Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No

SYNTHETIC EARTHQUAKE GROUND MOTIONS FOR SPECIFIED SEISMIC DESIGN SCENARIO

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ABSTRACT

A method for generating a suite of synthetic ground motion time-histories for a specified seismic design scenario is presented. Synthetic time-histories are modeled by a stochastic process developed in a previous study. This stochastic model has parameters that capture the main features of earthquake ground motions, including the evolutionary intensity and the time-varying predominant frequency and bandwidth. By identifying the parameters of the stochastic model for many recorded accelerograms obtained from the NGA database, predictive relations are constructed that empirically link the model parameters to a seismic design scenario that is specified by a set of earthquake and site characteristics. These characteristics include the faulting mechanism, earthquake magnitude, source-to-site distance and the site shear-wave velocity. For any specified seismic design scenario, the predictive relations are employed to randomly generate possible values of the model parameters, which are then used in the stochastic model to generate an ensemble of synthetic motions. The resulting synthetics can be used in conjunction with or in place of previously recorded motions in seismic design and analysis. They realistically represent the natural variability of ground motions for the specified design scenario. Furthermore, the statistics of their resulting elastic response spectra are in close agreement with the median and variability predicted by the NGA ground motion prediction equations.

Introduction

In seismic design and analysis of structures, development of ground motions is a crucial step because even with the most sophisticated and accurate methods of structural analysis, the validity of predicted structural responses depends on the validity of the input excitations. Several levels of ground motions are commonly considered for seismic assessment of a structure.

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For lower levels of intensity, when the structure is expected to remain elastic, response spectrum analysis is usually sufficient. This type of analysis only requires knowledge of the ground motion spectral values. One of the most practical approaches to obtain these values is to use empirically based ground motion prediction equations (GMPEs), also known as attenuation relations. Many GMPEs have been developed that predict the median and standard deviation of ground motion spectral values for a range of spectral periods. The most recent of them is the Next Generation Attenuation (NGA) relations (Abrahamson et. al. 2008). These GMPEs have been calibrated against observed data and are commonly used in practice.

For higher levels of intensity, when nonlinear structural behavior is likely, response-history dynamic analysis is necessary. This type of analysis requires knowledge of acceleration time-histories. It is common practice to use real recorded ground motions for this purpose. However, difficulties in this approach arise because ground motion properties vary for different earthquake and site characteristics, and recorded motions are not available for all types of earthquakes in all regions. As a result, the designer is often forced to modify the record (e.g., scale it or modify its frequency content) in ways that are often questionable and may render motions that are not realistic. Another alternative is to generate synthetic motions for specified earthquake and site characteristics. The resulting synthetics can be used in conjunction or in place of previously recorded motions in performance-based earthquake engineering design and analysis.

Many models have been developed in the past to synthetically generate ground motions. One group of models are physically-based seismological models that produce realistic accelerograms at low frequencies, but often need to be combined with stochastic models known to be more appropriate at high frequencies; the resulting combination is usually referred to as a hybrid model (Douglas and Aochi 2008). The physically-based seismological models tend to be too complicated for use in engineering practice as they require a thorough knowledge of the source, wave path, and site characteristics, which typically are not available to a design engineer. For these reasons and due to lack of calibration against observed data, these models are rarely used for engineering purposes. In this study, we employ a site-based (as opposed to modeling the seismic source) stochastic ground motion model instead, which focuses on realistically representing those features of the ground motion that are known to be important to the structural response (e.g. intensity and frequency content of the ground shaking at the site of interest). Our aim in this study is to develop a method for generating synthetic ground motions, which uses information that is readily available to the practicing engineer. Considering the success of GMPEs in practice, we develop predictive equations for the stochastic model parameters in terms of selected earthquake and site characteristics that are typically required as input arguments to GMPEs.

In this study, a previously developed site-based stochastic process is used to model acceleration time-histories. Predictive relations are constructed for the model parameters in terms of earthquake and site characteristics. Correlation analysis is conducted to empirically identify dependencies among the model parameters. The results are used to randomly generate sets of the model parameters for specified earthquake and site characteristics, which are used in turn in the stochastic model to generate an ensemble of synthetic motions. These synthetics account for the natural variability of ground motions for the given earthquake and site characteristics. Finally, the proposed method is validated by comparison against real earthquake ground motions.

Generating a Suite of Synthetic Motions

In this section, the development of a method to generate a suite of synthetic motions for a specified seismic design scenario (expressed in terms of earthquake and site characteristics) is briefly presented. First, a brief summary of the ground motion model used in this study is provided. This model employs six parameters that are related to the physical features of ground motions. The model parameters are identified for a database of recorded accelerograms with known earthquake and site characteristics. Parameter identification is done by matching certain statistical characteristics of the model and the target accelerogram, which represent the time-varying intensity, predominant frequency and bandwidth of an acceleration time-history. The resulting observational data are used to construct predictive equations for the model parameters. These predictive equations are then employed to generate a suite of synthetic motions for a specified seismic design scenario. More details of the proposed procedure can be found in Rezaeian and Der Kiureghian (2008, 2009).

Ground Motion Model

Earthquake ground motions have nonstationary characteristics both in time and frequency domains. Variation of the ground motion intensity in time is referred to as the temporal nonstationarity. Variation of the frequency content of the ground motion in time is referred to as the spectral nonstationarity. To simulate ground motions, a fully nonstationary stochastic model is developed that is based on time-modulating a filtered white-noise process with the filter having time-varying parameters (Rezaeian and Der Kiureghian 2008). Whereas the time-modulation provides temporal nonstationarity, the variation of filter parameters over time achieves spectral nonstationarity. This fully nonstationary process is high-pass filtered to assure zero residual velocity and displacement of the motion, as well as realistic response spectral values at long periods. Without such filtering, stochastically generated ground motions tend to overestimate response spectral values in the long period range (usually greater than 2 seconds). The entire procedure is detailed in Rezaeian and Der Kiureghian (2009), as illustrated in Fig. 1.

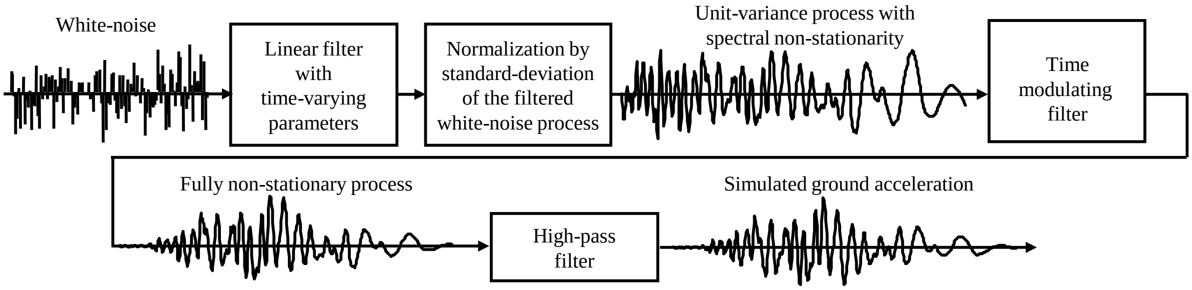

Figure 1. Procedure of simulating a single ground motion (Rezaeian and Der Kiureghian 2009).

The stochastic process model with temporal and spectral nonstationarity is defined by

$$x(t) = q(t, \boldsymbol{\alpha}) \left\{ \frac{1}{\sigma_h(t)} \int_{-\infty}^{t} h[t-\tau, \boldsymbol{\lambda}(\tau)] w(\tau) \mathrm{d}\tau \right\} \tag{1}$$

where $x(t)$ represents acceleration as a function of time, which can be interpreted as the superposition of filter responses to a sequence of statistically independent pulses with the time of application τ. In this expression, $q(t,\boldsymbol{\alpha})$ is the time-modulating function with parameters $\boldsymbol{\alpha}$; $w(\tau)$ is a white-noise process and is the source of stochasticity; the integral inside the curled brackets is a filtered white-noise process with $h[t-\tau,\boldsymbol{\lambda}(\tau)]$ denoting the impulse response function (IRF) of a linear filter with time-varying parameters $\boldsymbol{\lambda}(\tau)$; and $\sigma_h(t)$ is the standard deviation of the process represented by the integral inside the curled brackets. Due to the normalization by $\sigma_h(t)$, the process inside the curled brackets has unit variance so that $q(t,\boldsymbol{\alpha})$ represents the standard deviation of $x(t)$ and thus completely defines the temporal nonstationarity of the process, while the spectral nonstationarity is defined separately by the unit-variance process inside the curled brackets.

The stochastic model in Eq. 1 is completely defined by the formulation and the parameters of the modulating function and the filter IRF. The modulating function, $q(t,\boldsymbol{\alpha})$, is formulated according to Rezaeian and Der Kiureghian (2009), where the functional form (referred to as a "gamma" function) is chosen due to its flexibility and ease of relating its parameters to physical properties of accelerograms. For the filter, an IRF is selected that corresponds to the pseudo-acceleration response of a single-degree-of-freedom linear oscillator with $\boldsymbol{\lambda}(\tau)=(\omega_f(\tau),\zeta_f(\tau))$, where $\omega_f(\tau)$ represents the filter frequency and $\zeta_f(\tau)$ represents the damping ratio of the filter, both of which depend on the time of application of the pulse (see Rezaeian and Der Kiureghian 2008).

For the high-pass filter, a critically damped oscillator is selected. Therefore, the corrected acceleration record, denoted $z''(t)$, is obtained as the solution of the differential equation

$$z''(t)+2\omega_c z'(t)+\omega_c^2 z(t)=x(t) \tag{2}$$

where ω_c is the frequency of the high-pass filter. For more detail on the model and its implementation, which requires discretization in the time domain, see Rezaeian and Der Kiureghian (2008).

Model Parameters

The model parameters originate from two separate sources. The modulating function parameters, $\boldsymbol{\alpha}$, that completely characterize the temporal nonstationarity of the process, and the filter parameters, $\boldsymbol{\lambda}(\tau)$, that completely characterize the spectral nonstationarity of the process. If the model parameters are identified, ground motions can be simulated according to Eqs. 1 and 2.

The parameters of the selected modulating function are $\boldsymbol{\alpha}=(\bar{I}_a, D_{5-95}, t_{mid})$. $\bar{I}_a$ represents the expected Arias intensity, a measure of the total energy defined as the expected value of $(\pi/2\mathrm{g})\int_0^{t_n} x(t)^2 \mathrm{d}t$ where t_n denotes the total duration of motion and g is the gravitational acceleration. D_{5-95} represents the effective duration of motion defined as the time interval between the instants at which the 5% and 95% levels of $\bar{I}_a$ are reached. t_{mid} represents the time at the middle of the strong shaking phase of the motion, which in this study is assumed to occur at the 45% level of $\bar{I}_a$. This assumption is based on the observation that the time it takes to rise from zero to the strong shaking phase of an accelerogram is usually shorter that the time it takes to fall from the strong shaking phase back to zero.

After investigation of many recorded accelerograms, appropriate functional forms were selected for the filter parameters: $\omega_f(\tau)$ and $\zeta_f(\tau)$. The filter frequency is represented by a linear function: $\omega_f(\tau) = \omega_{mid} + \omega'(\tau - t_{mid})$, where ω_{mid} denotes the filter frequency at the middle of the strong shaking phase and ω' denotes the rate of change of the filter frequency over time. Due to the observed insignificant change in the damping ratio during the effective duration of motion, the filter damping is simply represented by a constant factor: $\zeta_f(\tau) = \zeta_f$.

In summary, the six physically meaningful parameters $(\bar{I}_a, D_{5-95}, t_{mid})$ and $(\omega_{mid}, \omega', \zeta_f)$ completely define the time modulation and the evolutionary frequency content of the nonstationary ground motion model. These parameters can be identified for a target recorded motion (this procedure is discussed in detail in Rezaeian and Der Kiureghian 2009). The complete separation of the temporal and spectral nonstationary characteristics of the process allows the modulating function parameters to be identified prior to and independently of the filter parameters. The modulating function parameters $(\bar{I}_a, D_{5-95}, t_{mid})$ are naturally matched with the corresponding measures of Arias intensity, effective duration, and the time at the middle of the strong shaking phase of the target accelerogram. The filter parameters $(\omega_{mid}, \omega', \zeta_f)$ are identified by matching the cumulative mean number of zero-level up-crossings (as a surrogate measure of the predominant frequency) and the mean rate of negative maxima and positive minima (as a surrogate measure of the bandwidth) to the corresponding measures of the target accelerogram. By identifying the model parameters for many recorded motions with known earthquake and site characteristics, empirical relations are constructed that facilitate generating samples of these parameters for a given set of earthquake and site characteristics.

Strong Motion Database

The strong motion database used in this study is a subset of the ground motions used in the development of Campbell-Bozorgnia NGA relations (Campbell and Bozorgnia 2008). The accelerograms in the database are representatives of "free-field" ground motions recorded in shallow crustal events in tectonically active regions. Four variables, F, M, R_{rup}, and V_{s30}, commonly available to a design engineer, are selected to represent earthquake and site characteristics. These variables respectively represent the faulting mechanism, the moment magnitude, the closest distance from the site to the ruptured area, and the shear wave velocity at the top 30 meters of the site. F assumes values of 0 and 1 for strike-slip and reverse types of faulting. The selected earthquakes in the database have $6.0 \leq M$, $10\text{km} \leq R_{rup} \leq 100\text{km}$, and $600\text{m}/\text{sec} \leq V_{s30}$. These limitations were enforced so the database represents motions that are capable of producing nonlinear behavior in structures, and also to exclude the effects of near-fault ground motions and soil nonlinearity. As a result, the database contains 103 pairs of horizontal recordings from 19 earthquakes. Fig. 2 demonstrates the magnitude-distance distribution of the data. (For specific records see Rezaeian and Der Kiureghian 2009).

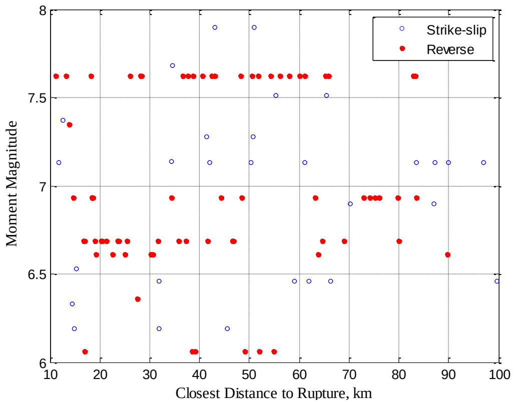

Figure 2. Magnitude and distance of database records (Rezaeian and Der Kiureghian 2009).

Empirical Predictive Equations for the Model Parameters

As mentioned previously, sample observations of the model parameters are obtained by fitting the stochastic ground motion model to the recorded motions in the database in terms of their time-varying intensity and their evolutionary frequency content. After obtaining these observational data, regression is used to construct empirical prediction equations for each model parameter in terms of earthquake and site characteristics. This procedure results in six predictive models of the form

$$\Phi^{-1}[F_\theta(\theta)] = \mu\big(F, M, R_{rup}, V_{s30}, \boldsymbol{\beta}\big) + \eta + \epsilon \tag{3}$$

where θ represents a model parameter, $\Phi^{-1}[.]$ is the inverse of the standard normal cumulative distribution function and $F_\theta[.]$ is the cumulative distribution function of θ as fitted to the observational data. As a result, the left hand side of Eq. 3 satisfies the normality criterion required for the response variable in regression analysis. To identify $F_\theta[.]$, probability distributions are assigned to the model parameters by maximum likelihood estimation. Fig. 3 shows the assigned marginal probability density functions (see Rezaeian and Der Kiureghian 2009 for details on the distribution boundaries and parameters).

In Eq. 3, the function μ represents the predicted mean of θ conditioned on earthquake and site characteristics and involving the set of regression coefficients $\boldsymbol{\beta}$. The summation of η and ϵ represents the total regression error defined as the difference between the observed and predicted values of the model parameter. The regression error is divided into two components because the database contains different numbers of recordings for different earthquakes. Therefore, to account for the specific effects of individual earthquakes on the database (since this effect is random among earthquakes, this type of regression is usually referred to as the random-effect modeling), η denotes the inter-event error (error among data belonging to different earthquakes) and ϵ denotes the intra-event error (error among the data belonging to records of an individual earthquake). η and ϵ are independent zero-mean normally distributed random variables whose variances must be identified based on the observed data.

Regression coefficients and variances of the error terms, which are different for each model parameter, are identified by calibrating each regression model against data using maximum likelihood estimation methods. Possible dependencies among the model parameters are then captured empirically by identifying the correlations between their errors. Details of the regression analysis and results are presented in Rezaeian and Der Kiureghian (2009).

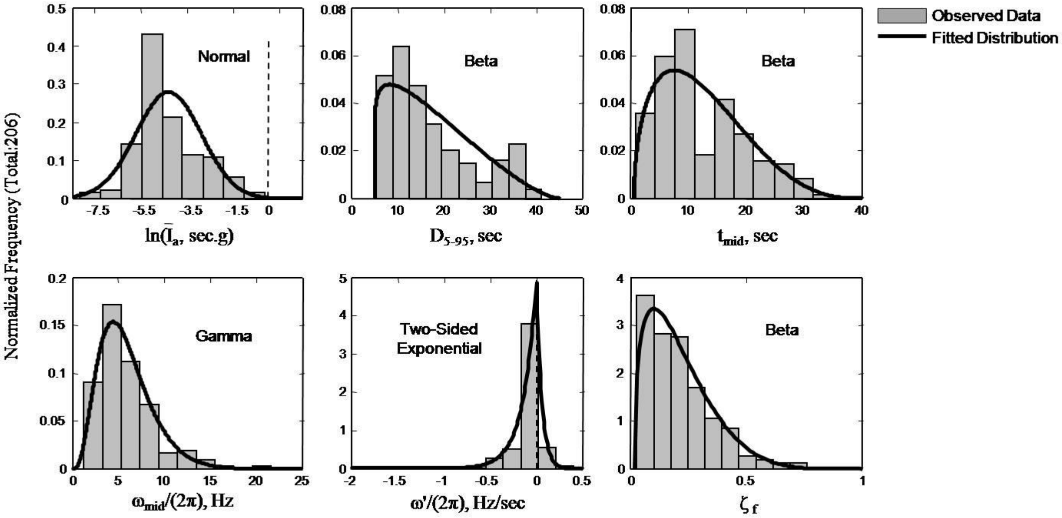

Figure 3. Distributions assigned to the model parameters (Rezaeian and Der Kiureghian 2009).

Given a seismic design scenario expressed in terms of F, M, R_{rup}, and V_{s30}, any number of model parameters can be simulated in the standard normal space as jointly normal random variables. Simulation of jointly normal random variables is possible because their mean values (conditioned on the seismic design scenario), variances and correlations are available. Using the assigned marginal distributions (see Fig. 3), model parameters are then transformed to their original spaces and each set of simulated $(\bar{I}_a, D_{5-95}, t_{mid}, \omega_{mid}, \omega', \zeta_f)$ values is used in the stochastic ground motion model to generate an artificial accelerogram that represents a possible realization of the future ground motion for the specified seismic design scenario.

Natural Variability of Ground Motions

The proposed simulation method accounts for the variability in the model parameters and, hence, maintains the natural variability of ground motions for a given set of earthquake and site characteristics. Many efforts have been made in the past to generate synthetics similar to a target recorded motion, in which case all the synthetics correspond to identical model parameters (i.e., those of the target motion) and do not provide a realistic representation of ground motion variability for a specified seismic design scenario. As an example, Fig. 4 shows a real recorded motion (recorded at Dayhook station during Tabas, Iran 1978 earthquake) that corresponds to the design scenario $F = 1$, $M = 7.35$, $R_{rup} = 14\text{ km}$ and $V_{s30} = 660\text{ m/sec}$. The simulated motions on the left side of Fig. 4 are generated using model parameters identical to those of the recorded motion; observe that even though they are different, they all have nearly identical overall characteristics, e.g., intensity, duration, frequency content. On the other hand, the simulated motions on the right side of Fig. 4 are generated for the seismic design scenario of the recorded motion but accounting for the variability of the model parameters. Thus, each simulation corresponds to a different set of model parameters. The variability observed in the intensity, duration, and frequency content of these motions is representative of the variability observed in recorded ground motions for the specified design scenario. The model parameters for the records in Fig. 4 are listed in Table 1.

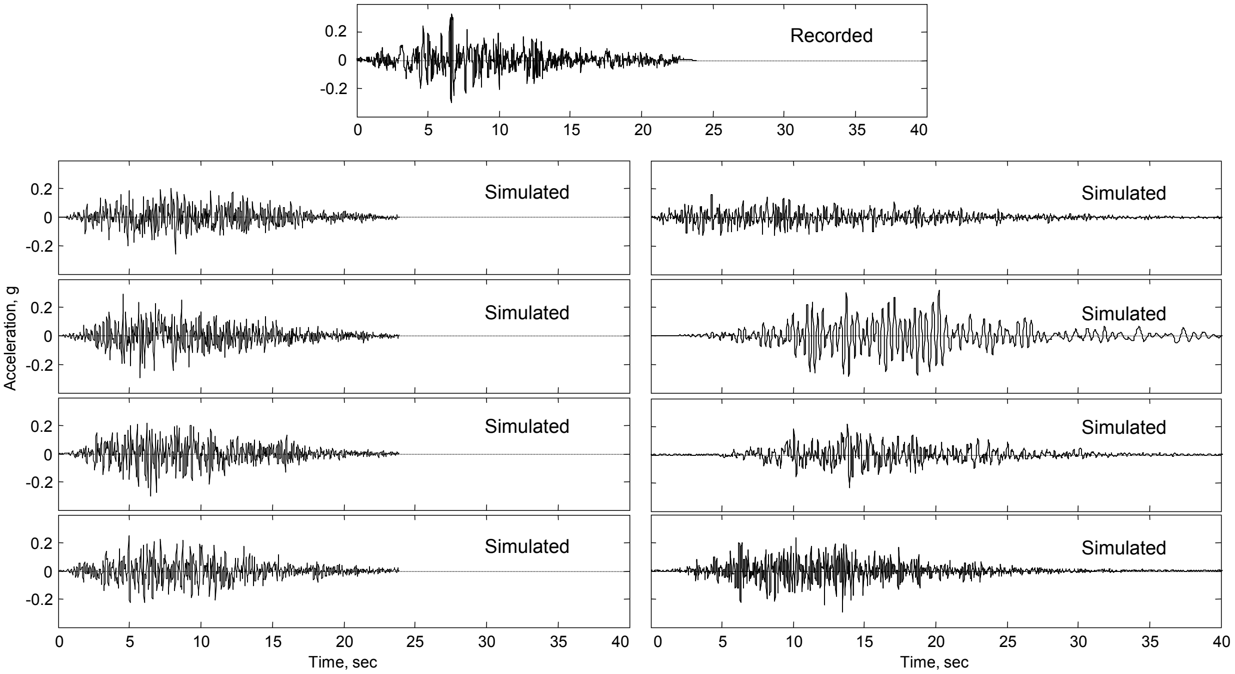

Figure 4. Top: recorded motion; Left: simulated motions with model parameters identical to the recorded motion; Right: simulated motions for design scenario of the recorded motion accounting for the variability of model parameters.

Table 1. Model parameters for records in Fig. 4.

	$\bar{I}_a$ (sec.g)	D_{5-95} (sec)	t_{mid} (sec)	$\omega_{mid}/2\pi$ (Hz)	$\omega'/2\pi$ (Hz/sec)	ζ_f (ratio)
Recorded motion and simulated motions on the left side of Fig. 4	0.145	12.3	6.8	5.90	0.12	0.18
Simulated motions on the right side of Fig. 4 (from top to bottom)	0.075	20.1	7.0	4.84	-0.012	0.25
	0.288	21.3	16.5	2.48	-0.054	0.12
	0.124	15.3	14.9	3.72	0.0039	0.40
	0.147	15.5	10.0	6.22	0.00046	0.18

It is possible to fix one or more of the model parameters to obtain conditionally simulated ground motions, e.g., synthetic motions with fixed Arias intensity. Care should be taken to account for the correlations among the fixed and variable parameters. See Rezaeian and Der Kiureghian (2009) for more detail.

Model Validation

The proposed method of generating a suite of synthetic motions for a specified seismic design scenario has been validated by comparing the resulting synthetics to real recorded motions and to previously existing GMPEs.

Synthetic acceleration, velocity and displacement time-histories are compared with real motions recorded during previous earthquakes. These comparisons indicate that not only acceleration, but also velocity and displacement time-histories of synthetic motions have similar characteristics (e.g., intensity, duration, spectral content, and peak values) and variability to real earthquake ground motions. Some examples of such comparisons are provided in Rezaeian and Der Kiureghian (2009).

In addition to time-histories, the elastic response spectra of synthetic motions are compared to those of recorded motions. This study indicated that for a given seismic design scenario, the variability of the response spectra of synthetic motions (at given spectral periods) is representative of the variability inherent in recorded ground motions for the same earthquake and site characteristics. For example, in Fig 5(a), for a specified seismic design scenario ($F = 1$, $M = 6.69$, $R_{rup} = 20.3$ km, $V_{s30} = 1223$ m/sec), the response spectrum of a previously recorded motion (regarded as just one realization of possible ground motions for the specified seismic design scenario) is within the range of the spectral values predicted by 50 synthetic motions.

The synthetic motions are intended for use in engineering practice as predictions of future earthquake ground motions at a given site. Therefore, a reasonable validation approach is to investigate how these motions compare with existing ground motion prediction equations used in practice. The statistics of the elastic response spectra of many synthetics for various design scenarios were compared to their corresponding predicted values by four of the NGA relations. In general, the median and variability of elastic response spectra (at given spectral periods) for synthetics are in close agreement with NGA models. An example scenario ($F = 0$, $M = 7.0$, $R_{rup} = 40$ km, $V_{s30} = 760$ m/sec) is shown in Fig. 5(b), where statistics of elastic response spectra for 500 synthetics are compared to their predicted values by NGA GMPEs.

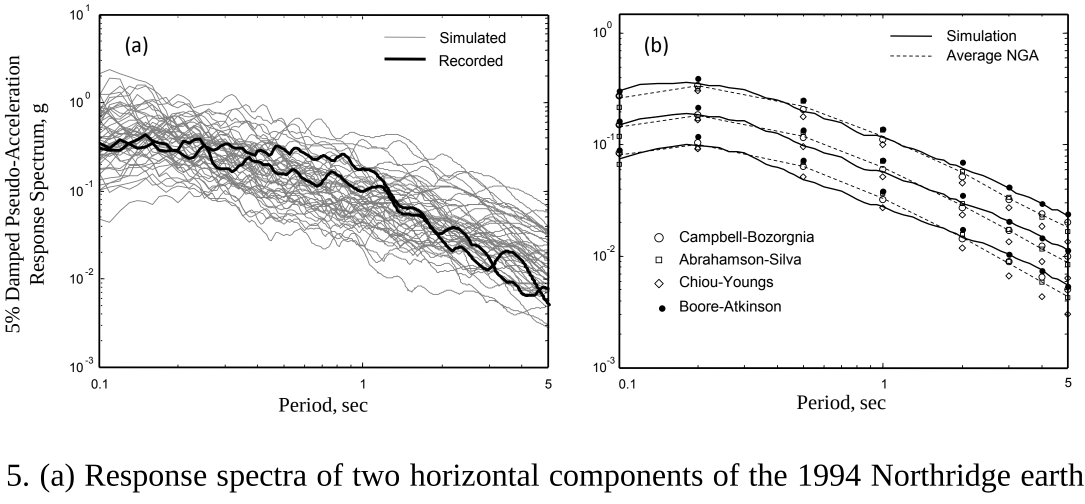

Figure 5. (a) Response spectra of two horizontal components of the 1994 Northridge earthquake

recorded at the LA-Wonderland Ave and 50 synthetic motions. (b) Median and median ± one logarithmic standard deviation of response spectra of 500 synthetic motions and corresponding values predicted by four of the NGA GMPEs. (After Rezaeian and Der Kiureghian 2009).

Conclusions

A new method for simulating an ensemble of far-field strong ground motions on firm ground for a given set of earthquake and site characteristics is presented. The resulting synthetics have similar characteristics as real recorded earthquake ground motions. An important achievement of the proposed method is that, by randomly generating the model parameters, the model provides a realistic representation of the natural variability of ground motions. This variability is comparable with the variability observed in existing ground motion prediction equations for elastic response spectrum that are based on the NGA database.

Further study is underway to simulate orthogonal horizontal components of ground motions by using the proposed methods and by identifying the correlations between the model parameters of the two ground motion components. Also, a companion study is currently underway for near-field ground motion simulation, where the directivity pulse and fling step are characterized in terms of the earthquake and site characteristics and added to the model in the fault-normal and fault-parallel directions respectively.

Acknowledgments

This work was supported by the State of California through the Transportation Systems Research Program of the Pacific Earthquake Engineering Research Center (PEER). Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the funding agency. The authors thank Y. Bozorgnia for his valuable advice and suggestions.

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