20.1s	1.26	1.10	0.88

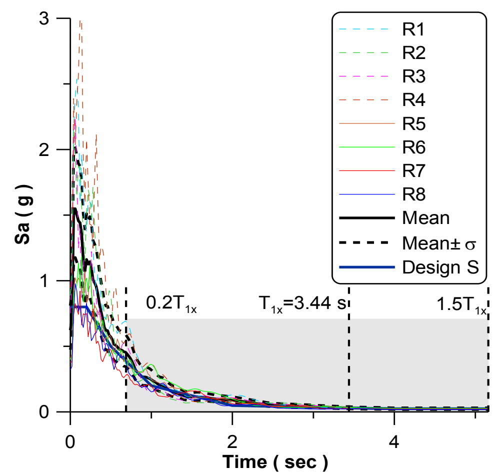

Figure 3. Absolute acceleration response spectra scaled for the Montreal area.

Seismic Assessment of Existing Building

A three-dimensional dynamic analysis was performed by using ETABS-Nonlinear software (Computer and Structures Inc.). Rigid diaphragm behavior was assumed at every floor and 2% modal damping was considered for the linear and non-linear time-history analysis. For each principal directions (X and Y), the computed first and second period of vibration are: T_{1X} =3.6s, T_{2X} =1.7s, T_{1Y} =2.9s and T_{2Y} =1.4s. The studied building was found to experience large interstorey drift under the design ground motions, especially in the X-direction. The lateral capacity of the building in X-direction was found to be only 50% of the required based shear (50% V) as calculated by following the NBC 2005 provisions. However, this value is lower than 60%V, which is the minimum lateral capacity of an existing building according to the Quebec Building Code. In addition, it was found that the envelope of the lateral force distribution over the structure height is significantly different than that resulted from the static equivalent method and the interstorey drift, especially in X-direction (3.9%h_s) is much larger than 2.5%h_s. In the Y-direction, the values of interstorey drift are smaller than those obtained in X-direction due to the structural configuration of the building. Analysing the behavior of building under the eight selected ground motions it was seen that the dominant contributor to the largest lateral deformation comes from the M_w7 magnitude earthquakes. This largest interstorey drift response of the existing building required retrofit action for seismic upgrade. The mean (Mean), mean+one standard deviation (Mean+σ) and the maximum value (Max) of the interstorey drift over the building height resulted from the eight selected ground motions applied in the X- and Y- direction are shown in Fig. 4.

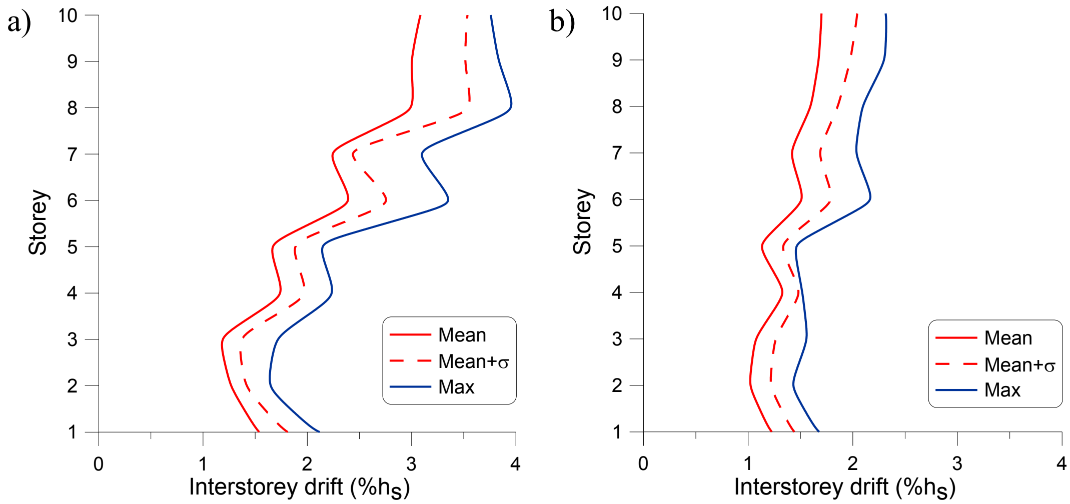

Figure 4. Interstorey drift of the existing building in a) X- direction, b) Y-direction.

Building Retrofit for Seismic Upgrade

It is known that adding stiffness to an existing building the interstorey drift is reduced while adding damping the magnitude of forces developed in all structural members are also reduced. In contrast with the conventional system, the friction-damped bracing system needs not to be vertically continuous in order to dissipate the amount of energy as required by design. In order to optimise the amount of supplemental damping added to an existing building, it is mandatory to verify the stiffness distribution over the structure height. However, the storey stiffness calculation is not well defined in the building code. By following Paulay and Priestley (1992) proposal, the calculation of the storey stiffness was made by dividing the storey shear to the interstorey drift. Storey shear and interstorey drift are the response of the building to an arbitrary horizontal force applied in the centre of masses of the roof level. Fig. 5 shows the normalised value of storey stiffness for the bare frame building in both X- and Y-direction. It can be seen that the 3rd, 4th, and 5th floors are stiffer than the 2nd and the ground floor which are prone to soft-storey mechanism. Furthermore, the shear force per floor exerted by the frame with braces $V_{f,j+br,j}$ and the bracing system alone $V_{br,j}$ (as per Eq. 8), the horizontal projection of the slip-load magnitude considered as $V_{br,j}/1.3$, the total number of devices per floor in one direction (X or Y), the assigned slip-load to each device P and the total slip-load per floor P_{storey} are given in Table 2 together with the ratio $k_{br,j}/k_{f,j+br,j}$. For the frame system with braces only and with braces plus dampers, the normalised storey stiffness over the structure height in X- and Y-direction is illustrated in Fig. 5. It is clearly shown that after retrofit, the stiffness of the each lowest storey exceeds or equates that of the storey immediately above.

From non-linear time-history analysis in X-direction (Fig. 6a) it was found that the interstorey drift over the building height is reduced to less than 2.5%h_s for the existing frame system equipped with staggered braces (BF) and to less than 1.0%h_s when dampers were added

Table 2. Shear force and the optimum dampers slip-load setting

St.	k_{braces}/ $k_{frame+br.}$	$V_{f,j+br,j}$ (kN)	$V_{br,j}$ (kN)	$V_{br,j}$ /1.3 (kN)	Total devices/ 2	P_{storey} (kN)	P (kN)
10	0.80	3303	1334	1026	4	1200	300
9	0.77	3829	1467	1128	4	1320	330
8	0.79	3919	1556	1197	4	1400	350
7	0.79	3891	1556	1197	4	1400	350
6	0.77	4012	1556	1198	4	1400	350
5	0.79	4259	1690	1300	4	1520	380
4	0.80	4629	1867	1436	4	1680	420
3	0.76	5282	2002	1540	4	1800	450
2	0.72	6150	2210	1700	4	2000	500
1	0.71	8046	2840	2184	6	3000	500

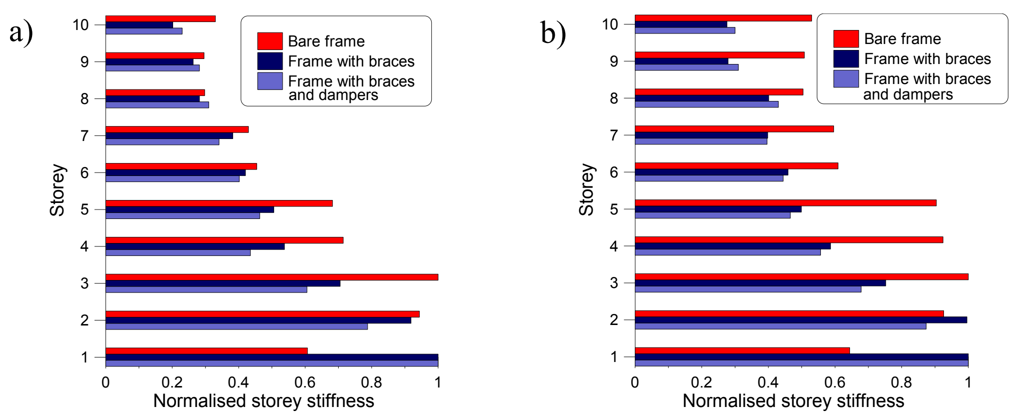

Figure 5. Normalised storey stiffness over the structure height: a) X-direction, b) Y-direction.

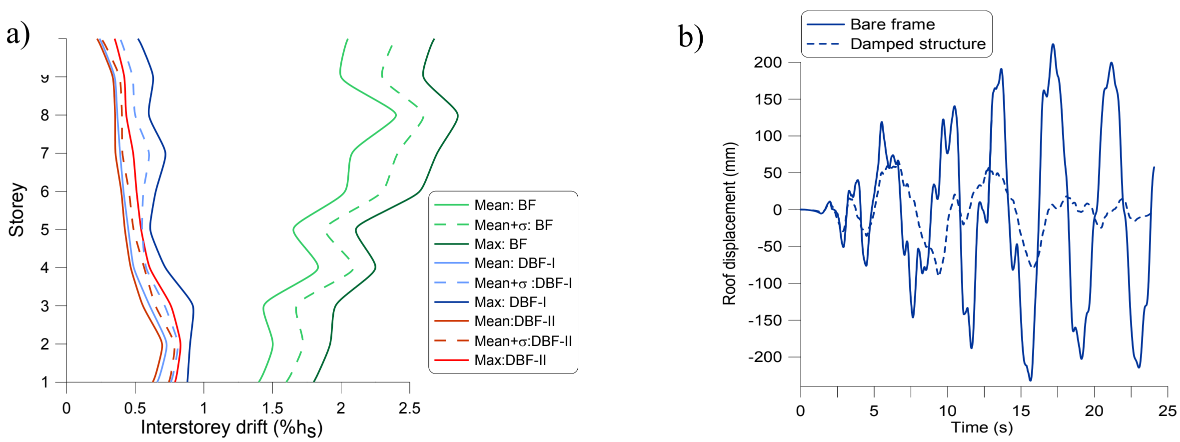

Figure 6. Building lateral deformation: a) interstorey drift in X-direction; b) roof displacement.

in line with braces (DBF-I). Since the period of the building with braces and dampers did not shift much (e.g. in X-direction from 3.6s to 3.4s) while the inter-storey drift (Mean+σ) was substantially reduced from 3.9%h_s to 0.7%h_s, it means that the floor acceleration was constrained. In addition, for finding the optimum location of staggered braces equipped with dampers, a second optimisation technique (DBF-II) is applied based on Eq. 9. Therefore, as long as the torsional effect is minimised, the torsional component of the storey shear and the interstorey drift are reduced (Fig. 6a). In order to emphasise the performance of the retrofitted building in term of time-history roof displacement, a comparison of building response before and after the structure was upgraded is illustrated in Fig. 6b under the R8 ground motion.

Conclusions

This paper presents a design procedure for optimal design of friction dampers incorporated in multi-storey buildings based on minimising the difference between the total energy input and the energy dissipated by friction. By following the proposed optimisation procedure, practicing engineers are able to calculate the total number of braces equipped with friction dampers per floor, to set dampers slip-load, and to estimate the location of staggered braces within the building such that the torsional effect to be minimised. In this study, it was found that keeping a constant ratio over the building height between the stiffness of braces and the stiffness of bare frame equipped with braces, the sensitivity of structure to soft-storey mechanism formation is avoided. The effectiveness of this retrofit technique in term of interstorey drift is clearly emphasised and the behavior of the retrofitted building is substantially improved. Further work is required to verify if in the final retrofit design, all existing frame members and connections perform in elastic range under the selected ground motions for a given building location.

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