



ENERGY-BASED DESIGN OF STEEL BUILDING FRAMEWORKS USING NONLINEAR TIME HISTORY ANALYSIS

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ABSTRACT

This research concerns the development of an energy-based design method for structural steel frameworks under seismic loads. The methodology is based on the capacity design approach, which requires a structure to resist severe earthquakes through inelastic response in confined regions. Specifically, the design procedure seeks to minimize structure weight and seismic energy input while it strives to maximize the hysteretic energy dissipation of fuse members under strong ground motions. Nonlinear time-history analysis is adopted to evaluate seismic response. Design constraints concern member-end plastic rotation and structure inter-storey drift. The proposed design method is illustrated for a simple steel moment-resisting frame example.

Introduction

In general, building structures in seismic zones are designed to deform far beyond the elastic limit during intensive ground motions. The capacity design principle, which is widely accepted in seismic design, requires that the inelastic deformations resulting from ground motions be confined to specific regions (or fuse members) to protect non-fuse members from additional loading effects. As earthquake response is a dynamic phenomenon, evaluating inelastic dynamic response is of central importance to proper design. To this end, nonlinear time-history analysis is a tool which can account for both the dynamic effect of earthquake loading and the inelastic behaviour of structural components.

Among various measures of seismic response, the limits of maximum displacement or force do not account for the damaging effects of cyclic seismic loads, which cause cumulative inelastic deformation reversals that can result in low-cycle fatigue failure of structural members. The potential duration-related damage, which is highly related with hysteretic energy dissipation, can be evaluated through employing a seismic energy analysis. The design of the structure concerns the balance of the seismic input energy with the dissipation of energy through cyclic inelastic behaviour and damping. The hysteretic energy dissipation of fuse members is a most important property of structural systems subjected to severe earthquakes.

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Energy-Based Design Method

It is well accepted (Zahrah 1984, Leger 1992) that a structure can survive a severe earthquake if its structural energy absorption capacity is not less than the input seismic energy. During a strong ground shaking, the seismic energy imparted to a building is dissipated through the movements and deformations of structural members in the forms of kinetic energy, damping energy, elastic strain energy and inelastic hysteretic energy.

Seismic Energy Equation

The equation of motion of a multi-degree-of-freedom system is (Chopra 2007),

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + \{F_r(t)\} = -[M]\{r\}\ddot{x}_g(t) \quad (1)$$

where $[M]$ is the global mass matrix; $[C]$ is the global viscous damping matrix; $\{F_r(t)\}$ is the global nonlinear restoring force vector at time t ; $\{\ddot{u}(t)\}$, $\{\dot{u}(t)\}$, $\{u(t)\}$ are the response vectors of acceleration, velocity and displacement respectively; $\{r\}$ is the support influence vector; $\ddot{x}_g(t)$ is the ground acceleration at time t . Equation (1) can be transformed into an energy balance equation as follows (Leger 1992):

$$E_k(t) + E_d(t) + E_a(t) = E_i(t) \quad (2)$$

where $E_k(t)$ is the kinetic energy and $E_d(t)$ is the damping energy, $E_a(t)$ is the absorbed energy comprised of recoverable elastic strain energy (E_s) and irrecoverable hysteretic energy (E_h), and $E_i(t)$ is the relative seismic input energy. Generally speaking, the kinetic energy and elastic strain energy are stored temporarily in the structure, and are dissipated through damping at the end of the earthquake.

Computation of Seismic Energy

Various schemes are available to carry out the computation of seismic energy. In order to utilize available nonlinear time-history analysis software, this paper proposes to compute seismic energies based on the history of internal forces and displacements of structural members. The hysteretic energy dissipation of a structural member $E_{h,k}$ at the end of an earthquake is expressed as the sum of the work of internal member forces and corresponding displacements, i.e.,

$$E_{h,k} = W_{Moment,k} + W_{Axial,k} + W_{Shear,k} \quad (k = 1, 2, \dots, n_e) \quad (3)$$

where $W_{Moment,k}$, $W_{Axial,k}$, $W_{Shear,k}$ are the work done by the bending moment, axial force and shear force for member k , respectively; and n_e is the number of the structural members.

The total hysteretic energy of fuse members E_{h_fuse} is expressed as:

$$E_{h_fuse} = \sum E_{h,l} \quad (l = 1, 2, \dots, n_{fuse}) \quad (4)$$

where $n_{fuse} < n_e$ is the number of fuse members.

Relative seismic input energy at time t is computed using the following discrete expressions,

$$E_{i,m}(t) = E_{i,m}(t - \Delta t) + \left(\frac{1}{2} (F_{t,m}^i + F_{t-\Delta t,m}^i) (u_{t,m} - u_{t-\Delta t,m}) \right) \quad (5)$$

in which

$$F_{t,m}^i = -M_m \cdot \ddot{x}_{g,t} \quad F_{t-\Delta t,m}^i = -M_m \cdot \ddot{x}_{g,t-\Delta t} \quad (6)$$

where: $E_{i,m}(t)$, $F_{t,m}^i$, M_m are the seismic input energy, the inertial force and the mass of the m^{th} degree of freedom, respectively; $\ddot{x}_{g,t}$ is the ground acceleration; Δt is the time-increment for dynamic analysis.

The seismic input energy $E_i(t)$ imparted to the seismic force-resisting structural system is then expressed as,

$$E_i(t) = \sum E_{i,m}(t) \quad , \quad (m = 1, 2, \dots, n_m) \quad (7)$$

where n_m is the number of the degree-of-freedom.

Design Objectives

The design variables are the sizes of the members for the seismic force-resisting structural system, which are to be selected from among commercially available steel sections. Expressed as functions of the sizing variables, this study considers three design objectives concerning: 1) structural cost; 2) seismic energy input; and 3) hysteretic energy dissipation.

Assuming member cost is proportional to its weight, the structural cost objective is expressed by the following *minimization* function,

$$OBJ_1(\mathbf{A}) = \min(\sum \rho \cdot L_k \cdot A_k) \quad (k = 1, 2, \dots, n_e) \quad (8)$$

where: A_k is the sizing design variable for member k ; L_k is the fixed length of member k ; n_e is the number of members; and ρ is the material mass density.

Since the smaller the amount of seismic energy imparted to the seismic force-resisting system, the smaller will be the amount of consequential damage, the seismic energy input objective is expressed by the following *minimization* function,

$$OBJ_2(\mathbf{A}) = \min(\hat{E}_i) \quad (9)$$

where \hat{E}_i is the mean value of the seismic input energies for n_g ground motions (i.e., the average value of the input energy for the ensemble of ground motion records considered by the time-history dynamic analysis).

Since the larger the magnitude of cyclic plastic deformations that fuse members can undergo, the larger will be the amount of energy dissipated for the structure, the hysteretic energy dissipation objective is expressed by the following *maximization* function,

$$OBJ_3(\mathbf{A}) = \max(\hat{\beta}) \quad (10)$$

where $\hat{\beta}$ is the mean value of $\beta_j = \left(\frac{E_{h_fuse}}{E_i}\right)_j$, ($j = 1, 2, \dots, n_g$) the hysteretic energy dissipation ratio of fuse members under n_g ground motions; in which E_{h_fuse} is the hysteretic energy of all fuse members and E_i is the total seismic input energy (It is shown in the following numerical example that this objective function effectively serves to confine plastic deformation to the fuse members of the structure).

Design Constraints

Constraints are imposed on member and structural deformations in order to ensure the integrity of the seismic force resisting structural system. Specifically, a design is deemed feasible if it satisfies the following constraints on member-end plastic rotation and inter-storey drift,

$$\hat{\varphi}_{k,p} \leq \varphi_0 \quad (k = 1, 2, \dots, n_e, p = 1, 2) \quad (11)$$

$$\hat{\delta}_s \leq \delta_0 \quad (s = 1, 2, \dots, n_s) \quad (12)$$

where: $\hat{\varphi}_{k,p}$ is the mean value of member-end plastic rotation for n_g ground motions; p is the number of member-end sections; $\hat{\delta}_s$ is the mean value of the s^{th} inter-storey drift for n_g ground motions; n_s is the number of stories; φ_0, δ_0 are specified allowable values for rotation and drift, respectively.

The frame members are selected from among available *compact* steel sections, which allow for the development of plastic rotation. Side constraints are imposed on the member sizing variables as follows,

$$A_k^l \leq A_k \leq A_k^u \quad (k = 1, 2, \dots, n_e) \quad (13)$$

$$A_k \in \{C_k\} \quad (k = 1, 2, \dots, n_e) \quad (14)$$

where: A_k^l, A_k^u are the lower and upper bound areas of commercial-standard *compact* steel cross-sections (e.g., from AISC 2005, CISC 2006); $\{C_k\}$ is the set of discrete sizes of available cross section areas for member k ; and n_e is the number of structural members.

Numerical Examples

A 3-story 4-bay moment-resisting frame shown in Figure 1 is employed as an example to demonstrate the proposed seismic design method. All four bays are each 30 ft (9.14 m) wide (centerline dimensions) and all three stories are each 13 ft (3.96 m) high. The frame is assumed to have rigid beam-to-column connections, with all column bases fixed at the ground level. All the columns have 50 ksi (345 MPa) steel wide-flange sections, while all the beams have 36 ksi (248 MPa) steel wide-flange sections. Dead and live load are 100 psf (4.79 kPa) and 50 psf (2.39 kPa) for both of the first and second storey, respectively. For the roof has dead load of 85 psf (4.07 kPa) and live load of 20 psf (0.96 kPa). The width of the tributary (floor/roof) area for the frame is 30 ft (9.14 m). The linear dead and live loads for the first and second storey beams are 3.0 k/ft (43.8 kN/m) and 1.5 k/ft (21.8kN/m), respectively, while the roof beams have linear dead loading of 2.6 k/ft (37.2 kN/m) and live loading of 0.6 k/ft (8.8kN/m).

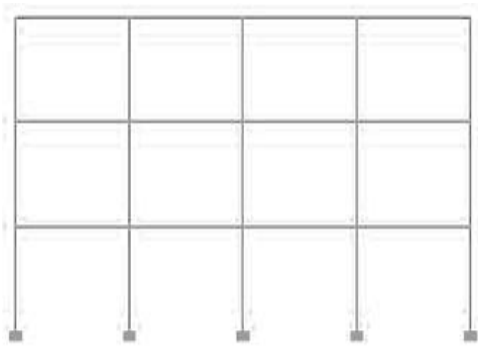


Figure 1. 3-Storey 4-Bay Frame

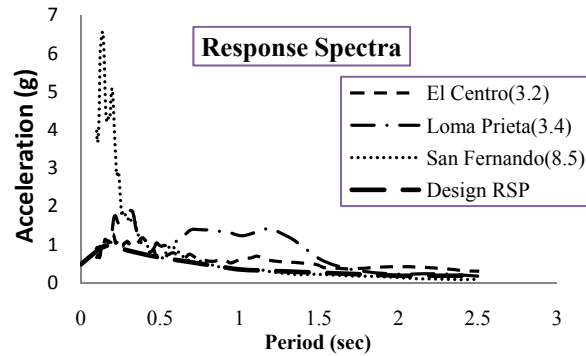


Figure 2. Response Spectra

Table 1. Earthquake data

Record Name	Earthquake Magnitude	PGA (cm/sec ²)
1979 Imperial Valley: El Centro Array #12	6.5	0.143
1989 Loma Prieta: Oakland Tile & Trust	6.9	0.195
1971 San Fernando: Santa Anita Dam 273	6.6	0.212

Nonlinear time history analyses are highly dependent on the characteristics of the individual ground motion records and subtle changes in these records can lead to significant differences with regard to the predicted response of the structure. Thus, a number of ground motions should be considered when adopting a time-history analysis as the evaluation tool for seismic structural design. Seismic Provisions (FEMA 2004) require that at least seven ground motion records should be used when adopting the average values of the structural response for design. With the view to mitigate calculation burden, however, only three ground motions are employed for this example. To be compatible with a response spectrum constructed from design spectral acceleration, each ground motion record is selected such as to be “spectrum-compatible”, i.e., its response spectrum equals or exceeds the target spectrum throughout the period range of interest (NRCC 2006).

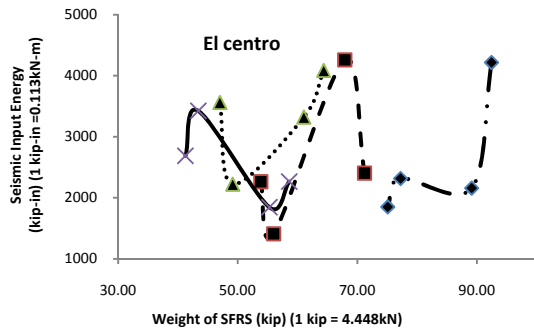
The three earthquake ground motions (PEER 2008) adopted for this example are given in Table 1. It is assumed that the frame is to be constructed at a location in Vancouver, B.C., Canada. The peak ground acceleration specified for the site class C is 0.48g; and the pseudo-acceleration spectral values are $A/g = 0.83, 0.97, 0.96, 0.84, 0.74, 0.66, 0.34, 0.18$ at period $T_1 = 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 1.0, 2.0$ sec, respectively. Figure 2 shows this design spectrum and the spectral values of the three scaled ground motions. To be compatible with the design spectrum range from 0.2 sec to 1.0 sec, scale factors 3.2, 3.4, and 8.5 have been used for the three ground motions.

Structural steel material behaviour is modelled by a bilinear elastoplastic stress-strain relationship with 5% strain hardening. Based on the assumptions of plane sections and uniaxial stress-strain relationships, the moment-curvature relationship of a W-section is obtained through the integration of 256 subdivided segments over the cross-section (each portion of web and flanges has been divided into 128 segments). A geometrically nonlinear beam-column element is employed to account for the second-order effect due to the interaction between axial force and bending deformation. The nonlinear beam-column effect element is modelled by five pre-defined W-sections at the Gauss-Labatto points along the length of element. The stiffness-proportional Rayleigh damping model is employed to construct the damping matrix. The proportionality constants for the initial stiffness matrix are computed from the frequencies of modes 1 and 3 with damping ratios of 0.05 (OpenSees 2008).

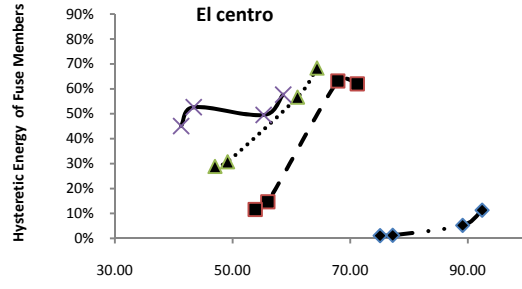
Table 2 Design Results

Design	Beam Section	Column Section	Weight (kip)	Period (sec)	Average δ_{max} (Maximum Inter-storey drift) (in)	Average Energy Values		
						Input Energy (kip-in)	Hysteretic Energy of beams (%)	Hysteretic Energy of columns (%)
G1-1	W24x162	W18x175	92.4	0.366	1.15	4164	24.1%	12.7%
G1-2	W24x162	W18x158	89.1	0.380	1.21	3066	15.2%	20.6%
G1-3	W24x162	W18x97	77.2	0.459	1.49	3704	4.1%	51.6%
G1-4	W24x162	W18x86	75.1	0.482	1.43	3527	3.7%	52.6%
G2-1	W24x103	W18x175	71.2	0.414	1.35	3614	42.7%	2.5%
G2-2	W24x103	W18x158	67.9	0.427	1.25	3486	33.7%	2.6%
G2-3	W24x103	W18x97	56.0	0.502	1.49	3670	16.0%	29.5%
G2-4	W24x103	W18x86	53.9	0.524	1.65	4531	12.5%	50.2%
G3-1	W24x84	W18x175	64.4	0.439	1.25	3570	44.7%	0.3%
G3-2	W24x84	W18x158	61.1	0.453	1.37	3998	46.9%	1.4%
G3-3	W24x84	W18x97	49.2	0.526	1.57	4310	30.8%	27.0%
G3-4	W24x84	W18x86	47.0	0.547	1.90	5537	27.3%	42.7%
G4-1	W24x68	W18x175	58.6	0.472	1.41	3660	50.7%	0.1%
G4-2	W24x68	W18x158	55.3	0.485	1.34	3645	46.8%	0.2%
G4-3	W24x68	W18x97	43.4	0.558	1.73	5315	46.6%	18.2%
G4-4	W24x68	W18x86	41.3	0.578	1.98	6147	39.3%	23.7%

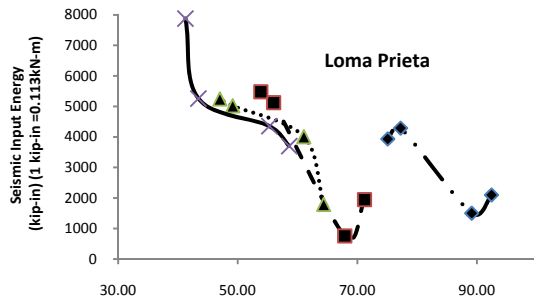
1 kip = 4.448 kN, 1 kip-in = 0.113 kN-m



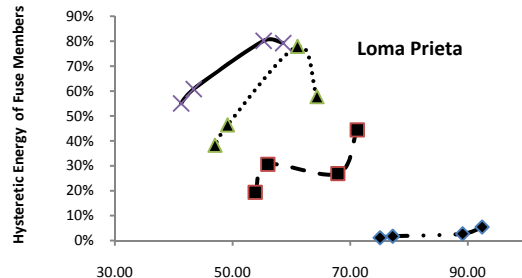
(a) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



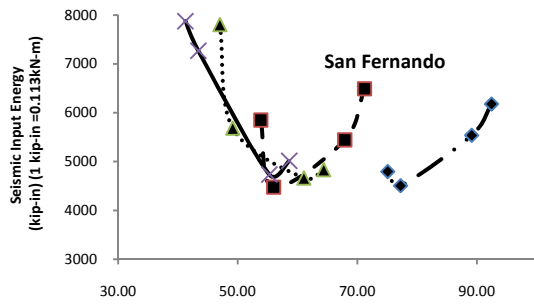
(b) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



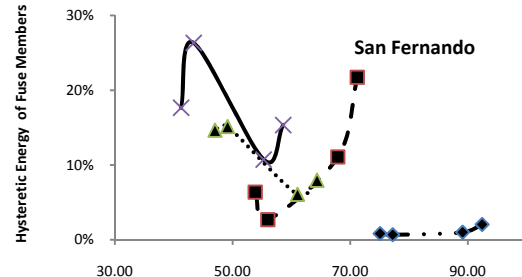
(c) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



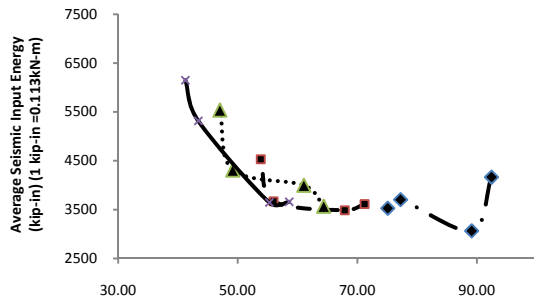
(d) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



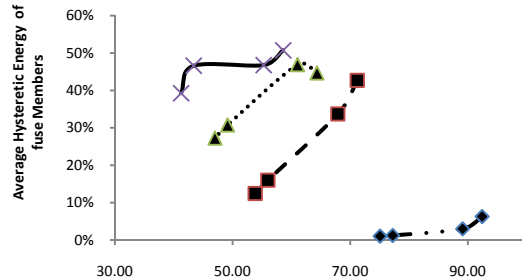
(e) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



(f) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



(g) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68



(h) G1:W24x162 G2:W24x103
G3:W24x84 G4:W24x68

Figure 3 Seismic Energy versus Weight

For this design example, the column sections are selected only from W18 sections and beams from W24 sections. Moreover, for any one design, all the columns are assumed to have the same cross-section and are linked together in a non-fuse member group, while all the beams are assumed to have the same cross-section and are linked in a fuse member group (i.e., each design involves only two design variables). As shown in Table 2, a selection of 16 trial designs is assessed for the proposed methodology. The 16 designs are divided into four groups where, for each group, the beam sections are identical while the column sections are different (e.g., Group G1 corresponds to the designs with W24x162 beam cross-section).

It is assumed that the plastic rotation capacity of the steel member sections is 0.015 radians. The allowable inter-storey drift is taken to be 2.5% of the storey height. Based on the results of nonlinear time history analysis of all 16 designs, the maximum plastic hinge rotations of the structural members are all less than 0.015 radians, while all of the maximum inter-storey drifts are less than $0.025 \times 13 \times 12 = 3.9$ in. (see Table 2). Therefore, all 16 designs are feasible for the given ground motions.

Among the 16 designs in Table 2, the design G4-4 best satisfies the cost objective because it is the minimum-weight design of the frame. However, from the viewpoint of the two energy objectives, this design is not acceptable because it incurs maximum seismic energy input and dissipates hysteretic energy through inelastic deformation of both beams and columns. The design G1-2 best satisfies the input energy objective because it incurs minimum seismic energy input, but it also unacceptable because it has the second-heaviest weight and dissipates hysteretic energy through inelastic deformation of both beams and columns. The design G4-1 is a strong-column weak-beam design that best satisfies the dissipation energy objective because it dissipates the largest percentage of hysteretic energy through plastic deformation of fuse members, while the columns essentially remain elastic. Overall, the design G4-2 is perhaps the ‘best’ design because it has lower weight than designs G1-2 and G4-1, incurs smaller seismic energy input than designs G4-1 and G4-4, and dissipates hysteretic energy similarly to design G4-1.

A note of caution is in order concerning the conclusions made in the previous paragraph, which are based on average response values for a suite of three different ground motions. Figures 3(a)-(f) present the results of nonlinear time-history analyses of all 16 designs (comprising the four design groups G1, G2, G3 and G4) for the three individual ground motions, while Figures 3(g)-(h) present the corresponding averaged results. It is evident that the seismic energy input and hysteretic energy dissipation for a given design are quite sensitivity to the nature of the ground motion. For example, from among the 16 designs in Table 2, design G2-2 (beams W24x103 / columns W18x158) incurs the maximum seismic energy input of 4258 kip-in (481 kN-m) when subjected to the El Centro ground motion record (Figure 3a), while it incurs the minimum seismic energy input of 756 kip-in (85 kN-m) for the Loma Prieta ground motion record (Figure 3c). For design G3-1 (beams W24x84 / columns W18x175), about 68% of the seismic input energy is dissipated as hysteretic energy by the fuse-beams for the El Centro ground motion record (Figure 3b), but only 8% is dissipated by the beams for the San Fernando ground motion record (Figure 3f); in both cases, no hysteretic energy is dissipated by the columns.

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