



THREE DIMENSIONAL NUMERICAL MODEL FOR NONLINEAR EARTHQUAKE RESPONSE OF SLOPES

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ABSTRACT

Traditional methods of seismic slope stability assessment are based on a quasi-static approach. In moderate to severe seismic regions this approach usually indicates slope failure. In reality, however, the slope experiences some permanent displacements without failure. A realistic three-dimensional simulation of the nonlinear material subjected to irregular earthquake loading is very complex and requires reliable and advanced computational tools. Such tools need to have capabilities for 3D analyses, dynamic loading, fully coupled solid-fluid interaction, handling the boundary conditions in a realistic way, and most importantly application of advanced plasticity models. In the present developments the constitutive behavior of the soil matrix is characterized using a Simple Anisotropic CLAY plasticity model with Destructuration (SANICLAY-D). The model is efficiently integrated in FLAC3D program and provides an advanced numerical tool with a reasonable level of sophistication for realistic modeling of clays in practical geotechnical earthquake engineering problems. An illustrative example describing the earthquake behavior of saturated clayey slope using the SANICLAY-D model is presented and discussed.

Introduction

Stability evaluation of slopes under earthquake loading is one of the most challenging issues in geohazard studies. Traditional methods of seismic slope stability assessment are based on a quasi-static approach. Earthquakes generate vibrations and mass inertia forces, which at times cause large shear stresses. In moderate to severe seismic regions the quasi-static approach usually indicates failure in slopes that have marginal static Factor of Safety (FoS). The duration of earthquake load, however, is short and in most such cases the slope experiences some permanent displacements without total failure. This means that the focus of the seismic slope stability assessment must be on estimating the earthquake-induced deformations, rather than

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computing a pseudo-static Factor of Safety as commonly done by many geotechnical engineers. A more realistic approach is therefore to allow for soil nonlinearity and set limits for acceptable displacements. Stress-deformation analyses with numerical tools are becoming more common because they can provide insight into the nonlinear behavior of the material.

Numerical methods (mainly finite difference and finite element) have been applied to several case studies. Pestana and Nadim (2000) introduced a finite element program for the solution of the one-dimensional wave propagation problem in the case of an infinite slope. Havenith et al. (2002, 2003), Bourdeau et al. (2004), Crosta et al. (2005) and Chugh and Stark (2006) presented two-dimensional models with finite difference method of cases from Kyrgyzstan, El Salvador, and California, using simple constitutive models such as Mohr Coulomb model. Bourdeau et al. (2004) noticed that two-dimensional numerical models give smaller failed areas than pseudo-static and static slope stability analyses. Chugh & Stark (2006) found similar displacements in their results as obtained with the Newmark displacement-based method. Loukidis et al. (2003) compared the finite element method in a linear approach with pseudo-static evaluations and obtained similar results for the selected cases. Azizian and Popescu (2006) used two- and three-dimensional non-linear finite element models for earthquake slope stability assessments for quantitative study on the limits of applicability of the 2D, plane strain analysis assumptions. Sigaran-Loria (2007) conducted detailed sensitivity analyses on the effects of earthquake frequencies and amplitudes in different slopes using a nonlinear finite element and Mohr-Coulomb model. Nadim et al. (2007) presented the main mechanisms contributing to instability of clay slopes under seismic loading. Through one-dimensional analyses they showed that an appropriate approach for assessing seismic stability of clay slopes is to focus on estimating the earthquake-induced shear strains.

In the present development, the nonlinear finite difference code FLAC3D (Itasca Consulting Group, Inc., 2006) is selected as the main computational platform. FLAC3D (Fast Lagrangian Analysis of Continua in three Dimensions) is a dynamic code that uses an explicit time integration scheme and large strain formulation, and is well suited for calculating dynamic stability problems. Constitutive behavior of the soil matrix is characterized using a Simple ANIsotropic CLAY plasticity model with Destructuration (SANICLAY-D). The formulation of this model is based on the proposed framework by Dafalias (1986) and its further developments by Dafalias et al. (2006) for rotational hardening and Taiebat et al. (2009a) for a destructuration mechanism. The employed constitutive model is used in numerical simulation of the seismic response of a generic slope in order to investigate its performance and features. Despite the fact that the implementation is within a general 3D framework, for simplicity of presentation a 2D section of the slope has been simulated under a sinusoidal-type acceleration time history. The importance of anisotropy and destructuration to the mechanism of accumulation of displacements in clay slopes is illustrated. The objective here is to illustrate the capabilities of the developed tool using the rigorous yet practical constitutive model that is introduced in this numerical framework. It is not intended to reproduce some measured response of a certain case history here. The resulting framework presents an advanced tool for simulation of fully coupled nonlinear dynamic behavior of soil in engineering problems and has a wide range of applications in seismic analysis of infrastructures and lifelines and other civil engineering constructions such as in design of dams.

Constitutive Model

An accurate estimation of the soil strength is crucial in the assessment of slope stability. The shear strength is not a unique property of soil, but is affected by a number of factors, each with different effects and variable influence. Advanced geotechnical design on soft clays has often been based on using isotropic and unstructured elastoplastic soil models, such as the modified cam clay (MCC) model. Natural soft clays, however, almost always have a significant degree of anisotropy and in several cases a natural structure formed during deposition and subsequent one-dimensional (oedometric) consolidation, and the development of chemical bonds between particles due to various reasons. The effects of anisotropy and destructuration on soil strength may be critical for design. The term destructuration refers to the progressive damage to bonding during plastic straining, and this is the sense in which it is used here. Destructuration manifests itself as a sudden post-yield increase in compressibility, or a post-peak decrease in strength under shearing. Damage to the bonding can be caused by both plastic volumetric and plastic shear strains, both of which involve slippage at inter-particle contacts and consequent breakage of bonds. The laboratory tests by Nadim et al. (1996) and Nadim and Kalsnes (1997) showed that the cyclic shear strains induced by the earthquake tend to reduce the shear strength in sensitive clays. If the earthquake-induced cyclic shear strains are large, a slope could undergo further displacements after the earthquake and experience a significant reduction of static shear strength. Neglecting the effect of anisotropy, structure and their evolution from soil behavior may lead to incorrect predictions of soil response under loading (see, e.g. Leroueil et al., 1979 and Zdravkovic et al., 2002).

Anisotropy can be accounted for by rotational hardening, which implies a rotation of the yield and plastic potential surfaces. Dafalias (1986) proposed what can be considered to be the simplest possible energetic extension of the Modified Cam Clay (MMC) model from isotropic to anisotropic response, introducing in the rate of plastic work expression a contribution coupling the volumetric and deviatoric plastic strain rates. The resulting plastic potential surface in the triaxial p - q stress space, which for associative plasticity serves also as a yield surface, is a rotated and distorted ellipse. The amount of rotation and distortion portrays the extent of anisotropy, and is controlled by an evolving variable α , which is scalar-valued in triaxial and tensor-valued in multiaxial stress space. Based on the work of Dafalias (1986), Dafalias et al. (2006) have proposed the SANICLAY model in which a non-associated flow rule allows the simulation of softening response under undrained compression following oedometric consolidation. More recently Taiebat et al. (2009a) presented the destructuration mechanism in a generic format appropriate for various forms of clay constitutive models. They also incorporated the developed destructuration mechanism into a slightly modified version of the SANICLAY model and used it to simulate the response of various loading paths of structured clays.

The employed formulation of the model in the present study takes advantage of the simple framework of MCC, and with perhaps the simplest possible approach adds the very important features of anisotropy and destructuration to the MCC model. Each one of these important constitutive features can be de-activated, if so desired by the user, simply by selecting appropriate values for certain model constants. In this way the developed model can be simplified back to the MCC model. The present formulation does not include some of the constitutive features that existed in earlier works of Taiebat et al. (2009a), such as a non-

associated flow rule and a frictional destructuration mechanism, trading simplicity for some accuracy in simulations. The resulting model is simple, convenient and rational yet significantly improves the MCC model in describing some essential features of response in natural clays. It is aimed to become a tool for solution of boundary value problems encountered in geotechnical engineering. In addition to the regular parameters of MCC model, the present form of SANICLAY-D has one constant for characterization of initial anisotropy (Lode angle dependency), two constant for anisotropy (rotational hardening), and one constant for isotropic destructuration. Similar to MCC the model has a variable size for the yield surface (p_0), and in addition has a tensor variable α for the degree of orientation of the yield surface (anisotropy), and a scalar variable $S_i \geq 1$ for the structuration factor ($S_i=1$ indicates no internal bonding). Details of the model formulation can be found in Taiebat et al. (2009b).

Model Implementation in FLAC3D

The SANICLAY-D model in its simple form as it was explained in the previous section, has been numerically implemented in the three-dimensional explicit finite difference program FLAC3D (Itasca Consulting Group, Inc., 2006) via its UDM option. The constitutive model is written in C++ and compiled as a DLL file (Dynamic Link Library) that can be loaded whenever it is needed. Accuracy of the numerical implementation of a constitutive model in a numerical framework is tied to the employed integration scheme. Various numerical techniques - explicit, refined explicit, and implicit - have already been proposed and extensively discussed in the literature. Implicit integration is the most accurate approach, however it could be computationally complex and run-time extensive in particular for more advanced constitutive models. The choice of the more efficient algorithm depends on the constitutive law and the numerical code where it will be implemented. Because of the explicit nature of the global solution algorithm used in FLAC3D, using implicit algorithms for constitutive integration should preferably be avoided, or else a simple analysis may become computationally very expensive. In fact, very small increments must be used anyway because this is a prerequisite for computational stability of the global solution. Therefore, the stress update algorithm should have a computational effort as low as possible. In the current work, an explicit integration scheme with a drift correction method and an optional substepping technique has been adopted for the model implementation. Details of the numerical implementation of the constitutive model in FLAC3D are presented in Taiebat et al. (2009b).

Verification and Validation

Prediction of mechanical behavior comprises the use of a computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated (Oberkampf et al., 2002). Confidence in predictions relies heavily on proper verification and validation (V&V) processes. Verification is the process of determining that a model implementation accurately represents the developer's conceptual description and specification, so it is a Mathematics issue. Verification provides evidence that the model is solved correctly. Verification is also meant to identify and remove errors in computer coding and verify numerical algorithms and is desirable in quantifying numerical errors in computed solution. Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of

the model; therefore, it is a Physics issue. Validation provides evidence that the correct model is solved. Validation serves two goals, namely, (a) tactical goal in identification and minimization of uncertainties and errors in the computational model and (b) strategic goal in increasing confidence in the quantitative predictive capability of the computational model.

The V&V procedures are the primary means of building confidence and credibility in modeling and computational simulations. The employed mechanisms in the SANICLAY model with different levels of complexity have been examined for capturing the results of a wide range of element tests. In particular, the SANICLAY models have been calibrated for Boston Blue clay (Papadimitriou et al., 2005), Lower Cromer Till (Dafalias et al. 2006), and Bothkennar clay (Taiebat et al., 2009a) and details of the performance of the model with different constitutive features have been examined. Taiebat et al. (2009a) also examined the response of the SANICLAY-D model with different levels of complexity for capturing the response of Bothkennar clay. It was observed that although with a lesser degree of complexity the model does not show the accuracy that was obtained in the complete form of the model, still the results are in acceptable level of accuracy from a practical point of view. The accuracy of the numerical implementation of the model in FLAC3D has been verified in Taiebat et al. (2009b) using a comprehensive and independent constitutive driver developed based on the proposed approach by Bardet and Choucair (1991).

Numerical Simulations

In order to illustrate various features of the SANICLAY-D model in a boundary value problem, the response of a saturated clay slope under an idealized earthquake excitation has been simulated using this constitutive model integrated within the framework of FLAC3D. Part of the results of these simulations are presented and discussed in this section. Figure 1 shows the geometry and finite difference mesh of a 5H: 1V slope of saturated clay in FLAC3D. The model is discretized using 1700 brick zones in FLAC3D. For simplicity, the grid points have been fixed in the y direction and thus the problem is reduced to only two dimensions (x-z). The figure also shows three monitoring points at depths of 0, 3 m and 5 m in the middle section of the slope.

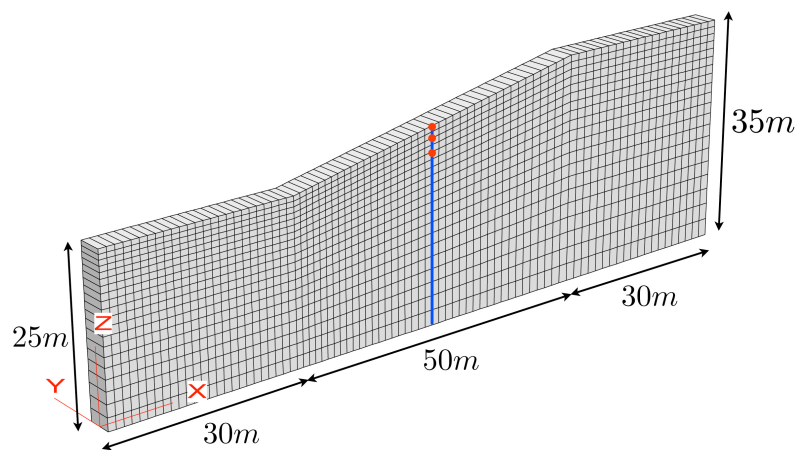


Figure 1. Model geometry, the finite difference mesh, and the positions of the monitoring points.

The analysis starts with applying the initial stresses and pore pressures under self-weight loading using a K_0 value of 0.6. Here is where the unique capability of the SANICLAY model in realistic prediction of the K_0 value becomes important. This has become possible with the aid of the rotational hardening mechanism and using the flexibility that the constant x brings to the model. This flexibility allows SANICLAY to be calibrated via the parameter x to whatever the measured value of K_0 , unlike the MCC model, which is known for overestimating the K_0 value. The slope remains stable under self-weight. The input excitation is then applied in form of horizontal acceleration at the base of the model. Figure 2 shows the time history of the idealized input acceleration with a frequency of 2 Hz and maximum amplitude of 0.25g. Ghosh and Madabhushi (2003) concluded that responses with simple input motions are easier to understand than to motions with wide frequency ranges, such as real earthquakes.

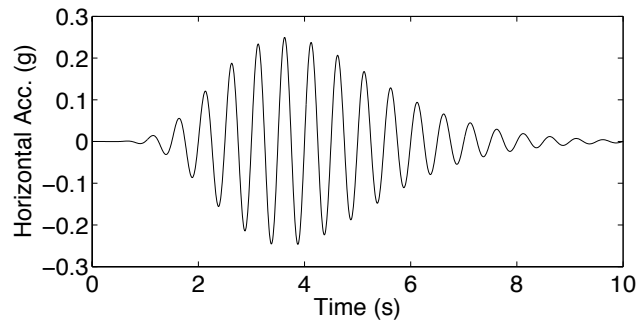


Figure 2. Time history of input motion, a_x , at the base of the model.

The newly implemented SANICLAY-D model has been used to characterize the response of clay. The SANICLAY-D model parameters that are used in the present simulation are presented in Table 1. These parameters are adopted from Dafalias et al. (2006) where the SANICALY model with non-associated flow rule was calibrated for Lower Cromer Till (Gens, 1982). With the present assumption for associated flow rule, the value of N is set equal to the critical stress ratio M . The initial deviatoric stress state (\mathbf{s}), i.e. the deviatoric stress under self-weight loading, is used to estimate the initial value of the internal tensor variable $\boldsymbol{\alpha}$ as \mathbf{s}/x at each zone.

Table 1. Model constants used in the simulation.

Model constant		Value
Elasticity	κ	0.009
	ν	0.2
Critical state	M_c	1.18
	m	0.73
	λ	0.063
Hardening	C	16
	x	1.56
Destructuration	k_i	0.6

The initial size of yield surface is set to $p_0 = Rp_{0,n}$ with $R = 1.2$ and $p_{0,n}$ the corresponding value of p_0 for normally consolidated state at the present stress state, i.e. having the stress point on the yield surface. Therefore a small value of overconsolidation has been introduced for the material

state at the beginning of the shaking phase. The values of the model internal variables at the beginning of the shaking phase are presented in Table 2. In order to examine the effect of destructuration mechanism in SANICLAY-D the destructuration parameter k_i and the structuration factor S_i are set to 0.6 and 3, respectively.

Table 2. Initial values of the model internal variables used in the simulations.

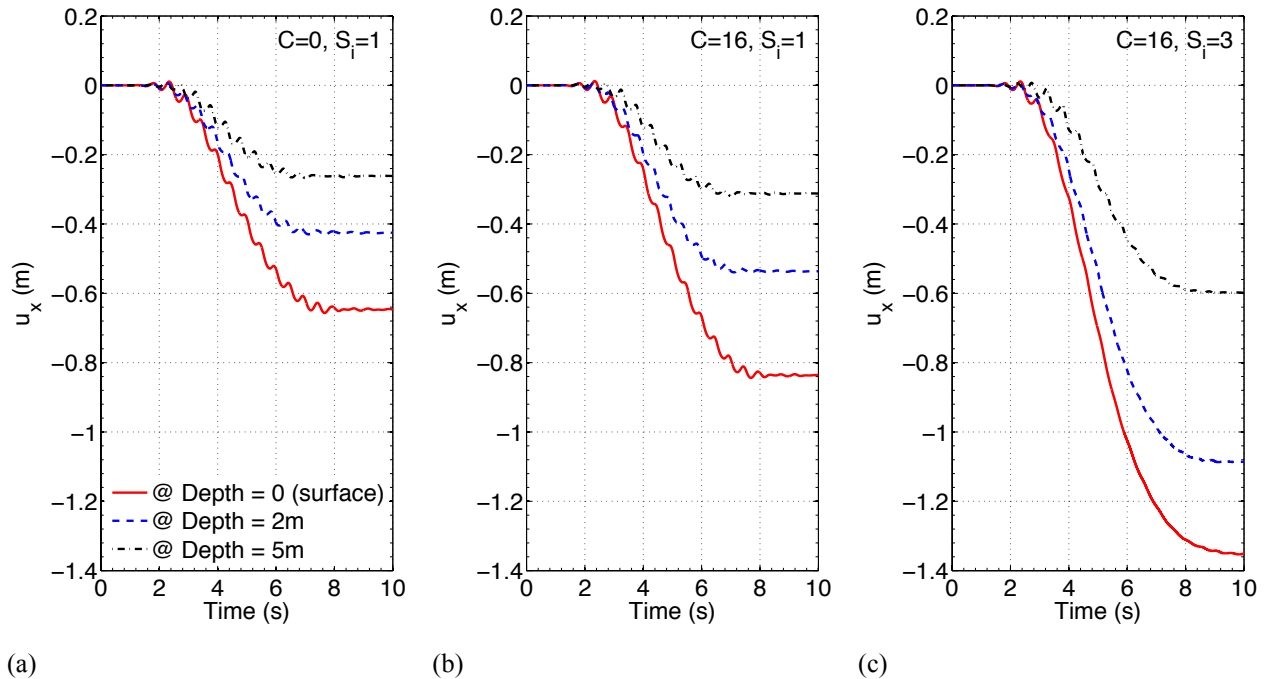
Model internal variable		Value
Size of the YS	p_0	$1.2p_{0,n}$
Orientation of the YS	α	s/x
Structuration factor	S_i	3

Formulation of FLAC3D allows for conducting any sophisticated solid-fluid interaction analysis. The examined problem in this study, i.e. seismic response of a saturated clayey slope, is essentially an undrained problem and this allows switching off the water flow in the analysis. A proper constitutive model together with the built-in equations for modeling of solid-fluid interaction allows the time-dependent pore pressure changes in clayey ground. For the dynamic analysis specifying the Free-Field boundary condition feature of FLAC reduces wave reflections at the boundaries of the mode. This approach enforces the free-field motion in such a way that boundaries retain their non-reflecting properties - i.e., outward waves are properly absorbed.

Time histories of the horizontal displacements at the three monitoring points in the middle section of the slope are presented in Fig. 3. Results of the following three sets of simulations are presented and compared in this figure:

- (a) Simulation with the simplest form of the SANICLAY-D model without incorporation of the effects of anisotropy and destructuration. This is achieved by setting $C=0$ and $S_i=1$.
- (b) Simulation using the anisotropy feature in SANICLAY-D, but without the effect of destructuration. This is achieved by setting $C=16$ and $S_i=1$.
- (c) Simulation using both anisotropy and destructuration features of SANICLAY-D, with $C=16$ and $S_i=3$.

Presented results in this figure show the accumulation of lateral displacements at the monitoring depths during the 10 s of shaking. The amount of accumulated displacement increases towards the surface, as expected. The figure shows the importance of the anisotropic (rotational) hardening and the destructuration mechanism in the SANICLAY-D model. The analyses show that in absence of anisotropy and destructuration features in the model the maximum lateral displacement at the surface in the middle section of the model does not exceed 0.64 m while in presence of anisotropy it reaches up to 0.84 m, and in presence of both anisotropy and destructuration it reaches up to 1.36m. In other words, in this case the lateral displacement at the surface monitoring point shows an increase of about 30% in presence of anisotropy, and an increase of 112% in presence of destructuration.



(a) (b) (c)
 Figure 3. Time histories of horizontal displacement u_x at different depths in the middle part of the slope using the SANICLAY-D model: (a) without any anisotropy and destructuration ($C=0$, $S_i=1$), (b) with anisotropy but without destructuration ($C=16$, $S_i=1$), and (c) with both anisotropy and destructuration ($C=16$, $S_i=3$).

More detailed investigation on different features of the model and their importance on numerical simulation of seismic effects in clay slopes are presented in Taiebat et al. (2009b). It is worth noting that in all cases while generation of permanent lateral displacement is a result of yielding in the slope, it is not necessarily an indication of slope failure. The size of permanent displacement, however, is used by some engineering guidelines to decide on stability of a slope under earthquake loading.

Summary

The ultimate goal of constitutive modeling is its application to practical problems. To this end the corresponding constitutive model should be comprehensive enough in order to account for main features of material response, and at the same time be simple enough for understanding, calibration, and application. In addition the model should be efficiently implemented in a proper numerical framework that can handle different types of loading and boundary conditions. For certain problems of interest in geotechnical engineering the numerical program also needs to have a three dimensional formulation, and properly handle the solid-pore fluid interaction and dynamic problems.

A simple and practical version of the advanced SANICLAY-D model is employed for characterization of the response of clays. This model includes a number of key mechanisms that are essential in prediction of response in clays, such as Lode angle dependency, anisotropic hardening, and destructuration mechanism -in perhaps the simplest possible theoretical

approach- with the hope to make the model attractive for real applications. In terms of model constants, the use of just three additional ones than what is required in the MCC, namely the m , C and x , the model can capture the important feature of anisotropic hardening. Besides the more accurate evaluation of plastic shear strains, the anisotropy feature allows the correct prediction of the K_0 by the model, something that is missing in the MCC model. One additional parameter k_i and the initial value of isotropic strcuturation factor S_i enable the model in capturing the strain softening as a result of destructureation. Capabilities of the main features of the model have been already validated against a number of laboratory results. The model has is efficiently integrated in FLAC3D program that is well known in both research and practical communities of geotechnical engineering and especially in the field of geotechnical earthquake engineering. The implementation details have been extensively verified in the numerical framework. The resulting computational model is used for numerical simulation of a slope under seismic loading in order to show the effect of different features in SANICLAY-D. More refinements of the model to account for cyclic loading applicable to seismic problems while maintaining its simplicity are underway. However the present framework meets a good portion of the practical needs in many problem of interest in Geotechnical Engineering.

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