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CORRELATION BETWEEN STRENGTH AND STIFFNESS OF REINFORCED CONCRETE ELEMENTS AND ITS IMPLICATIONS ON DISPLACEMENT BASED SEISMIC DESIGN OF BUILDINGS

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ABSTRACT

This paper investigates the correlation between the initial stiffness (cracked) and the strength of reinforced concrete beams and columns sections when the reinforcement steel ratio is varied. Based on the results obtained from fiber analyses of the sections with various reinforcement steel ratios, all between the allowable limits, the variation of the yield curvature, ultimate curvature, deformation capacity and the relationship between the initial and the post yielding stiffness is studied to define a consistent bilinear model which can be used in a rational procedure for the displacement based seismic design and evaluation of reinforced concrete structures.

Introduction

Most existing seismic evaluation and design codes are based on a design philosophy based on forces (FBSD); however, with this philosophy it is not possible to guarantee the structural performance under the seismic design demands, as it has been shown by the effects of recent destructive earthquake, in which the excessive structural and non-structural elements damage have led to large human and economic losses. With the aim of predicting and guarantying structural performance under seismic design actions, several investigations have been carried out oriented to develop rational seismic evaluation and design methods under this context, as current tendencies in seismic evaluation design methods centre around the so called performance based design philosophy (PBSD), within which displacement based design (DBSD) methods have been developed (Chopra and Goel, 2001; Panagiotakos and Fardis, 2001; Priestley *et al.* 2007); however these methods use concepts and assumptions which validity and generality still are under discussion within the research and professional community; one of these assumptions, used by Priestley *et al.* (2007), suggests that the yield curvature (φ_y) of reinforced concrete elements depends on the depth of the section and the yield strain of the steel used as reinforcement, something that is not rigorously true. However, within the context displacement based design, if this assumption is accepted as valid the definition of design methods is facilitated.

This paper reports the results of a parametric investigation on the effects of the

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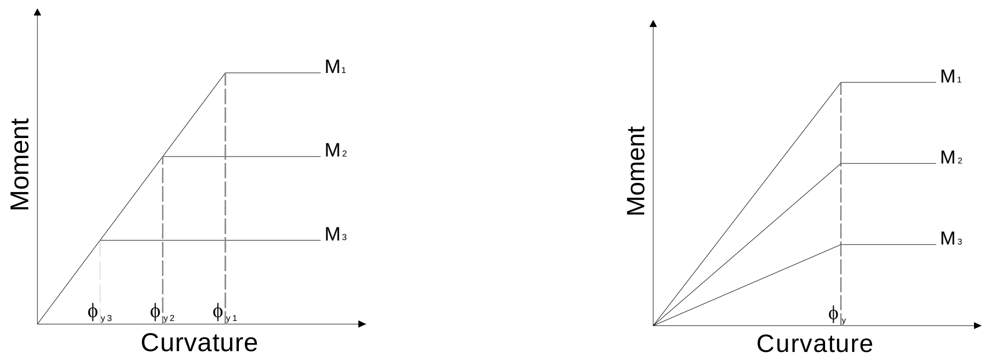
reinforcement ratio (ρ) on the yield curvature of sections of code designed reinforced concrete beams and columns. The yield curvatures obtained are compared with those obtained from other approximations available in the literature. The effect of changes in strength produced by changes in the reinforcement ratio on initial (cracked) and post-yielding stiffnesses is also investigated. From the analysis of the results obtained, the range of reinforcement ratios for which the variation of φ_y is minimum, and therefore the consideration of it as constant valid, is found.

Background

The deficiencies presented by the FBSD have been demonstrated in recent intense seismic events. The differences between calculated and observed performances are due, among other things, to the inconsistencies of some assumptions used in the FBSD (Priestley, 2003; Smith and Tso, 2002); such as the assumption that the stiffness and strength of an element section are independent variables. However, it is a well know fact that the stiffness and the strength of a section are correlated variables, and that, this correlation depends on how the stiffness and / or strength is varied, either by changing the geometry of the section, the reinforcement steel ratio, the mechanical properties of the constitutive materials, or combinations of them.

The assumption that the initial stiffness (cracked) and the strength of a section of a structural element are independent properties, *i.e.,* when the strength of a section is modified, its stiffness remains constant, leads to a situation in which, the yield curvature reduces as the strength reduces (figure 1-a), however, it has been analytically and experimentally shown (Paulay, 1997; Smith and Tso, 2002; Priestley *et al.*, 2007) that the stiffness and the strength of the reinforced concrete structural elements are correlated properties.



a) Design Assumption (constant stiffness - variability strength and yield curvature)

b) Realistic Conditions (constant yield curvatures – variability stiffness and strength

Figure 1. Influence of strength on moment- curvature relationship (Priestley *et al.* 2007)

Within the PBSD, special interest has been placed on the DBSD methods, as with them it is possible to relate lateral displacements and/or interstorey drifts with the local deformations in the elements and in this way also with structural damage. Thus, by controlling displacements structural damage may be also directly controlled. In different DBSD methods it is accepted that the yield curvature depends only on the depth of the section and on the yield strain of the reinforcement steel. In these methods, the design process starts with an estimate of the geometry of the sections of the elements, obtained from a preliminary design of the structure; thus, for a section of given dimensions and material properties, Priestley (2007) postulates an equation in which φ_y remains constant (Eq. 1), and its strength is proportional to its stiffness (figure 1-b). However, if a moment–curvature analysis of the sections is carried out increasing only the ratio of reinforcement steel, it is observed that φ_y also depends on it; however, it is also shown that there is a range of ρ values for which the variation of φ_y is insignificant and thus it may be approximated as constant.

$$\phi_y = k\, \varepsilon_s / h_b \tag{1}$$

where k is a coefficient which depends on the type of element, being 2.1 for rectangular columns, from 1.9 and 1.7 for T-section beams whether the effects of strain hardening are or are not included, correspondingly, and a value 10% higher than those for T-section beams for beams of rectangular-section, ε_s= fy/Es is the yield strain of the reinforcement steel and h_b is the depth of the element.

Procedure and considerations for the analysis of sections

It is a well known fact that the characterization of reinforced concrete elements is a complex task; as the interaction between the materials (heterogeneity) and the effects of actions acting (bending, shear and axial), determine the type of behavior presented by the element (ductile, non ductile or semi-ductile). The ductile and non ductile behaviors are two types of response of the reinforced concrete elements subject to cyclic loading. The ductile behavior is initially present in the element whose shear strength is larger than its bending strength although there is a large quantity of elements which end showing a shear dominated behaviors after several load cycles, condition mainly due to the buckling of longitudinal reinforcement steel, strain hardening and strength degradation induced by the alternation of bending load cycles. The non ductile behavior is shown by elements whose bending strength is higher the its shear strength,

The ductile behavior of an element is present when the acting loads do not reach its shear strength; this implies that this element will have a bending dominated behavior. The non ductile behavior of an element is shown when the acting loads reach its shear strength, and the longitudinal steel reinforcement does not yield. An intermediate case occurs in the element, a semi-ductile behavior, when the acting loads reach the shear strength and the longitudinal reinforcement steel incursions into a yield state.

To reproduce the behavior of an element several models have been developed, however the models aimed to reproduce complex behaviors are difficult to use and of doubtful results. Furthermore, the fact that the majority of codes have design guidelines oriented to produce bending dominated elements, clearly avoiding any other type of behavior under design actions, oriented this work to investigate only sections with bending dominated behavior, always verifying that the demand levels were such that no other types of behavior were possible.

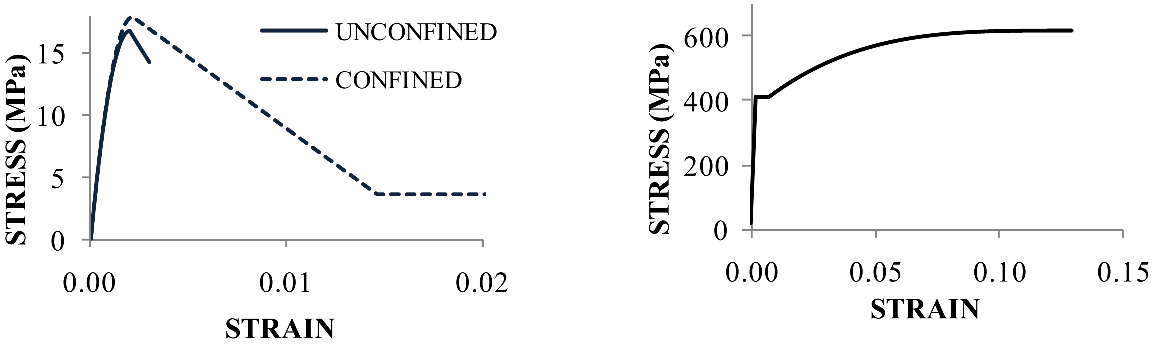
As a part of a research project aimed to develop a consistent DBSD method, a parametric study was carried out to identify the effects of the structural variables upon yield curvatures, stiffnesses, strengths and deformation capacities of reinforced concrete sections typically used in building structures (beams and columns). The structural variables evaluated were: concrete and steel strengths, aspect ratio of the sections, reinforcement ratio (tension and compression), confinement level of concrete and acting shear and axial loads.

This paper reports only the effects of variations of the ρ value of reinforced concrete sections of rectangular beams and columns with different aspect ratios on their yield curvatures and on the correlation between their stiffness and strength. For these sections their strength was varied by increasing the reinforcement steel ratio calculating the moment curvature diagram for each considered section and verifying that the section did not present shear failure. The statistics of the results obtained were interpreted identifying the variations of stiffness, strength, and yield curvatures of the sections and suggesting a range of reinforcement ratios for which the variation of the yield curvature in a given section was minimal and the assumption of constant yield curvature could be approximately valid.

To attain this objective, aspect ratios, steel reinforcement ratios and shear and axial load levels were chosen within the limits specified in the Construction Code of the Federal District, RCDF-04.in Mexico, (GDF, 2004). In beams the longitudinal reinforcement steel was considered placed in lower and upper beds, with different ratios of compression to tension steel, and, in the case of columns the longitudinal steel was considered placed in symmetric perimetral patterns.

Geometry of sections and mechanical properties of materials

Geometry and reinforcement ratios of the sections employed in this study are shown in tables 1 to 3, for each beam section 10 values of ρ were used combining each of them with three different ratios of compression steel reinforcement, ρ′, in such a way that that the ratios ρ/ρ′ were equal to 0.75, 0.85 and 0.95; for the case of column sections, 12 values of ρ were used, each of these sub-sections with three different values of axial load defined in such a way as to produce values of N/agf′c equal to 0.1, 0.2 and 0.5. Concrete type I, with compression strengths $f'c=16.67$ MPa and $f'c_{CR}=17.78$ MPa for unconfined and confined concrete respectively (figure 2-a), and A-42 steel reinforcement, with a yield stress fy=411.88 MPa and an elastic modulus Es=196140.00 Mpa (figure 2-b), were considered for the analyses.



a) For concrete (compression)

b)For reinforcing steel

Figure 2. Stress-strain relationships

Analysis and interpretation of results

To identify the variations of yield curvatures of the studied sections, moment-curvature diagrams, M-ϕ, were obtained from fiber analyses of the sections with various reinforcement steel ratios. The behaviors observed were approximated by bilinear curves which meet the equal energy criterion when compared to the analytical (figure 3). Some of the diagrams M-φ are illustrated in figures 4 and 5.

Table 1. Beam and column geometry

Section	**Geometry**	
	Width (mm)	**Depth (mm)**
BEAM		
b2	300	450
b3	300	600
b4	300	750
b5	300	900
COLUMN		
C1	400	400
C2	800	800
C3	1200	1200
C4	400	600
C5	400	800
C6	400	1000

Table 2. Steel reinforcement ratios for beams

Section	**Sub-section**	**Ratio of tension reinforcement (ρ)**	**Ratio of compression reinforcement (ρ')**		
			ρ/ρ'=75%	**ρ/ρ'=85%**	**ρ/ρ'=95%**
BEAM	1	0.00264	0.00190	0.00216	0.00248
	2	0.00512	0.00369	0.00420	0.00482
	3	0.00761	0.00548	0.00624	0.00715
	4	0.01009	0.00727	0.00828	0.00949
	5	0.01258	0.00906	0.01031	0.01182
	6	0.01506	0.01084	0.01235	0.01416
	7	0.01755	0.01263	0.01439	0.01649
	8	0.02003	0.01442	0.01643	0.01883
	9	0.02252	0.01621	0.01846	0.02116
	10	0.02500	0.01800	0.02050	0.02350

Table 3. Steel reinforcement ratios for columns

Section	**Sub-section**	**Ratio reinforcement (ρ)**
COLUMN	1	0.00480
	1	0.01000
	3	0.01500
	4	0.02000
	5	0.02500
	6	0.03000
	7	0.03500

	8	0.04000
	9	0.04500
	10	0.05000
	11	0.05500
	12	0.06000

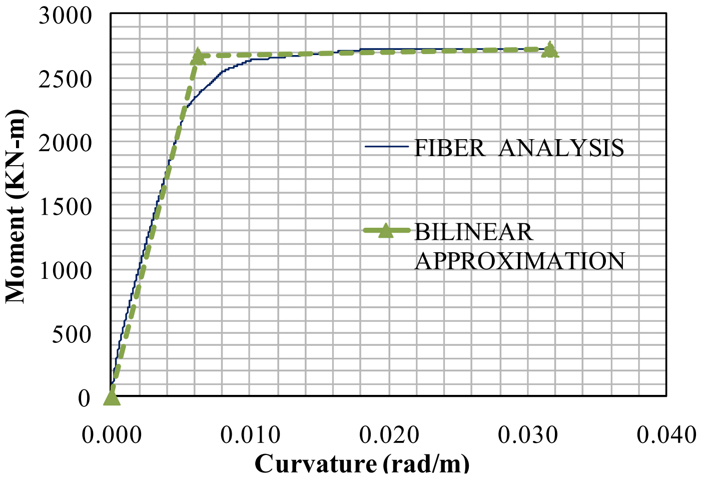
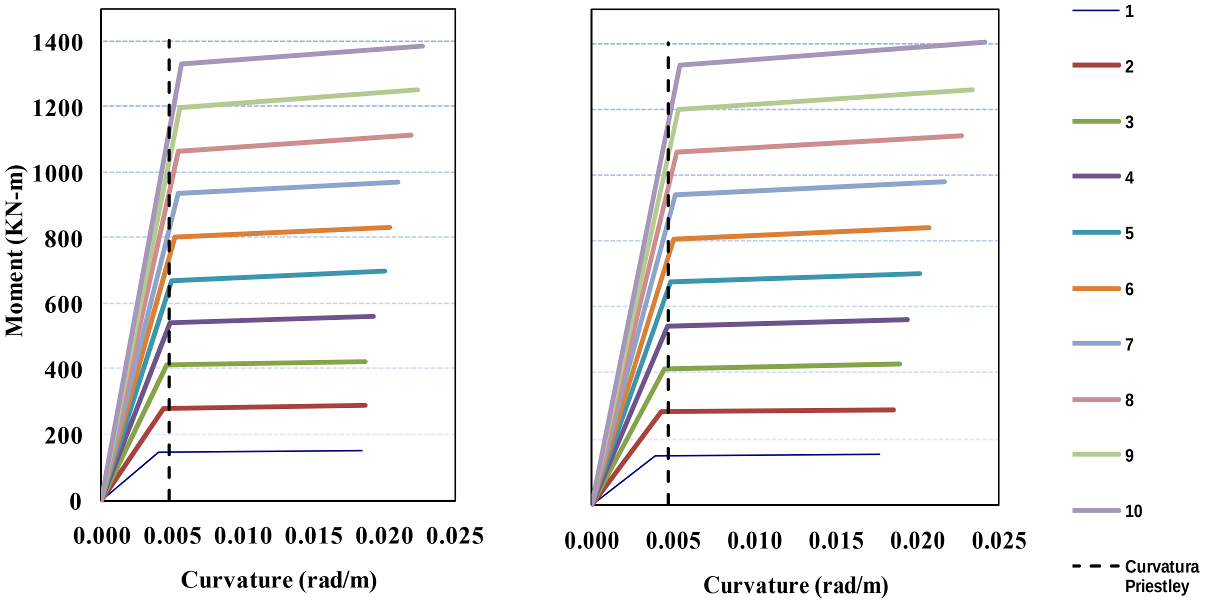
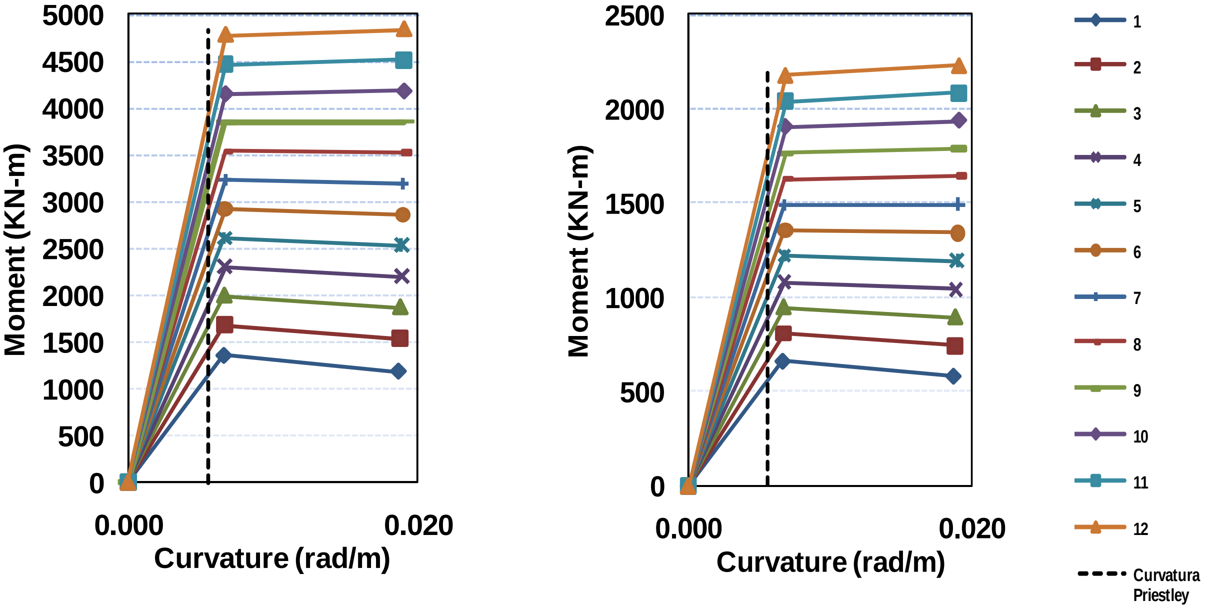


Figure 3. Diagrams moment curvature with bilinear approximation



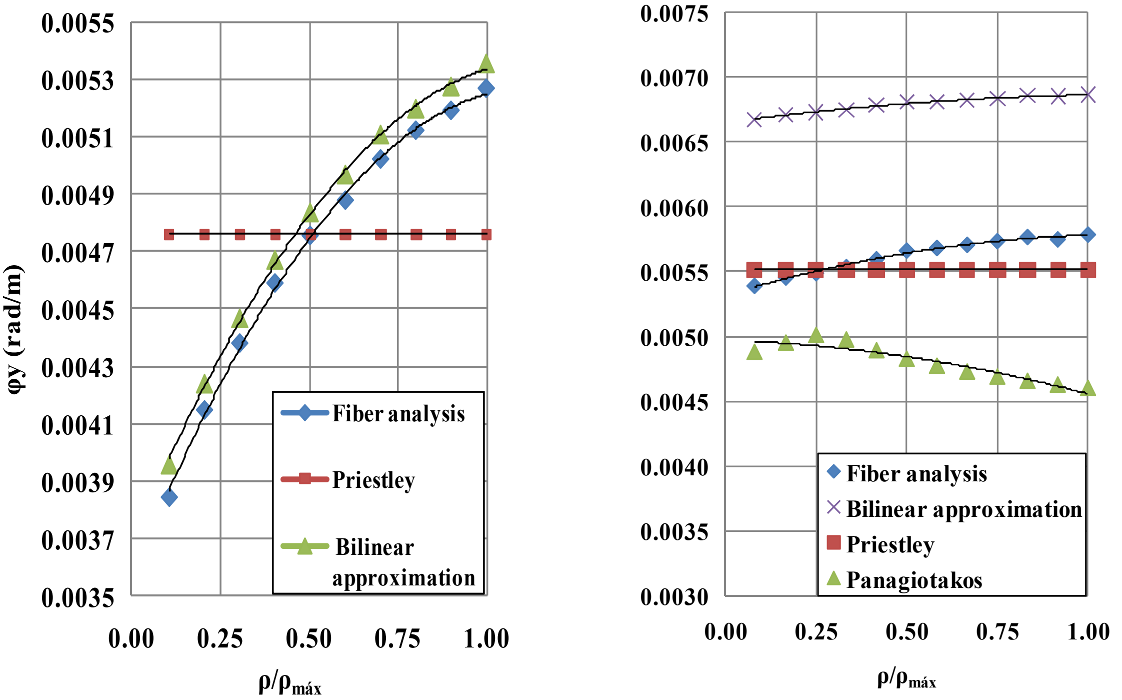
a) Beam section b4 (ρ / ρ´ = 85%) b) Beam section b4 (ρ / ρ´ = 95%)

Figure 4. Diagrams moment curvature for beam b4



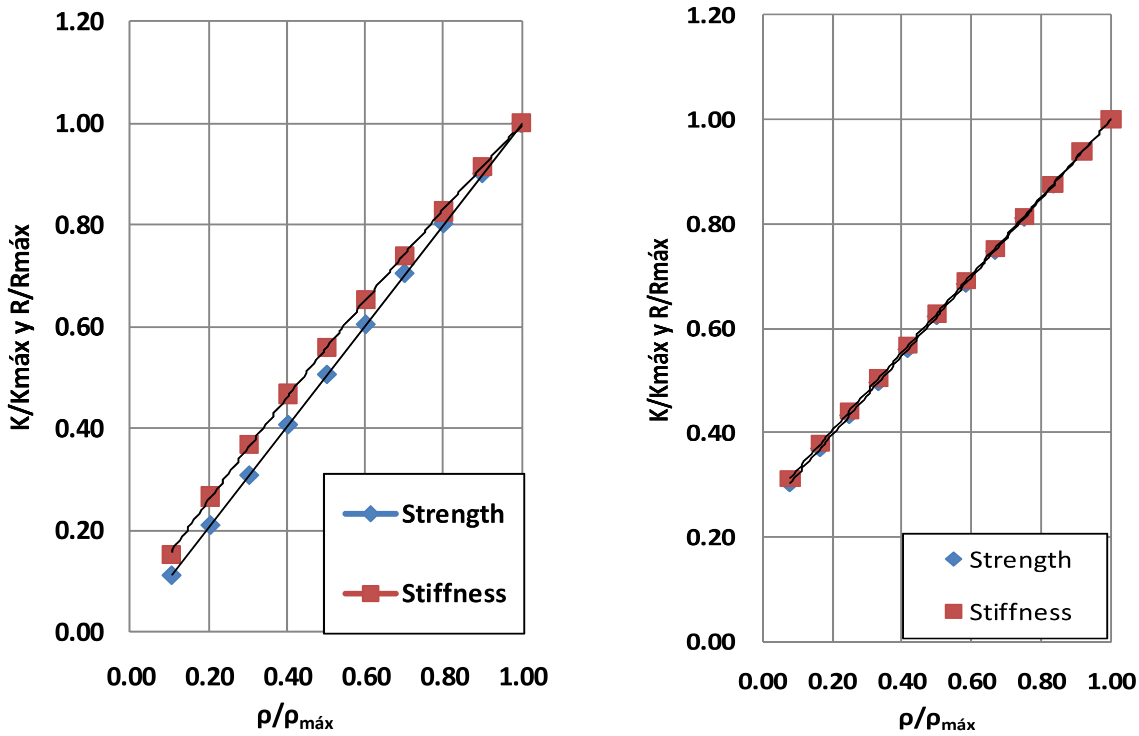
a) Section C2 (N/agf´c=0.2) b) Section C5 (N/agf´c=0.2)

Figure 5. Diagrams moment-curvature for columns C2 and C5



a) Beam section b4 (ρ / ρ´ = 95%) b) Column section C5 (N/agf´c=0.2)

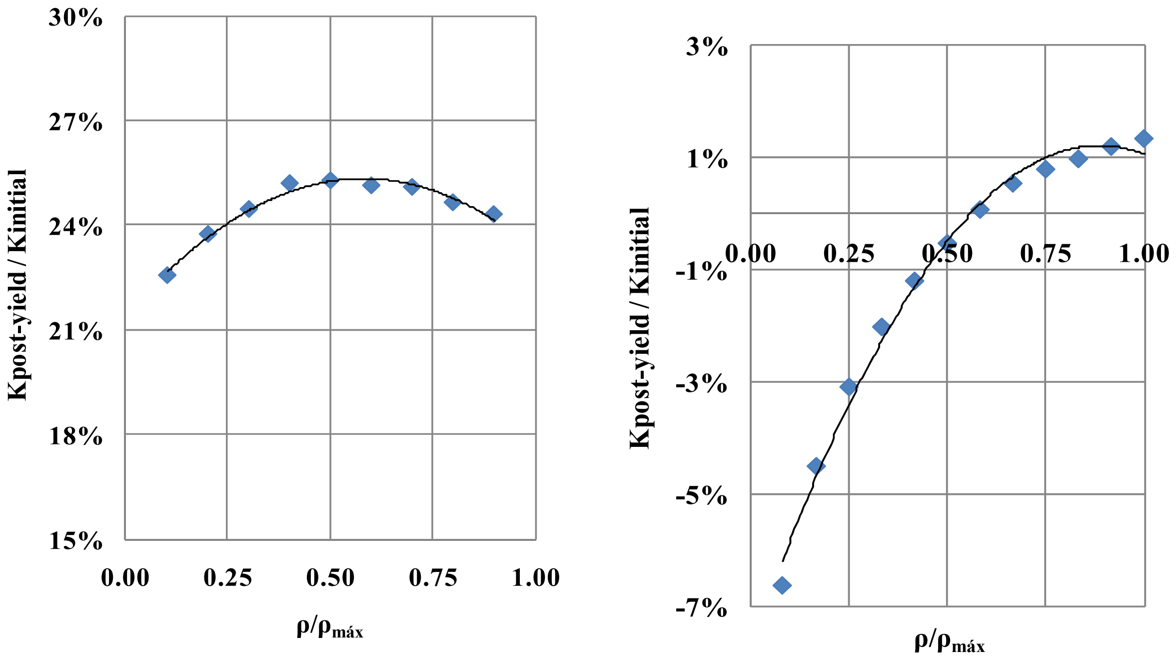
Figure 6. Comparison of approximations for yield curvature



a) Beam section b4 (ρ / ρ´ = 95%)

a) Column section C5 (N/agf´c=0.2)

Figure 7. Correlation between initial stiffness (cracked) and strength



a) Beam section b4 (ρ / ρ´ = 95%)

b) Column section C5 (N/agf´c=0.2)

Figure 8. Variation of post-yield to initial stiffness ratio

Figure 6 shows for different approximations the variation of the yield curvature of a section with steel reinforcement ratio. Figure 7 shows the variation of initial stiffnesss (cracked) and strength with steel reinforcement ratio and figure 8, shows the variation of post- yield to initial stiffnesss ratio.

Observations and Conclusions

From the results obtained, the following observations are made:

- BEAMS: The compression zone depth is a function of the steel reinforcement ratio: for high amounts of steel the compression steel stresses increase and vice versa. Because of this, for steel reinforcement ratios lower than 60% of ρ_{max}, stiffness before yield presented higher increments in strength: as compression zone depth increased, concrete area and thus stiffness increase also. For steel reinforcement ratios higher than 60% of ρ_{max}, deeper compression zones and thus high compression steel stresses and strains occur, this causes the effective stiffness of the section being provided mainly by the tension reinforcement, and the increments of stiffness and strength to be approximately proportional. The variation of the ratio of post-yield to initial stiffness, α, was relatively low, being the maximum increment obtained in the order of 6%.
- The maximum variation in yield curvature for the full range of steel reinforcement ratios for sections of the structural elements considered was in the order of 30%. However for steel reinforcement ratios ranging from 60% to 95% of ρ_{max}, the variations in yield curvature were less than 10%, which means that Priestley's equation gives good results for sections with relatively high amounts of steel, as it would be expected in seismic design.
- COLUMNS. Yield curvature presents high variations in columns because compression depth is a function of steel reinforcement and axial force, so in order to take into account these variables an extensive study to define ranges of reinforcement and of axial force level must be done.

From de interpretation of the results of the parametric study presented in this paper the following conclusions may be drawn:

When the strength of a reinforced concrete section is increased by exclusively increasing the reinforcement ratio, besides an increment of stiffness, a change in the yield curvature and, as a consequence of its deformation capacity is found, important fact to be considered in the displacement based design procedures, as this result affects in a clear way the validity of its foundations.

The increments of stiffness and strength with steel reinforcement ratio were approximately proportional and the variation of the ratio of post-yield to initial stiffness, α, was relatively low, being the maximum increment obtained in the order of 8%.

It is possible to consider the assumption of constant yield curvature valid for sections with reinforcement ratios larger than 50% of the maximum permissible. Thus as high steel ratios are generally used in seismic design, to have a better use of the capacity of reinforced concrete; it is possible to formulate design procedures considering yield curvatures as constant. In the case of columns, it is convenient to conclude from the parametric study to find the reliable limits for this assumption, establishing in the procedures the criterion that the yield curvature in a structure be defined by the beam elements.

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