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SENSITIVITY OF NBCC 2005 BASE SHEAR TO THE FUNDAMENTAL PERIOD

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ABSTRACT

The 2005 edition of the National Building Code of Canada (NBCC 2005) provides empirical equations to predict the fundamental lateral period of a building to help structural designers calculate design seismic loads. The code allows designers to use other methods to estimate the period, but it imposes restrictions on the deviation from the value calculated from the code equations. However, these equations are not accurate, and therefore the period of a building once it is constructed may differ significantly from the value used in design. On the one hand, when the code period is used in design, the period can be expected to be more than 75 percent longer in certain cases. Conversely, when the seismic design is based on the maximum period allowed by the code, the period can be expected to be more than 65 percent shorter in certain cases. This uncertainty in predicting the period leads to inaccurate design loads. In certain cases, the design base shear may be overestimated by more than 50 percent (if the code period is used), or underestimated by up to 300 percent (if the maximum allowable period is used).

1. Introduction

The fundamental lateral period of a building plays an important role in its behavior during an earthquake. Therefore, when engineers try to predict the seismic forces that a building is likely to experience during its design life, they must first estimate its fundamental period. However, the fundamental period is difficult to estimate before a building is constructed. It is a function of the building's stiffness and mass, which depend on several variables; most notably the building's material properties, the type of seismic force-resisting system (SFRS), and height. Therefore, the 2005 edition of the National Building Code of Canada (NBCC 2005) provides five different equations to predict the fundamental lateral period of buildings, which depend on these three variables. These equations were calibrated based on data from buildings whose periods were measured during the 1971 San Fernando earthquake (Goel and Chopra 1997, 1998). However, all of the equations fit the data rather poorly (Saatcioglu and Humar 2003; Tremblay 2005).

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Since the seismic loads used to design a building are a function of its fundamental lateral period, it is important for structural engineers to understand the effect that poorly predicting the period has on the prediction of these design loads. To the best of the author's knowledge, the uncertainty in the design seismic forces that results from the poor prediction of the fundamental period has not been rigorously investigated. To address this question, we first studied period data for the more common types of SFRS to obtain a measure of the uncertainty in predicting the fundamental period using the provisions of the NBCC 2005. We identified two scenarios that may regularly arise in practice: in the first, a structural designer may underestimate the period, and thereby overestimate the design seismic base shear (conservative scenario); and in the second, a structural designer may overestimate the period, and thereby underestimate the design seismic base shear (unconservative scenario). For the first scenario, the uncertainty in predicting the period was estimated by comparing the period used in design to the period that may be exceeded in approximately 20 percent of cases, i.e. the 80th percentile period; whereas for the second scenario, the design period was compared to the 20th percentile period. Considering this uncertainty, we performed an "error propagation" analysis to estimate the relative error in design seismic base shear ($\Delta V/V$) that results from poorly predicting the fundamental period. We considered various soil conditions and SFRS, as well as different cities across Canada. We found that, in certain cases, the relative error in base shear can be as large as 50 percent for the first scenario (conservative), and 300 percent for the second (unconservative), which shows that poorly predicting the fundamental period has an important effect on the reliability of design seismic loads.

2. Overview of NBCC 2005 Seismic Provisions

In the NBCC 2005, the design ground motion (DGM) is specified as a set of spectral acceleration values, having a 2 percent in 50 year probability of exceedance (Adams and Atkinson 2003), and is referred to as a uniform hazard spectrum (UHS). For design purposes, the code suggests to evaluate the response of a building to the DGM using dynamic analysis methods. However, in certain cases, an equivalent static force procedure (ESFP), based primarily on the fundamental sway mode response, may be employed. Note that even when dynamic analysis is used, the design base shear is not permitted to be taken lower than 80 percent of the static base shear specified in the NBCC (NRC/IRC 2005).

In the ESFP of the NBCC 2005, the design seismic base shear, V, is calculated as

$$V = \frac{I_E\, W}{R_d\, R_o} \cdot \left[S(T_a) \cdot M_v \right], \qquad (1)$$

where I_E is the importance factor for earthquake loads, W is the weight of the building, R_d and R_o are the ductility- and overstrength- related force modification factors, $S(T_a)$ is the design spectral acceleration, and M_v is the higher mode factor (NRC/IRC 2005).

The design spectral acceleration, $S(T_a)$, is calculated at the fundamental period of the structure and is meant to account for the site-specific seismic hazard. It is calculated using the UHS spectral acceleration values for the appropriate location. To account for amplification effects on soft soils, the spectral accelerations at specific period values are first multiplied by site factors, which vary with seismic hazard, as well as with the class of soil at the site (Finn and Wightman 2003). Soil classes vary from A (hard rock) to E (soft soils), with an additional class for problematic soils, such as liquefiable soils (site class F).

As mentioned previously, the ESFP is based on first-mode response. However, in long-period structures, higher modes play a significant role in the structural response during seismic ground motions, which tends to increase the base shear (Humar and Mahgoub 2003). The NBCC 2005 includes a higher mode factor, which depends on seismicity and the type of SFRS, to capture this phenomenon. The remaining terms of Eq. 1 are not relevant to this study, and will not be discussed further here. The reader may refer to Heidebrecht (2003) for more details.

3. Uncertainty in Predicting the Fundamental Period

The fundamental period plays an important role in the calculation of design seismic loads since it determines the value of the design spectral acceleration. To help structural designers estimate the fundamental lateral period of a structure before it is built, the NBCC 2005 provides the following five equations for different SFRS:

$T = 0.085\, h^{3/4}$ for steel moment-resisting frames (steel MRF), (2)

$T = 0.075\, h^{3/4}$ for reinforced concrete moment-resisting frames (RC MRF), (3)

$T = 0.1\, N$ for other moment-resisting frames (MRF), (4)

$T = 0.025\, h$ for braced frames (BF), and (5)

$T = 0.05\, h^{3/4}$ for shear walls (SW) and other structures (Others), (6)

where T, h, and N represent the fundamental period (in s), building height (in m), and number of storeys, respectively. The code permits designers to use other accepted methods (for example, eigenvalue analysis of building models) to estimate the fundamental period; however it limits the deviation from the value calculated with the appropriate code equation. When dynamic analysis is used, no restriction is imposed on the value of the fundamental period per se; but the base shear to be used in design cannot be less than 80 percent of the base shear calculated from the ESFP. So, in either analysis procedure, seismic design depends greatly on the fundamental period calculated from Eqs. 2 to 6.

Equations 2, 3, and 6 were derived in the 1970s from regression analyses using data from 40 instrumented buildings in California (17 Steel MRF, 14 RC MRF, and 9 RC SW), whose periods were measured during the 1971 San Fernando earthquake (Applied Technology Council 1978; Saatcioglu and Humar 2003). Goel and Chopra (1997, 1998) evaluated the accuracy of these equations, considering period data from more recent earthquakes. For RC SW structures, they suggested that an adequate fit could not be achieved using building height alone. They proposed a more complex equation which includes the dimensions of shear walls. For Steel and RC MRF, they suggested that the period could be estimated more accurately by increasing the exponent in Eqs. 2 and 3. However, these recommendations were not adopted in the NBCC 2005.

Equation 5 was derived from an analytical study of the periods of steel braced frames. A number of analytical models of braced frame structures were generated, and their periods were estimated using the Rayleigh method. Regression analyses were then performed, and Eq. 5 was found to provide a conservative estimate of the fundamental period for these types of structures (Tremblay 2005). Lamarche et al. (2009) later evaluated this equation using experimental data from ambient vibration studies on 22 single-storey braced steel frame buildings in Canada. They found that Eq. 5 was inaccurate for single-storey buildings and suggested an equation having an additional parameter: the maximum distance between two consecutive lateral load-resisting braced bays. However, this equation is not likely to apply to taller steel braced frames.

Finally, Eq. 4 is disappearing from building codes as more research is performed on the more common types of lateral load-resisting systems, so it will not be discussed further here.

Figure 1 shows the period data from Goel and Chopra (1997, 1998) and Lamarche et al. (2009) plotted against building height. Each of these graphs also includes a curve corresponding to the appropriate code period equation, T_{code} (solid black line), and the maximum period value allowed by the code for that type of SFRS, T_{max} (dash-dot blue line). The two remaining curves represent the upper limit, T_u (dashed black line), and lower limit, T_l (dotted blue line); these equations are of the same form as the code equations, but the coefficients were adjusted such that 20 percent of the data points lie above (T_u) or below (T_l) the curve. Thus T_u represents the 80th percentile period value, for which 80 percent of the data points fall below the curve; and T_l represents the 20th percentile value, for which 20 percent of the data points fall below the curve. Note that for steel BF, the lower limit period equation coincides with the code equation. To help identify T_l on the graph, X markers were added in Fig. 1d.

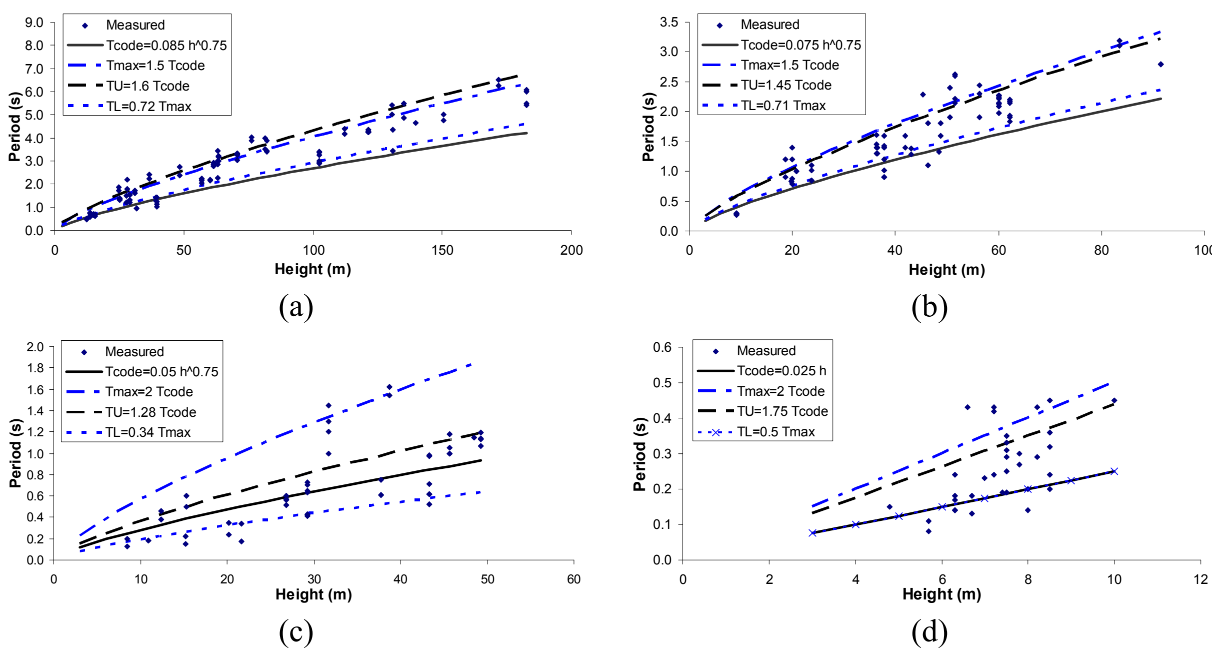

Figure 1. Measured periods and period equations for (a) Steel MRF; (b) RC MRF; (c) RC SW; and (d) Steel BF

Figure 1 clearly indicates that the measured fundamental period of structures may be significantly different from the period calculated from the NBCC equations. Recognizing that it is possible to either overestimate or underestimate the period, we targeted two design scenarios to evaluate the propagation of the error in predicting the period.

Scenario 1: Underestimating the Fundamental Period

In the first scenario, a structural designer may choose to use the code equation (T_{code}) directly to estimate the fundamental period of a building. However, the actual period of the building may be significantly longer, as evident in Fig. 1. In fact, in roughly 20 percent of cases, the actual period may exceed T_u. To quantify the potential period underestimation associated with this design scenario, we compared the upper limit period (T_u) equation to the code equation (T_{code}) for each SFRS. By taking the ratio of the coefficients in these equations, we obtained a measure of the period underestimation that should be exceeded in approximately 20 percent of cases where the code period is used in design. This shall be referred to as the *20th percentile period underestimation*. The value of 20 percent is somewhat arbitrary, but it provides an objective basis to compare the potential errors in predicting the period for the different SFRS. Scenario 1 is illustrated by the black (solid and dashed) lines in Fig. 1. In approximately 20 percent of such cases, the period would be underestimated by more than 60 percent for steel MRF, 45 percent for RC MRF, 30 percent for RC SW, and 75 percent for BF. This would lead to overestimation of the design seismic base shear, a conservative error, since the spectral acceleration tends to decrease with increasing period. However, this period underestimation could lead to underestimation (unconservative) of storey drifts, since stiffness would likely be overestimated.

Scenario 2: Overestimating the Fundamental Period

In the second scenario, a structural designer may use a numerical model (or another acceptable method) to estimate a building's fundamental lateral period. There is a concern that these structural models often overestimate the flexibility of the system, and consequently overestimate the fundamental period (Heidebrecht 2003). In many cases, the estimated period may be larger than the maximum period allowed by the building code (T_{max}), in which case T_{max} would be used to compute design seismic loads. However, T_{max} may be significantly longer than the actual period of the building. Similarly to Scenario 1, to quantify the potential period overestimation, we divided the coefficient of the maximum allowable period equation (T_{max}) by the coefficient of the lower limit period equation (T_l), for each SFRS; this ratio shall be referred to as the *20th percentile period overestimation*. This scenario is illustrated by the blue (dash-dot and dotted) lines in Fig. 1. In this case we cannot claim that the estimation error will exceed the 20th percentile value in 20 percent of cases where T_{max} is used in design, since proper structural modeling should limit the error associated with predicting the period. Considering the potential overestimation of flexibility in structural models, which is impossible to validate with physical data at the design stage, the estimation error may exceed the *20th percentile period overestimation* in many cases. Fig. 1 shows that, when T_{max} is used in design, the period may be overestimated by over 30 percent for steel and RC MRF, 66 percent for RC SW, and 50 percent for BF. This would generally lead to underestimation of design base shear, and potentially to unsafe building designs.

Table 1 summarizes the 20th percentile period errors for Scenarios 1 (underestimation of *T*) and 2 (overestimation of *T*) for the different types of seismic force resisting systems. Let us now examine the uncertainty associated with predicting the design base shear, considering the uncertainty associated with predicting the period.

Table 1. 20th percentile period errors for Scenarios 1 and 2

SFRS	Scenario 1		Scenario 2	
	Underestimation	c*	Overestimation	c*
Steel MRF	60%	1.60	30%	0.70
RC MRF	45%	1.45	30%	0.70
RC SW	30%	1.30	66%	0.34
Steel BF	75%	1.75	50%	0.50

* c is the ratio of the 20th percentile period to the design period

4. Error Propagation Analysis

To evaluate the uncertainty in design seismic base shear that results from poorly predicting the fundamental period, we performed an error propagation analysis. Making use of the 20th percentile period errors for the different types of SFRS, we estimated the associated errors in predicting design base shear. From Eq. 1, the relative error in base shear can be expressed as:

$$\frac{\Delta V}{V} = \frac{\left[S(T)\cdot M_v\right]_{T=T_a} - \left[S(T)\cdot M_v\right]_{T=c\cdot T_a}}{\left[S(T)\cdot M_v\right]_{T=T_a}}, \tag{7}$$

where T_a is the fundamental lateral period used in design, c is a constant that reflects the 20th percentile period errors for Scenarios 1 and 2 (see Table 1), and the relative error in base shear, $\Delta V/V$, is expressed as a fraction of the design base shear V. Note that the "true" value (denominator in Eq. 7) is calculated at the design fundamental period. Also, for simplicity, the value of c for underestimation of T (Scenario 1) was assumed to be 1.45 for both steel and reinforced concrete MRF.

It is clear from Eq. 7 that the relative error in base shear is a function of the modified design spectrum, which itself depends on seismicity, soil conditions, and the type of SFRS. Therefore we considered 23 cities across Canada: 13 in Eastern Canada and 10 in Western Canada. These are the same cities that Adams and Atkinson (2003) selected to represent the different levels of seismicity across the country. However, to ease the presentation of results, we considered a subset of these 23 cities. We first selected 3 cities in each region (East and West), which approximately capture the minimum, average, and maximum relative errors in base shear: Saint-John's, Ottawa, and Trois-Rivières in the East; and Prince Rupert, Tofino, and Victoria, in the West. We also included Montréal and Vancouver, as these two metropolitan areas account for nearly 2/3 of the total urban seismic risk in Canada (Adams et al. 2002). Further, although we considered different soil classes; results are only presented for class C soil, as these represent approximately average values.

5. Results

To study the potential error in predicting base shear using current standards, we plotted the relative error in base shear against the design fundamental period (Figures 2 to 4). For each type of SFRS considered, two plots were generated (a and b): the first considering underestimation of the fundamental period (Scenario 1); and the second considering overestimation (Scenario 2), for the eight selected cities.

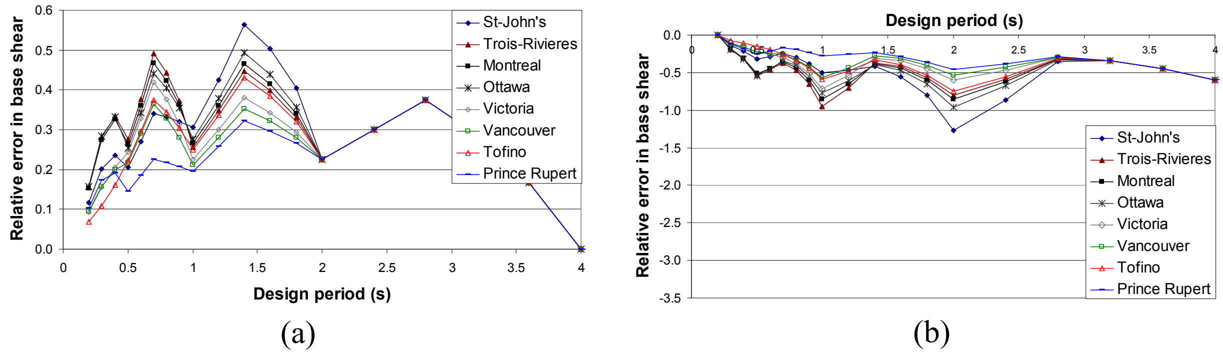

Figure 2. Relative error in base shear for MRF, for (a) Scenario 1 – 45% underestimation of period; (b) Scenario 2 – 30% overestimation of period

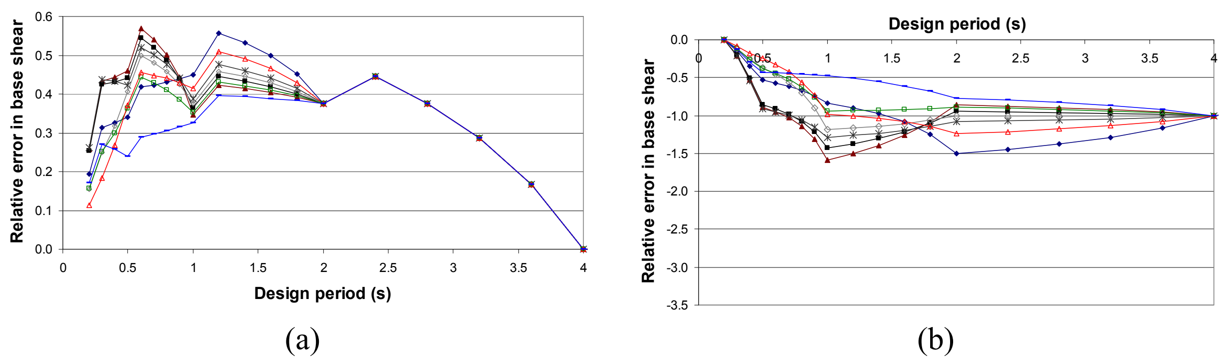

Figure 3. Relative error in base shear for steel BF, for (a) Scenario 1 – 75% underestimation of period; (b) Scenario 2 – 50% overestimation of period

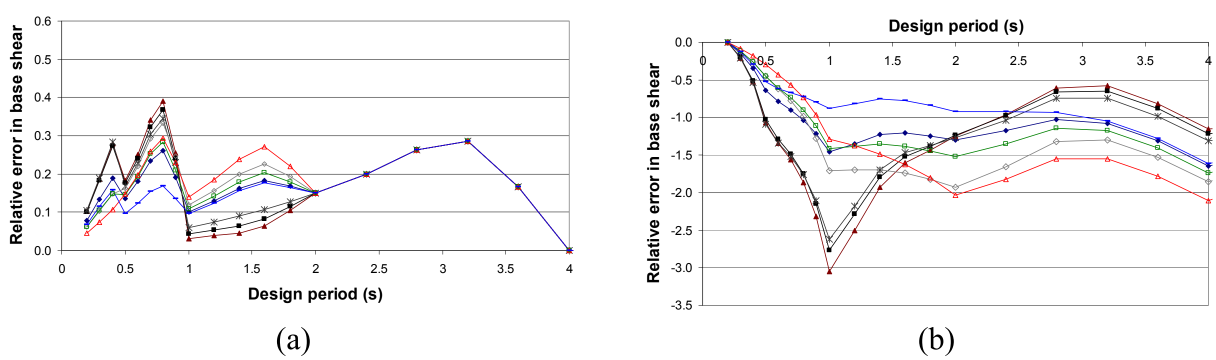

Figure 4. Relative error in base shear for RC SW, for (a) Scenario 1 – 30% underestimation of period; (b) Scenario 2 – 66% overestimation of period

For Scenario 1, the relative error values are positive since the base shear is overestimated in design. The errors are identical beyond a period of 2 s, considering each type of SFRS separately. Since the period is underestimated in Scenario 1, both the design period and the upper limit period lie beyond 2 s. Though the pseudo-acceleration design spectra for different cities are different, their variation is identical beyond a period of 2 s. That is, the spectral acceleration at 4 s is exactly half the value at 2 s, and remains constant beyond 4 s. Therefore, when we normalize by the modified spectral acceleration at the design period to find the relative error in base shear (Eq. 7), the values are identical for all cities. Further, the relative error in base shear is zero for values of the design period greater than 4 s. This is obvious since both the design period and the upper limit period lie in the flat tail of the spectrum beyond 4 s. It should be noted that few buildings in Canada have a fundamental period as long as 4 s.

Considering the first portion of the graphs between 0.2 s and 2.0 s, the relative error varies for different cities though the shapes of the different curves are similar. The values also vary widely according to the type of SFRS, mainly due to differences in the errors associated with the prediction of the fundamental period. If the code period is used to design an MRF building in St-John's on class C soil, the design base shear may be overestimated by as much as 56 percent. In contrast, if the same building is to be located in Prince Rupert, the design base shear may only be overestimated by 40 percent. Similarly, for steel BF and RC SW, the base shear may be overestimated by as much as 56 percent and 40 percent, respectively. This would lead to inefficient building designs in cases where the seismic loads govern the design of the lateral load-resisting system. Given that the errors are not overly large, and that the seismic design loads are conservative, many structural designers may deem these errors acceptable, especially in cases where wind loads govern the design of the lateral load resisting system. Nevertheless, it is clear that improving the estimation of the period would improve the reliability of the design seismic loads.

For Scenario 2, the relative error values are negative since the base shear is underestimated in design. The error in base shear for all locations and all types of SFRS is identically zero below a period of 0.2 s, indicating that overestimating the period in this range has no influence on the calculated design base shear. This is evident since Scenario 2 involves overestimation of the period, and thus both the design period and the lower limit period lie in the initial flat portion of the response spectrum. Similarly to Scenario 1, considering each SFRS independently, the relative error is identical for all cities beyond a threshold period value. For each type of SFRS, this threshold value depends on the magnitude of the overestimation of the period. It corresponds to the design period for which the lower limit period is equal to 2 s. For steel BF, for example, the threshold value is 4 s, for which 50 percent overestimation corresponds to a lower limit period of 2 s. As for Scenario 1, when both the design and lower limit periods are greater than 2 s, the relative errors in base shear are identical for all cities.

Between 0.2 s and the threshold period value, the relative errors vary according to location, with the peak value typically occurring at a design period of 1 or 2 s. Note that the errors are generally smallest for MRF; while they are largest for RC SW. This is mainly due to the relative accuracy of the different equations used to predict the period. If the maximum allowable period is used to design an MRF building in St-John's on class C soil, the design base shear may be underestimated by as much as 128 percent. And, if the same building were to be constructed in Prince Rupert, the underestimation of the design base shear might be reduced to 46 percent. In the extreme case, we may underestimate the design base shear by up to 300 percent for RC SW buildings in Trois-Rivières. This means that the design base shear calculated with the actual building period could be 4 times larger than that calculated using the maximum allowable period. This could have serious implications for the seismic level of safety of these buildings. Though proper structural modeling should limit the possibility of incorrectly using the maximum allowable period in design, there is a concern that these models often overestimate flexibility. For this reason, structural designers should be very vigilant when using the maximum allowable period in design.

In general, for both scenarios, the relative errors in base shear tend to be larger in Eastern Canada than in Western Canada, suggesting that cities in the East are more sensitive to the accurate prediction of the fundamental period. Note also that these relative errors are not constant throughout the spectrum, which indicates that poor prediction of the fundamental period has a more pronounced effect on the prediction of the design base shear for certain period values. Finally, it is important to note that the relative errors may be larger on other types of soils.

6. Conclusion

When estimating design seismic loads, every step of the process is riddled with uncertainty, from predicting the likely ground motions to evaluating the amplification of seismic waves on soft soils. In this study, only the uncertainty associated with the prediction of the fundamental lateral period of a building, and its effect on the accuracy of the design seismic loads, were investigated. The goal was not to rigorously assess the accuracy of the design seismic base shear, but rather to show that poorly predicting the fundamental period of a building has a significant impact on its seismic design. The results show that current formulae in NBCC 2005 lead to inaccurate estimates of the fundamental period, particularly for RC SW and steel BF structures, and this uncertainty leads to inaccurate design seismic loads. For example, when structural designers use the maximum allowable period to calculate design seismic loads, they may underestimate these by as much 300 percent, in the worst case scenario. This suggests that structural engineers should be very careful when using the maximum allowable period in design. The interested reader should refer to Saatcioglu and Humar (2003) for more details.

This study dealt solely with design seismic forces; while the satisfactory performance of a building during an earthquake depends also on serviceability and functionality aspects related to inter-story drift demands. Also, in many cases, the design of the lateral load-resisting system may be governed by wind effects rather than seismic loads. In such instances, as long as minimum seismic detailing is provided, the error made in predicting the design seismic loads would not affect the final structural design.

Despite the limitations mentioned above, this study shows that the equations provided in the NBCC 2005 are not accurate predictors of the fundamental lateral period of structures, and that this poor prediction leads to inaccurate design seismic loads. Therefore, these equations should be improved to ensure that buildings are designed to resist the loads induced by earthquakes in a safe and efficient manner. This is particularly important for RC SW and steel BF structures, for which the period equations are the least accurate. Lamarche et al. (2009)

suggested improved equations for single-storey steel BF structures, but these are not likely to be applicable to taller BF structures. For RC SW buildings, Goel and Chopra (1998) suggested improved equations that account for the dimensions of shear walls. However, in the interest of finding simpler equations based on global building geometry, a period database is currently being compiled by performing ambient vibration tests in RC buildings in Montréal. Preliminary results suggest that the period equations can be improved by incorporating the plan dimensions (Gilles and McClure 2008).

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