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DAMAGE-BASED SPECTRAL MATCHING

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ABSTRACT

Common methods of scaling the earthquake records for time history analyses are mostly based on linear response. However, there is no guarantee that the resulting inelastic response will be close to the expected response. This paper uses wavelet transformations and the damage index proposed by Park and Ang for inelastic spectral matching. Each selected record is decomposed into a set of time histories referred to as components with non-overlapping period bands. Each component, which contributes mainly to the response within its period band, is modified as many times as needed so that the response spectrum obtained from the superposition of modified components will match the target spectrum. The modified damage spectra show close agreement with the target spectra. The method also preserves the characteristics of the frequency content of the original records. It supersedes the traditional methods developed for linear spectral matching and can be utilized in damage-based design methodologies.

Introduction

Performance-based design requires more detailed information on the displacements, drifts, and inelastic deformations of a structure than traditional methods (Bozorgnia 2004). As a result of developments in performance-based methods and rapid progress in computer technology, the nonlinear time history analysis of structures has become more common. Current seismic evaluation guidelines such as FEMA 356 Prestandard and Commentary (FEMA 2000) contain detailed provisions for the nonlinear time history analysis.

In a time history analysis an ensemble of ground motions compatible with the design spectrum should be selected. Because of the time-consuming nature of computations, specially for nonsymmetrical buildings, and the limited number of records available, the structure cannot be analyzed under a large number of records consistent with the design scenario. The common approach in design codes is to select and scale a small number of records to obtain an estimate of the response that would be obtained under the ideal scenario where no compromise was required (Hancock 2008). Some of the methods introduced for this purpose are: selection on the basis of spectral shape proximity and amplitude-scaling at a single period or over a range of periods (Bommer 2004; Iervolino 2005), simultaneous selection and scaling of records using genetic algorithms (Naeim 2004), scaling on the basis of vector-valued intensity measures (Baker 2005; Baker and cornell 2006), and methods based on wavelet transformations (Gupta 2002;

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Mukherjee 2002).

The current record selection and scaling methods rely mostly on elastic Intensity Measures (IMs). However, considering that the inelastic response is not usually proportional to the excitation amplitude, there is no guarantee that the inelastic response estimated from records scaled to match an elastic design spectrum will be close to the expected inelastic response. This paper uses a damage index as an inelastic IM for the selection and scaling of records. Wavelet transformations will be used for inelastic spectral matching.

The Inelastic Intensity Measure

Considering the shortcomings with elastic IMs and the fact that most buildings experience inelastic behavior in major earthquakes, it seems that basing the selection and scaling of records on inelastic IMs is more appropriate. While damage indices are better predictors of structural performance, compared to other IMs such as inelastic pseudo spectral acceleration, the damage index of Park and Ang (Park 1985) will be used in this study. This will assist with the selection of records to match a predetermined level of damage.

A challenge with using new IMs is the computation of ground motion hazard. Generally, the design (target) spectrum can be from a code, from a site-specific analysis, or from a regression analysis. In the case of a damage index, neither code-based damage spectra nor attenuation relationships exists. While the derivation of empirical equations for damage prediction is currently under development by the authors, for this paper the mean damage spectrum of an ensemble of records is calculated and used as the target spectrum. The resulting spectrum represents the mean damage that an SDF system designed for a given ductility would sustain under the set of un-scaled records.

Park and Ang Damage Model

Park and Ang damage index (DI_{PA}) is widely used to quantify the damage under cyclic loading. It considers the effects of maximum displacement and hysteretic energy dissipated in an SDF system. The modified model is as follows:

$$DI_{PA} = \frac{U_{max} - U_y}{U_{umon} - U_y} + \beta \frac{E_{hys}}{U_{umon} F_y} \tag{1}$$

where U_{max} is the maximum deformation under cyclic loading, U_{umon} is the ultimate deformation capacity under monotonically increased loading, F_y is the yield strength, E_{hys} is the total hysteretic energy dissipated, and β is a parameter controlling strength deterioration. The ultimate deformation is evaluated using the following relationship (Bertero 2002):

$$U_{umon} = \theta_{umon} \times h/2 \tag{2}$$

where θ_{umon} is the rotational capacity of the system under monotonic loading (e.g. 0.05) and h is the equivalent height of the building. Empirical equations can be used to find an equivalent height for the natural period of an SDF system. The dimensionless parameter DI_{PA} ranges from zero for elastic to one representing the collapse.

Complications of Inelastic Spectral Matching

The damage index of Park and Ang is a function of two response parameters: E_{hys} and U_{max}. While amplifying the record by α results in the amplification of E_{hys} by α^2, it does not create any predictable variation in U_{max}. Therefore, if reaching a specific value of DI_{PA} at a given period is desired, scaling the entire record will not be helpful. This is a challenge with most inelastic IMs. To overcome this issue, wavelet transformations will be used.

Wavelet Transformations

A wavelet is a function used to divide a given time signal into different components each corresponding to a period band. Wavelet transforms have advantages over traditional Fourier transforms for representing functions with discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals such as earthquakes. The acceleration time history of an earthquake $f(t)$ of duration L can be decomposed into N time histories using (Basu 2000; Mukherjee 2002):

$$f(t) = \sum_{j=1}^{N} f_j(t) \tag{3}$$

Each individual time history is obtained through

$$f_j(t) = \frac{K\Delta b}{a_j} \sum_i W_\psi f\left(a_j, b_i\right) \psi\left(\frac{t-b_i}{a_j}\right) \tag{4}$$

where

$$\psi(t) = \frac{1}{\pi\sqrt{\sigma-1}} \frac{sin(\sigma\pi t) - sin(\pi t)}{t} \tag{5}$$

The function $\psi(t)$, called the basis function, is a fast-decaying oscillating waveform of which the wavelets are scaled and shifted using the scale parameter a_j and the shift parameter b_j:

$$a_j = 2^{j/4} \tag{6}$$

$$b_i = (i-1)\Delta b \tag{7}$$

In the above relationships, σ equals $2^{1/4}$ and Δb equals 0.02 s. The wavelet coefficient for (a_j, b_i) is calculated using:

$$W_\psi f\left(a_j, b_i\right) = \frac{1}{\sqrt{a_j}} \int_0^L f(t) \psi\left(\frac{t-b_i}{a_j}\right) dt \tag{8}$$

The formulation for calculating K in Eq. 4 may be found in Basu (2000) and Mukherjee (2002).

Using the basis function displayed in Eq. 5 (a modified form of Littlewood-Paley function), the coefficients $W_\psi f(a_j, b_i)$ contribute mainly to the response of oscillators with natural period in $(2a_j/\sigma - 2a_j)$ (Basu 1998). Hence, $f_j(t)$ can be scaled to match the target spectrum within its period band. Noting that $f_j(t)$ also affects the response at periods not in $(2a_j/\sigma - 2a_j)$, the modification factor at iteration i should consider the response to the superposition of all modified components ($f^i(t) = \sum_{j=1}^{N} f_j^i(t)$). This is shown in the denominator of the following equation:

$$f_j^{(i+1)}(t) = f_j^{(i)}(t) \frac{\int_{\frac{2a_j}{\sigma}}^{2a_j} [DI_{PA}(T)]_{trg} dT}{\int_{\frac{2a_j}{\sigma}}^{2a_j} \left[DI_{PA}^{(i)}(T)\right]_{calc.} dT} \quad j = 1,2,\ldots,N \tag{9}$$

The criterion for terminating the modification is to reach an average error smaller than a desired limit:

$$Error_{ave}^{i} = \left(\sum_{j=1}^{N} \sum_{k=1}^{M+1} \frac{\left|(DI_{PA}(T_{jk}))_{trg} - (DI_{PA}{}^{i}(T_{jk}))_{calc}\right|}{(DI_{PA}(T_{jk}))_{trg}}\right) / (N \times M) \tag{10}$$

In the above equation, i is the iteration number, j is the component index, N is the number of components generated through decomposition of the original record, M is the number of divisions considered for each period interval, k is the period number within component j, T_{jk} is the kth period for component j, and trg and $calc$ refer to the target and calculated, respectively. In addition, $DI_{PA}{}^{i}(T_{jk})$ is the damage at period T_{jk} computed for the modified composed time history $f^i(t)$.

The characteristic of component $f_j(t)$ affecting the response primarily in its period band can help with inelastic spectral matching. Since the period band for each component is usually small, it seems feasible to scale that component in order to obtain an average inelastic response acceptably close to the average target response within the period band.

Ground Motions and Target Spectra

The ensemble of ground motions used for generating the mean damage spectrum (used as one of the target spectra) consists of 50 acceleration time stories recorded on Site Class C using the IBC 2006 soil classification (IBC 2006). Site Class C consists mainly of very dense soil and soft rock with the shear wave velocity in the top 100 ft layer of soil being in the range 1200-2500 ft/s (366-762 m/s).

To test the appropriateness of the method for matching target spectra with sharp peaks and valleys, the damage spectrum of the N22E component of the Jensen Filter Plant of the 1994 Northridge earthquake is also used as the target damage spectrum.

Noting that the period band for each component can be extremely small, the integral in the numerator of the modification factor (Eq. 9) cannot capture the area under the target spectrum correctly, unless an adequate number of periods are used for generating the spectrum. For this reason, three period increments (.001, .01, and .05 s) were used in three spectral regions.

For a target ductility μ_{trg} of 4, the target damage spectra calculated for a bilinear SDF model with 2% strain hardening ratio and 5% damping ratio are shown in Fig. 1.

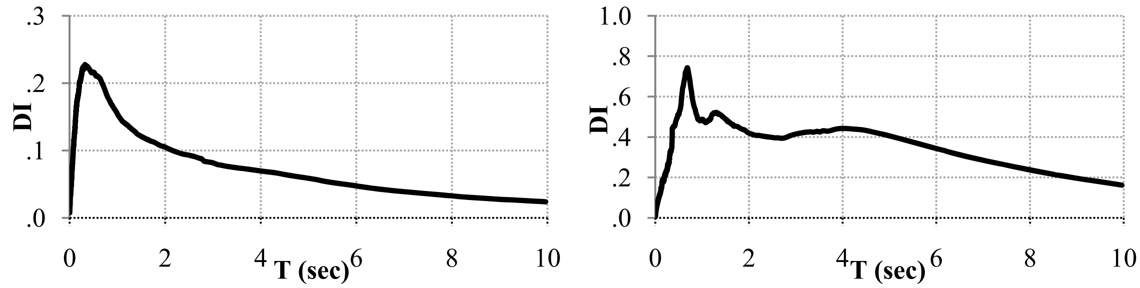

Figure 1. The mean damage spectrum of the 50 records in the ensemble (left) and the damage spectrum of the N22E component of the Jensen Filter Plant of the 1994 Northridge earthquake (right).

Results

The modification scheme was applied to four records with the damage spectra shown in Fig. 2 for the target ductility of 4. Attempt was made to select records with dissimilar duration, frequency content, spectral shape, etc.

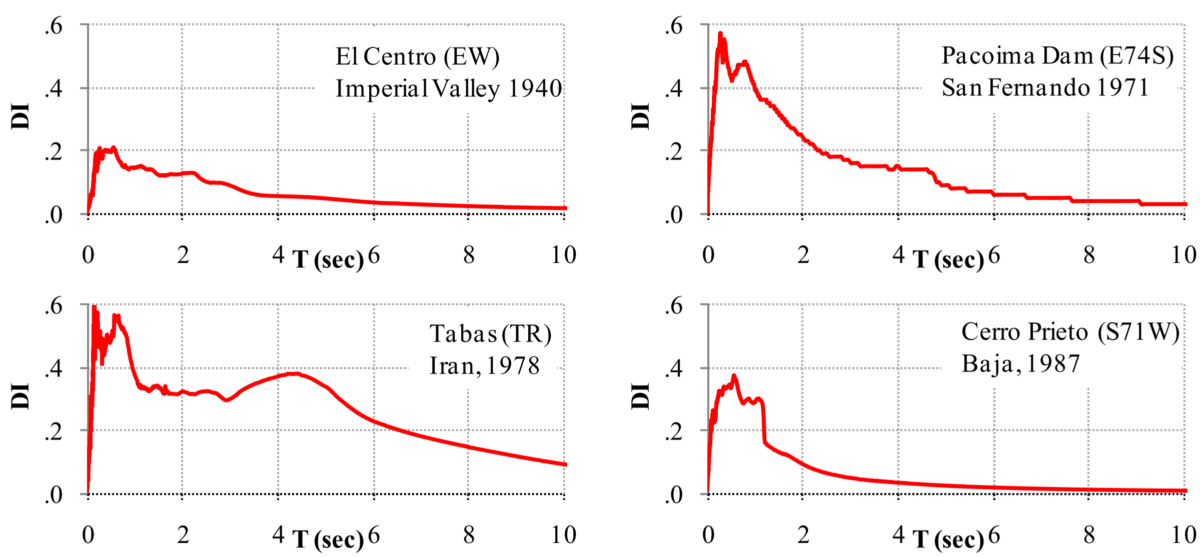

Figure 2. Damage spectra of the records used for spectral matching.

The number of components required to decompose a record depends on characteristics such as frequency content and strong motion duration. The number of components to be used for spectral matching may be different from the number that is required to precisely reconstruct the record. For example, for the Pacoima Dam record using 43 components with j spanning from -42 to 0 (periods in .001-2.000 s) results in an accurately reconstructed record. But, a scale parameter as small as $2^{-42/4}$ will result in the infinitesimal period interval 0.0012-0.0014 s, which is an impractical region in the spectrum. On the other hand, taking 31 components from -20 to 10 (periods in .053-11.314 s) covers the practical period range in the spectrum. In addition, the analyses have shown that using components with j less than, say -20, may lead to non-converging results. While a record reconstructed from a reduced number of components may have somewhat different characteristics from the original record, the primary purpose of the analyses, which is to match the inelastic target spectrum, is met using components from -20 to 10.

The mean damage spectrum of the 50 records used as the target spectrum, the damage spectrum of the original record, and the damage spectrum of the modified record are given in Fig. 3. For each record, 31 components with j ranging from -20 to 10 are considered. The plots illustrate a good agreement between the modified and the target spectra.

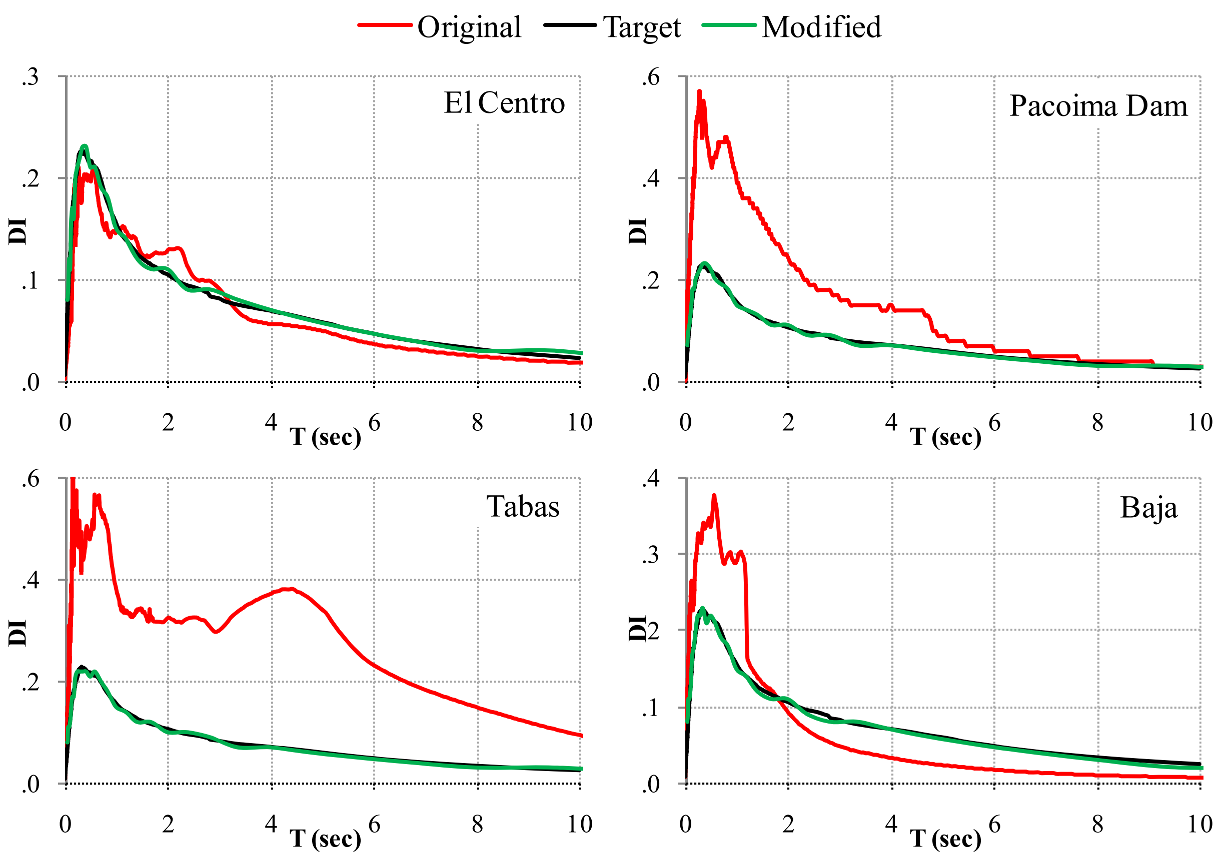

Figure 3. Comparison of the original and modified damage spectra of the selected records with the mean damage spectrum of the 50 records in the ensemble.

It should be noted that the modification scheme used cannot guarantee reaching any predetermined limit for the error neither for an individual period band nor for the overall period range of the spectrum. On the other hand, it is possible that an acceptable error obtained for a period band is not improved, or even approached, in further iterations. On this basis, for non-converging cases, if an error less than a certain limit, say 10%, is reached for component j, the corresponding modification factor is kept unchanged in further iterations.

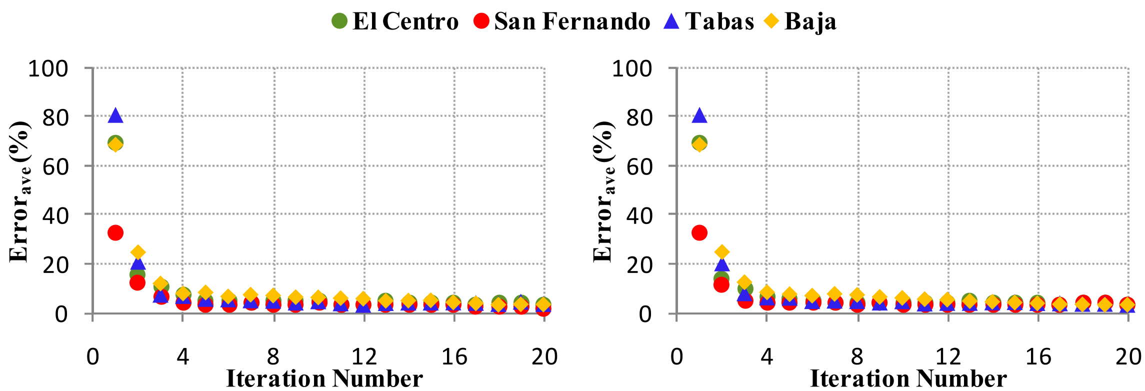

Figure 4. Average error versus iteration number for $M = 1$ (left) and $M = 5$ (right).

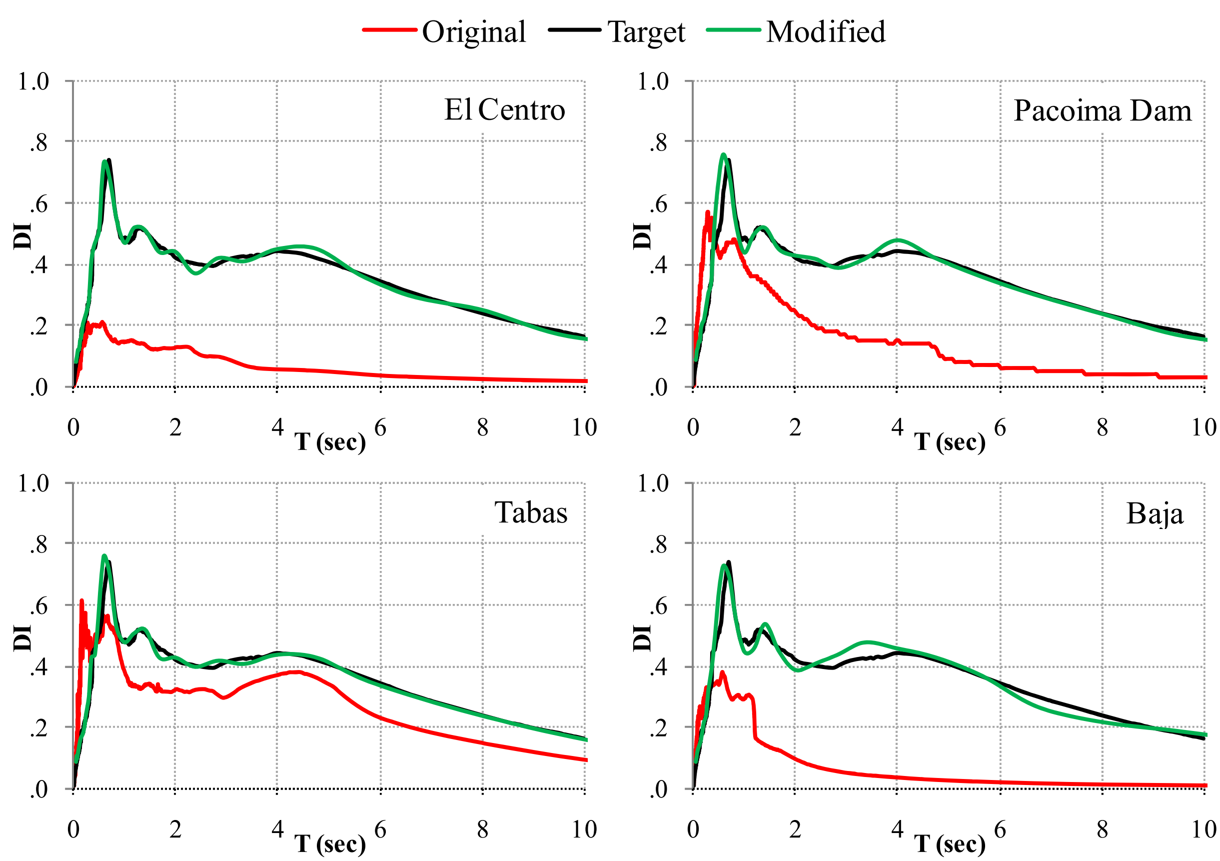

Figure 5. Comparison of the original and modified damage spectra of the selected records with the damage spectrum of the N22E component of the Jensen Filter Plant of the 1994 Northridge

earthquake.

Considering that the period band increases toward larger values of j, the corresponding number of divisions may need to be increased. Fig. 4 displays the average error for twenty iterations of modification. It is observed that the error drops rapidly, being as small as 2% at the end of the iterations. Additionally, it can be seen that the number of divisions M used for each period band has no effect on the error. This is expected considering that the target and the modified spectra overlap closely toward larger periods (Fig. 3) where the response is dominated by components with larger period bands.

Fig. 5 demonstrates the results of the analyses performed for matching the damage spectrum of the N22E component of the Jensen Filter Plant of the 1994 Northridge earthquake. It is observed that the modified damage spectra match the target satisfactorily with the results being the most successful for the Tabas record. Fig. 6 illustrates the variation in average error for the four records. The weakest matching is obtained for the Baja record, with the minimum error being around 8%.

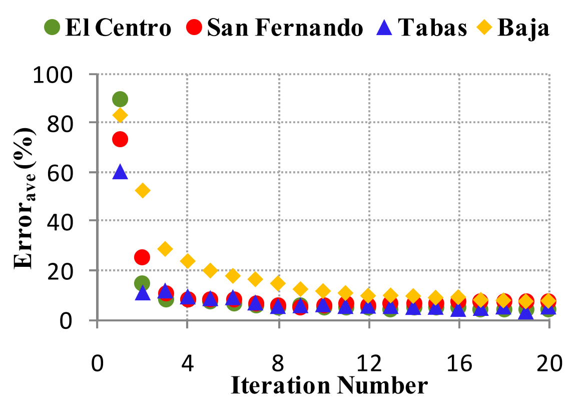

Figure 6. Average error versus iteration number ($M = 1$).

The original and the modified records corresponding to the modified spectra in Fig. 5 are displayed in Fig. 7. It is observed that the modified records have similar temporal characteristics to the original records. However, the modified records are generally of lower frequency as compared to the original records. This can be attributed to the components neglected in the process of modification.

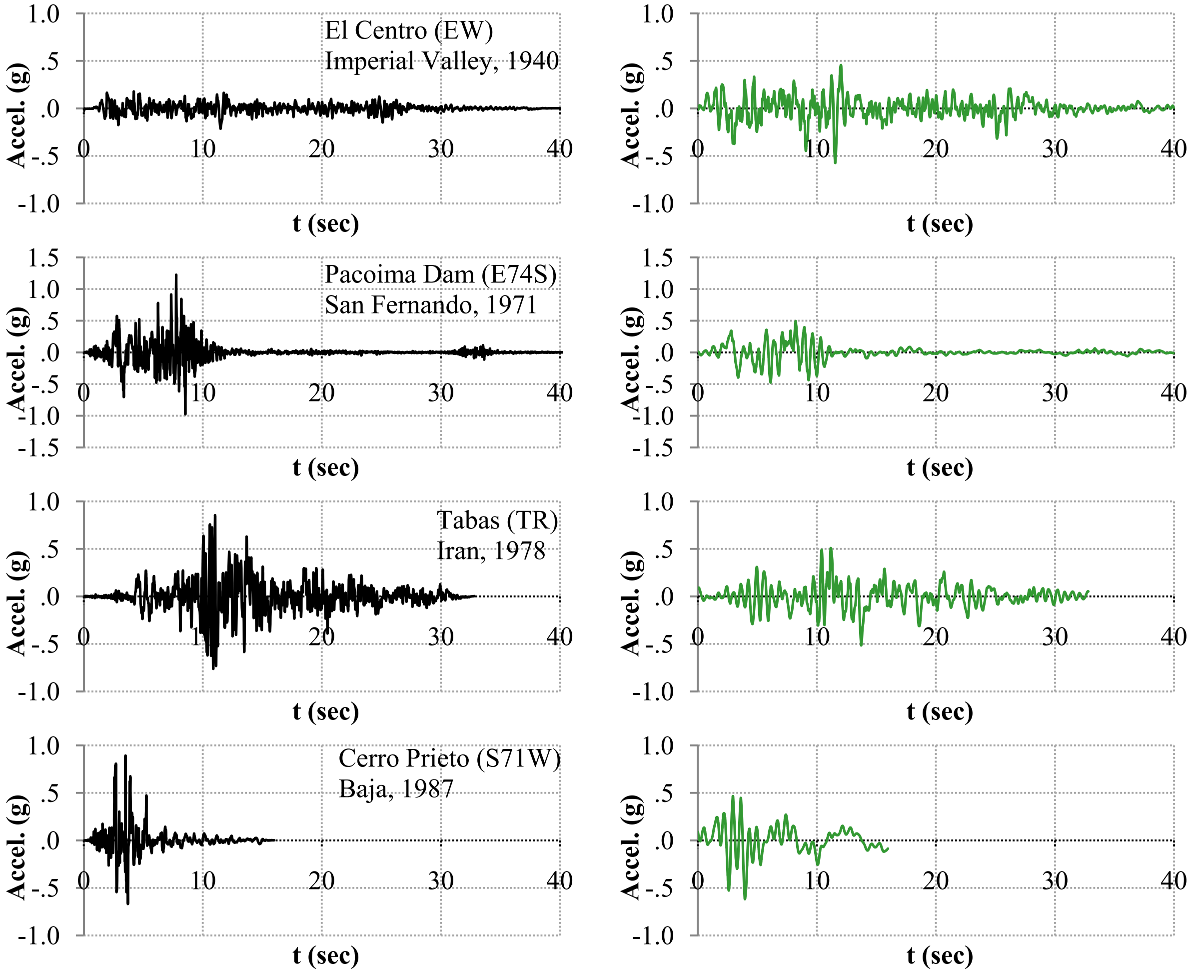

Figure 7. Original and modified acceleration time histories.

Conclusions

Damage-based spectral matching was accomplished using a modification scheme based on wavelet transformations. The damage index proposed by Park and Ang was used as the Intensity Measure. Each of the selected records was decomposed to 31 acceleration time histories. The modified damage spectra were found to be in good agreement with the target spectra. While the magnitude of the error depends mainly on the characteristics of the record and the target spectrum, the convergence rate is fast and the worst case resulted in 8% error. The modified records possess similar temporal characteristics to the original ones, but contain lower frequencies. They can be applied to induce a predetermined level of inelastic response in the structure. The method can supersede traditional methods developed mainly for elastic spectral matching and be utilized in damage-based design methodologies. Additionally, since the target spectrum is matched over the entire range of periods rather than at a single period, it can capture the response of higher modes in an MDF system as well as the increased period of an inelastic system.

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