Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 620

IMPROVED EFFECTIVE DAMPING EQUATION FOR EQUIVALENT LINEAR ANALYSISOF SEISMIC-ISOLATED BRIDGES

M.Karalar[1]and M.Dicleli[2]

ABSTRACT

In this paper, an improved effective damping (ED) equation is proposed to obtain more reasonable estimates of the actual nonlinear response of seismic-isolated bridges (SIB) using equivalent linear (EL) analysis procedure. For this purpose, first, the EL analysis results using AASHTO's ED equation is evaluated. The effect of several parameters such as substructure stiffness, isolator and ground motion properties are considered in the evaluation. It is found that AASHTO's ED equation should incorporate the effective period of the SIB and isolator properties for a more accurate estimation of the seismic response quantities. A new ED equation that includes such parameters is formulated and found to improve the accuracy of the EL analysis.

Introduction

An elastic analysis procedure for the design of seismic-isolated bridges (SIB) is presented in AASHTO (American Association of State Highway Transportation Officials) Guide Specifications for Seismic Isolation Design (AASHTO 1999). Since the behavior of seismic isolators is non-linear in nature, equivalent linear (EL) properties need to be defined for the elastic analysis of SIB. The EL properties of SIB are expressed in terms of an effective stiffness, an effective period and an effective damping (ED) ratio to account for the hysteretic energy dissipation of the isolators. Using these EL properties, an EL analysis procedure is followed to estimate the absolute maximum seismic responses in the isolators and in other components of the bridge.

The accuracy of the EL analysis results for SIB has been studied by several researchers (Hwang 1996; Franchin et al. 2001). Although these studies were very useful in identifying the imprecisions in the EL analyses results, they focused on specific design code procedures. Perceptibly, this type of an approach may not allow for a rational verification of the EL analysis results due to the approximations involved in the code procedures. Furthermore, most of these studies have not explicitly considered the effects of all the isolator, substructure and ground

---

[1] Graduate Research Assistant, Department of Engineering Sciences, Middle East Technical University, Ankara, Turkey, E-mail: memduh@metu.edu.tr
[2] Professor, Department of Engineering Sciences, Middle East Technical University, Ankara, Turkey, E-mail: mdicleli@metu.edu.tr,.

motion properties as well as the effective period of the bridge on the accuracy of the EL analysis results Therefore, a comprehensive evaluation of the EL analysis results for SIB is required to rationally identify the deficiencies within the analysis procedure and to suggest improvements to AASHTO's ED equation for a more accurate prediction of the actual nonlinear responses.

The evaluation of the EL analysis results mainly involves the comparison of the seismic response quantities obtained from EL analyses with those obtained from nonlinear time history (NLTH) analyses. The NLTH analyses are conducted using harmonic and seismic ground motions with various frequency characteristics. The effect of several parameters such as bridge substructure stiffness, isolator and ground motion properties are considered in the evaluation of the EL analysis results. The effect of bridge superstructure mass on the ED ratio and hence on the accuracy of the EL analysis results is also investigated. This is followed by regression analyses of the acquired data to incorporate additional empirical relationships in AASHTO's ED equation for improving the accuracy of the EL analysis results. At the end, the accuracy of the EL analysis results using the improved ED equation is assessed and conclusions are outlined.

Isolator and Ground Motion Proporties Used in the Analyses

The force-displacement relationship of most isolators is idealized as bilinear for design purposes as shown in Fig. 1(a). In the figure, Q_d = characteristic strength, k_u = elastic stiffness, k_d = post-elastic stiffness, F_y and d_y are respectively the yield force and displacement and F_i and d_i are respectively the maximum (design) force and displacement of the isolator.

Ground motions are characterized by their peak ground acceleration (A_p) to peak ground velocity (V_p) ratios, which represent their dominant frequency and energy content (Kramer 1996). Ground motions with intense long-duration acceleration pulses have low A_p/V_p ratios whereas those with high frequency, short-duration acceleration pulses have high A_p/V_p ratios. As the seismic response of a structural system may differ as a function of the A_p/V_p ratio, ground motions with various A_p/V_p ratios are used in the evaluation of the EL analysis procedure.

Two sets of ground motions are used for the verification of the EL analysis procedure. The first set consists of harmonic ground motions with $A_p/V_p=2\pi/T_g$, where T_g is the excitation period. The main reason for using harmonic ground motions for the verification of the EL analysis procedure is to have a clear understanding of the effect of the frequency characteristics (or the A_p/V_p ratio) of the ground motion on the accuracy of the EL analysis results. Harmonic ground motions with A_p/V_p ratios ranging between 5.23 s^{-1} and 20.0 s^{-1} are used in the analyses. The second set of ground motions involves a suite of 15 earthquakes with A_p/V_p ratios ranging between 5.50 s^{-1} and 21.5 s^{-1}. The details of the ground motions are presented in Table 1. These ground motions are used for further verification of the EL analysis procedure and for improving the AASHTO's ED equation for a more accurate prediction of the seismic response quantities.

The effect of the intensity of the seismic ground motion on the accuracy of the EL analyses results is studied using a dimensionless term, A_pW/Q_d (W: weight acting on the isolator), which represents the ratio of the seismic inertial force of a rigid bridge superstructure to the characteristic strength of the isolator.

Equivalent Linear Properties Used in Elastic Analyses

The EL analysis procedure requires the EL properties of the SIB, which are the effective stiffness, K_{eff}, the effective period, T_e and the ED ratio, β. The effective stiffness, k_{eff}, of a typical isolator is illustrated in Fig. 1(a). K_{eff} for a typical SIB (Fig. 1(b)) is then calculated by considering each substructure, and the isolators as springs connected in series. T_e is calculated using K_{eff} and the bridge mass, m. Neglecting the inertial effects, the ED ratio, β, of the SIB is expressed as (AASHTO 1999);

$$\beta = \frac{2\sum_{j=1}^{n_s} Q_d (d_i - d_y)}{\pi \sum_{j=1}^{n_s} K_{eff,j} (d_i + d_{s,j})^2} \tag{1}$$

Table 1. Important features of earthquake records used in the analyses

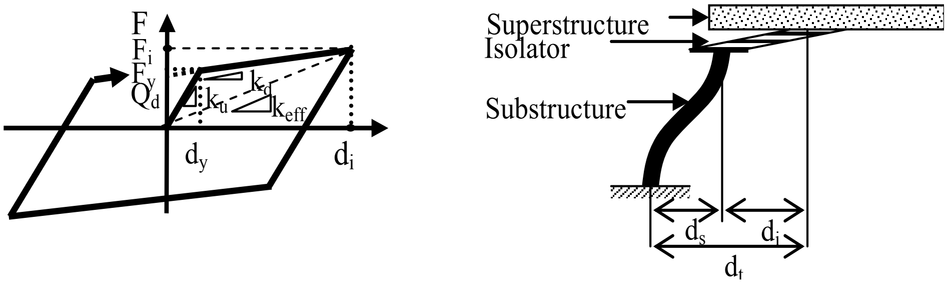
Earthquake	**Station / Component**	**A_p (g)**	**V_p (cm/s)**	**A_p/V_p (1/s)**
San Fernando, 1971	8244 Orion Blvd. / 180°	0.13	23.9	5.5
Imperial Valley, 1940	El Centro / 180°	0.21	36.5	5.8
Loma Prieta, 1989	Oakland Outer Wharf / 0°	0.22	35.4	6.1
Loma Prieta, 1989	Oakland Outer Wharf / 270°	0.28	37.6	7.2
Northridge, 1994	Arleta and Nordhoff Fire Station / 90°	0.34	40.4	8.4
Kern County, 1952	Taft Lincoln Tunnel / 69°	0.16	15.7	9.7
Imperial Valley, 1940	El Centro / 270°	0.35	2.3	10.6
Santa Barbara, 1978	283 Santa Barbara Courthouse / 222°	0.20	16.3	12.2
Coalinga, 1983	36227 Parkfield - Cholame 5W / 270	0.15	10.8	13.4
Northridge, 1994	Santa Monica City Hall Grounds / 0°	0.37	24.9	14.6
Whitter Narrows, 1987	24401 San Marino, SW Academy / 360°	0.20	12.8	15.6
Whitter Narrows, 1987	90079 Downey Birchdale / 90°	0.24	13.7	17.4
San Fernando, 1971	Pacoima Dam / 196°	1.08	57.5	18.4
Northridge, 1994	Santa Monica City Hall Grounds / 90°	0.88	41.8	20.7
Parkfield, 1966	Cholame, Shandon / 40°	0.24	10.8	21.5

Evaluation of the Equivalent Linear Analysis Results

Substructure Stiffness

To study the effect of the substructure stiffness on the accuracy of the EL analysis results, a single bridge substructure and an isolator supporting a tributary bridge superstructure is considered. The NLTH and EL analyses of the SIB are then conducted for A_pW/Q_d=6 using the seismic ground motions considered in this study. In the analyses, the ratio, k_u/k_s, of the elastic stiffness of the isolator to the stiffness of the bridge substructure is varied between 0.025 and 1.0. Analyses results have revealed that the effect of the substructure stiffness on the accuracy of the

EL analysis results is negligible. Accordingly, for the remainder of the research study, the SIB are represented by isolators placed on rigid supports and supporting a rigid mass.



(a) (b)

Figure 1. (a) Idealized hysteresis loop of a typical isolator, (b) Typical seismic-isolated bridge substructure

Elastic and Post-Elastic Stiffness of the Isolator

The effects of elastic and post-elastic stiffness of the isolator on the accuracy of the EL analysis results are studied by varying the post-elastic stiffness of the isolator while keeping its elastic stiffness constant and vice versa. Figs. 2(a) and (b) display the ratio, d_E/d_{NL}, of the maximum isolator displacements obtained from EL analyses to those obtained from NLTH analyses as a function of respectively the elastic and post-elastic stiffness of the isolator for the seismic ground motions considered in this study. The solid lines in the figures represent the average variation of the d_E/d_{NL} ratio and reveal that the accuracy of the EL analysis results depends on the stiffness properties of the isolator.

A_p/V_p Ratio of the Ground Motion

The effect of the A_p/V_p ratio of the ground motion on the accuracy of the EL analyses results is presented in Fig. 3. The figure displays the, d_E/d_{NL} ratio as a function of the A_p/V_p ratio of the seismic ground motions. It is observed that the d_E/d_{NL} ratios are generally smaller than 1.0 with only a few exceptions. Thus, the EL analysis procedure produces unconservative estimates of the displacement response ranging between 0.64 and 1.11 times the actual nonlinear response. Generally, the d_E/d_{NL} data become more scattered as the A_p/V_p ratio increases. This clearly demonstrates that the difference between the EL and NLTH analyses results becomes larger for ground motions with high excitation frequency. This indicates that the frequency characteristics (or A_p/V_p ratio) of the ground motion affect the accuracy of the EL analysis results.

Effect of A_pW/Q_d Ratio

Fig 4 displays the variation of the d_E/d_{NL} ratio as a function of the A_pW/Q_d ratio. The solid curve representing the average of the data indicates that the EL analysis produces more reasonable estimates of the displacement response for larger A_pW/Q_d ratios. This indicates that the A_pW/Q_d ratio affect the accuracy of the EL analysis results.

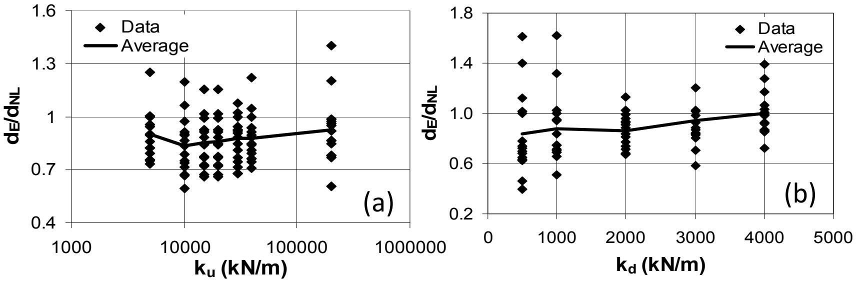


Figure 2. d_E/d_{NL} ratio as a function of (a) elastic stiffness of the isolator (b) post-elastic stiffness of the isolator

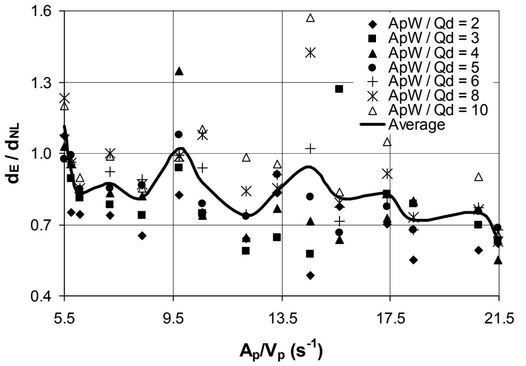


Figure 3. d_E/d_{NL} ratio for seismic ground motions

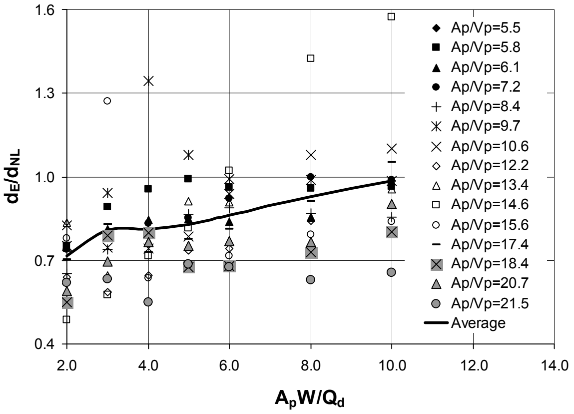


Figure 4. d_E/d_{NL} ratio as a function of A_pW/Q_d ratio for the seismic ground motions

Effect of Inertial Forces on Effective Damping Ratio

For an isolator supporting no mass and subjected to a harmonic cyclic displacement with a period, T_g and amplitude d_i,, the ED ratio, β, of an equivalent elastic system simulating the nonlinear behavior of the isolator is simply calculated by setting the area under its force-displacement hysteresis loop equal to the area under the hystresis loop of a viscous damper with the same displacement. Thus:

$$\beta = \frac{2Q_d(d_i - d_y)}{\pi k_{eff} d_i^2} \tag{2}$$

However, for an isolator supporting a rigid mass, m, and subjected to the same harmonic cyclic displacement the ED ratio is expressed as (Makris and Chang 1998);

$$\beta_1 = \frac{T_g}{T_e}\beta \tag{3}$$

This clearly demonstrates that AASHTO's ED ratio must somehow involve the effective period of the bridge and dominant period (or frequency content) of the ground motion. The EL analyses of the same SIB subjected to harmonic ground motions are conducted using Eq. 3 instead of Eq. 1 to demonstrate the effect of including T_g and T_e in the ED equation on the accuracy of the EL analyses results. The analyses results have revealed that including T_g and T_e in the ED equation resulted in considerable improvement to the accuracy of the EL analysis results.

Relationship Between Effective Damping Ratio and Period Shift

To picture the variation of the actual ED ratio as a function the effective period and to assess the accuracy of AASHTO's ED equation, first, the ED ratios, β_r, required to produce EL analysis results equal to those of the NLTH analyses are obtained for the cases considered in this study. AASHTO's ED ratios, β, are also calculated using Eq. 1. Next, the β_r/β ratios are calculated and displayed in Fig. 5 as a function of the period shift, T_e/T_i, where T_i represents the initial elastic period of the SIB calculated using the elastic stiffness, k_u, of the isolator. Fig. 5 reveals that AASHTO's ED equation generally yields damping values larger than those required to accurately estimate the nonlinear response by EL analysis. This obviously produces unconservative estimates of the actual nonlinear responses as demonstrated earlier. Although there is a considerable scatter in the data, it is observed that, the β_r/β, ratio approaches unity with increasing period shift (or increasing effective period). This indicates that the EL analyses may yield more reasonable estimates of the actual nonlinear response of SIB with larger effective periods. Furthermore, it was observed that larger effective periods are associated with lower A_p/V_p and larger A_pW/Q_d ratios. Thus, the effective period implicitly includes the effect of the A_p/V_p ratio or frequency characteristics and intensity of the ground motion. Accordingly, Fig 5 implicitly represents the variation of β_r/β ratio as a function of the intensity and frequency characteristics of the ground motion.

Improved AASHTO Effective Damping Equation

Including the Effective Period in the Effective Damping Equation

Figure 5 is used to incorporate the effective period in AASHTO's ED equation to improve the prediction of nonlinear responses by EL analysis. For this purpose, the logarithms of the β_r/β, ratios presented in Fig. 5 are first plotted as a function of the logarithms of the relative period shifts (T_e/T_{i-1}) of the SIB considered in this study. Then a minimum least square fit of the log-log data is performed to obtain the following equation for the β_r/β ratio as;

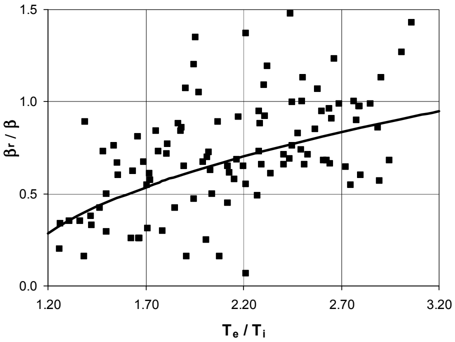


Figure 5. Ratio of actual ED to that calculated using AASHTO's ED equation as a function of T_e/T_i

$$\frac{\beta_r}{\beta} = \sqrt{0.41\left(\frac{T_e}{T_i} - 1\right)} \tag{4}$$

The above equation is plotted in Fig 5. It is the analytical representation of the variation of the actual ED ratio relative to AASHTO's ED equation (Eq. 1). Substituting Eq. 1 in place of β in Eq. 4 and solving for β_r, the improved ED ratio is expressed as:

$$\beta = \frac{2\sum_{j=1}^{n_s} Q_d (d_i - d_y)}{\pi \sum_{j=1}^{n_s} K_{eff,j} (d_i + d_{s,j})^2} \sqrt{0.41\left(\frac{T_e}{T_i} - 1\right)} \tag{5}$$

Including the Isolator Properties in the Effective Damping Equation

In this section, the elastic (k_u), and post-elastic (k_d) stiffness of the isolator are included in Eq, 5 to improve the prediction of the actual nonlinear responses by EL analysis for various isolator properties. Fig 6 displays the variation of β_r/β ratio as a function of the k_d/k_u ratio where β is calculated using Eq. 5. The figure reveals that the β_r/β ratio increases logarithmically as a function of the k_d/k_u ratio. Using a logarithmic least square fit, the β_r/β ratio is obtained as:

$$\frac{\beta_r}{\beta} = 2.21\left(\frac{k_d}{k_u}\right)^{0.32} \tag{6}$$

Substituting Eq. 5 in place of β in Eq. 6 and solving for β_r, the further improved ED ratio is expressed as:

$$\beta = \frac{2\sum_{j=1}^{n_s} Q_d (d_i - d_y)}{\pi \sum_{j=1}^{n_s} K_{eff,j} (d_i + d_{s,j})^2} \sqrt{2\left(\frac{T_e}{T_i} - 1\right)\left(\frac{k_d}{k_u}\right)^{0.64}} \tag{7}$$

Verification of the Proposed Damping Equation

The proposed ED equation and those proposed by AASHHTO, CALTRANS (California Department of Transportation), Iwan (1983) and Hwang (1996) are plotted in Fig. 7 together with the data representing the actual ED ratios. The figure demonstrates that the improved ED equation proposed in this study generally yields reasonable estimates of the actual ED ratio for all ranges of period shifts compared to other equations.

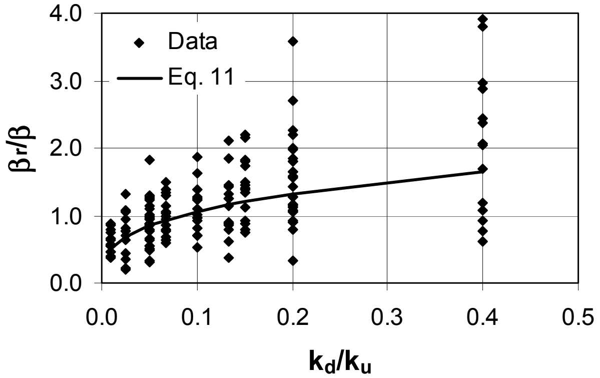


Figure 6. Variation of the β_r/β ratio as a function of k_d/k_u (A_pW/Q_d=6)

EL analyses of the SIB considered in this study are re-conducted using the proposed ED equation. The d_E/d_{NL} ratio and the dispersion of data with respect to 1.0 for the EL analyses results using AASHTO's and the proposed ED equations are illustrated in Figs 8(a), (b), (c), (d). Figs. 8 (a) and (b) demonstrate that the proposed ED equation yields more conservative and reasonable estimates of the actual nonlinear response of SIB. Furthermore, Figs 8(c) and (d) reveal that the proposed damping equation produces less dispersed data in most cases. This indicates that the proposed damping equation also improves the reliability of the individual EL analysis results for various A_p/V_p and A_pW/Q_d ratios.

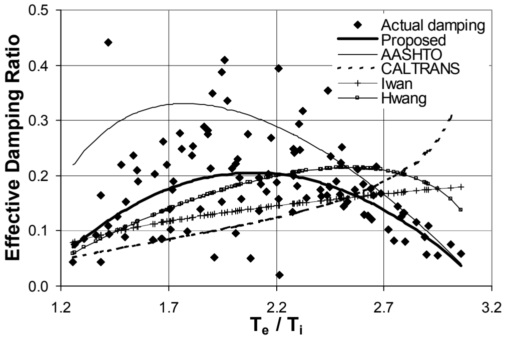
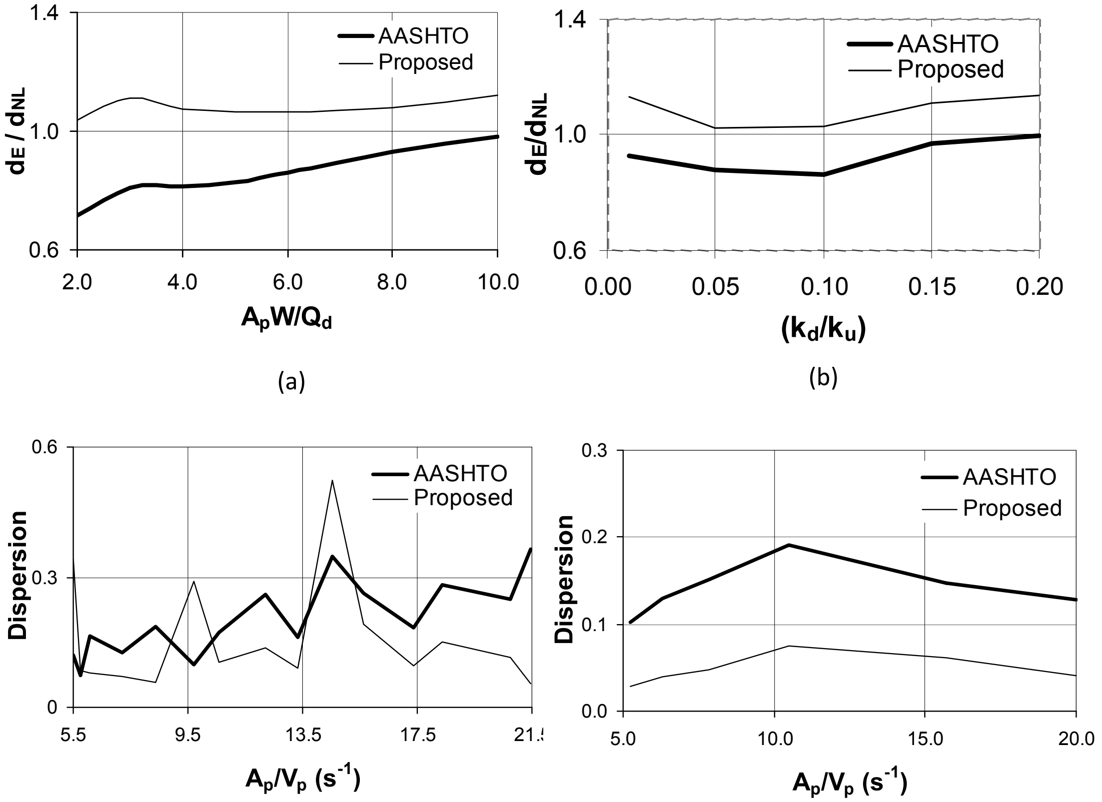


Figure 7. Comparison of results from various damping equations with actual effective damping data



(a) (b)

(c) (d)

Figure 8: Comparison of d_E/d_{NL} ratios and data dispersion obtained using AASHTO and proposed damping equations (a) d_E/d_{NL} vs A_pW/Q_d ratios for average of all A_p/V_p cases for seismic ground motions (b) d_E/d_{NL} vs k_d/k_u ratios for average of all A_p/V_p cases for seismic ground motions ($ApW/Q_d = 6$) (c) dispersion vs. A_p/V_p ratio for seismic ground motions (d) dispersion vs. A_p/V_p ratio for harmonic ground motions.

Conclusions

In this study, the accuracy of the EL analysis in predicting the actual nonlinear response of SIB is evaluated. AASHTO's ED equation is modified to improve the accuracy of the EL analysis. Followings are the conclusions drawn from this research study: It is observed that the effect of the substructure stiffness on the accuracy of the EL analysis results is negligible. The EL analysis generally produces unconservative estimates of the actual maximum displacement response of seismic isolators using AASHTO's damping equation. Analysis results have revealed that the difference between the EL analysis results and the actual nonlinear responses becomes larger for ground motions with high frequency content (high A_p/V_p ratio). It is also observed that for ground motions with larger intensity and lower A_p/V_p ratio, which produce larger effective periods; the EL analysis yields more reasonable estimates of the actual nonlinear responses. It is demonstrated that AASHTO's ED equation must involve the effective period of the bridge, dominant period (or frequency content) of the ground motion and k_d/k_u ratio of the isolator for a more accurate estimate of the actual nonlinear response of the SIB. Accordingly, an improved ED equation that incorporates the effective period and properties of the isolator is proposed. The impacts and benefits of the proposed (improved) ED equation are as follows: The proposed damping equation yields more reasonable estimates of the actual ED compared to several other damping equations found in the literature. The proposed ED equation yields more reasonable estimates of the actual nonlinear responses regardless of the type of ground motion used in the analyses. Furthermore, the proposed ED equation reduces the dispersion of the d_E/d_{NL} data. This indicates that the proposed ED equation improves the reliability of the individual EL analysis results for various A_p/V_p and A_pW/Q_d ratios. Based on the above remarks, it may be stated that the proposed (improved) ED equation may produce more reliable SIB designs.

References

AASHTO, 1999. *Guide Specifications for Seismic Isolation Design*, Washington, D.C.

Franchin, P., Monti, G. and Pinto, P. E., 2001. On the accuracy of simplified methods for the analysis of isolated bridges. *Earthquake Engineering and Structural Dynamics* 30 (3), 363-382.

Hwang, J. S., 1996. Evaluation of equivalent linear analysis methods of bridge isolation, *ASCE Journal of Structural Engineering* 122 (8), 972--976.

Iwan, W. D.,1980. Estimating inelastic response spectra from elastic spectra. *Earthquake Engineering and Structural Dynamics* 8 (4), 375-388.

Makris, N. and Chang, S., 1998. Effect of damping mechanisms on the response of seismically isolated structures, *PEER Report 1998/06, Pacific Earthquake Engineering Research Center, Collage of Engineering, University of California*, Berkeley