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A METHODOLOGY FOR CONSTRUCTING SEISMIC FRAGILITIES BASED ON EXPERIMENTS AND MECHANICAL MODELS

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ABSTRACT

A general method is developed for system fragility analysis based on mechanical models, a Bayesian formulation for model calibration, and seismic site characterization by actual and simulated ground motion accelerations. Simple examples illustrate the implementation of the proposed methodology for model calibration and fragility analysis. The extension of the method to realistic mechanical models for nonstructural systems is discussed using gypsum wall mechanical models.

Introduction

A key challenge in performance-based design is the development of validated tools and guidelines to assist with understanding and predicting the seismic response of nonstructural components and systems. Subsystems of particular interest, and the focus of a NEES research grand challenge project, include the ceiling-piping-partition and the gypsum partition walls nonstructural systems. These subsystems are highly interconnected, and account for a significant fraction of all hazards and losses associated with NCS seismic performance.

In this paper, we present an update of the simulation efforts of the NEES grand challenge team, with particular focus on model validation against emerging partition wall experiments and development of a fragility framework. The former efforts use simple mechanical models implemented in OpenSees, while the later adopts a Bayesian analysis to combine various sources of information, including for example, tests recording system responses and damage to specified loading protocols, field damage data if available, expert opinions, and general knowledge on material properties, system geometry, and/or boundary conditions.

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The proposed methods for fragility analysis involves three steps. In the first step, a functional form is selected for a mechanical model of the system under consideration and the parameters of this form are estimated. The selected model has to (1) be consistent with the physics, for example, the functional form of a mechanical model for gypsum walls has to degrade under cyclic loading, and (2) depend on a relatively small number of parameters that can be estimated reliably from the available information. In the second step, damage states are defined and relationships are developed between damage states and various system responses. Both the definition of damage states and the development of relationships relating responses to damage states are based on experimental observations and heuristic arguments. In the third step, the mechanical model and the damage state-model response relationship are used to construct fragility curves to arbitrary hazards, that generally differ from the loading protocols used in experiments. The proposed method provides a rational framework for extrapolating system performance beyond the available experimental results. The methodology is illustrated with a system examples.

The first part of the paper presents the proposed Bayesian method for calibrating mechanical models to data. An elementary mechanical model is used as a vehicle for illustrate the proposed methodology. The application of the Bayesian calibration method to calibrate a gypsum wall model is also discussed. The second part outlines a Bayesian framework for constructing system fragilities based on data and mechanical models.

Mechanical models

Mechanical models of increasing complexity are used to characterize the behavior of gypsum partition walls, for example, linear and nonlinear springs and finite element. Irrespective of their complexity, the models define mappings $a(t) \mapsto r(t)$ from seismic ground accelerations $a(t)$ to system responses $r(t)$ depending on vectors $\theta = (\theta_1, \ldots, \theta_q)$ of uncertain parameters.

Our objective is to calibrate postulated models to available information, that is, find values of θ such that model outputs be consistent with laboratory experiments and/or any other type of information that may be available. Suppose that n nominally identical gypsum partition walls have been subjected in laboratory to a seismic ground acceleration $a(t)$ and that $(r_{\max,1}, \ldots, r_{\max,n})$ are the largest responses of these specimens. It is assumed that there is a one-to-one correspondence between $(r_{\max,1}, \ldots, r_{\max,n})$ and wall damage states $(d_1, \ldots, d_n)$, so that response levels can be related simply to damage; the assumption is needed for fragility analysis. Let $r(t; \theta)$ be the response at time t of a gypsum wall mechanical model under $a(t)$, and denote by $r_{\max}(\theta) = \max_t\{r(t; \theta)\}$ the corresponding maximum response.

We consider two methods for estimating the uncertain parameters of a mechanical model. The simplest method is to set $\theta = \hat{\theta}$, where the estimate $\hat{\theta}$ is that value of θ that minimizes the discrepancy between observed and calculated responses, for example, $\hat{\theta}$ may be required to minimize the mean square error $\sum_{i=1}^{n} \left(r_{\max,i} - r_{\max}(\theta)\right)^2$. An alternative method is to view the uncertain vector of parameters θ as a random variable and determine its posterior probability law from both prior information and data within a Bayesian framework. This method is less simple but has the advantage that it accounts for all available information and provides not only an estimate $\hat{\theta}$ but also a measure of the uncertainty in this estimate. We use a simple example to illustrate the methodology for model calibration within a Bayesian framework.

Consider a rod with strength R following a shifted exponential distribution, that is, the density and distribution functions of R are $f_R(x) = \lambda \exp(-\lambda(x-\alpha))\, 1(x \geq \alpha)$ and $F(x) = \left[1 - \exp(-\lambda(x-\alpha))\right] 1(x \geq \alpha)$, respectively, where $\alpha > 0$ is known, $\lambda > 0$ is uncertain, and $1(\cdot)$ denotes the indicator function ($1(A) = 1$ if statement A is true and $1(A) = 0$ if A is false). We note that the functional form of the distribution of R is determined by a particular mechanical model for rod strength that in our illustration has a single uncertain parameter, that is, the vector of uncertain parameters θ reduces to a single parameter denoted by λ.

Suppose that n nominally identical rods have been tested at a load of magnitude $\xi > \alpha$ and $n_1(\xi)$ out of n rods failed under this load. The strengths $(R_1, \ldots R_n)$ of these rods are independent copies of R. Without loss of generality we assume that the first $n_1(\xi)$ rods failed, that is, $R_i < \xi$ for $i = 1, \ldots, n_1(\xi)$, and the last $n - n_1(\xi)$ rods survived, that is, $R_i \geq \xi$ for $i = n_1(\xi) + 1, \ldots, n$. The latter type of data is referred to as censored data. Since α is known it is convenient to shift the origin to this constant, so that the density and distribution functions of R become

$$
\begin{aligned}
&f_R(\tilde{x}) = \lambda\, e^{-\lambda \tilde{x}}\, 1(\tilde{x} \geq 0) \quad \text{and} \\
&F_R(\tilde{x}) = \left[1 - e^{-\lambda \tilde{x}}\right] 1(\tilde{x} \geq 0),
\end{aligned} \tag{1}
$$

where $\tilde{x} = x - \alpha$. In this new coordinate, the magnitude of the applied load is $\tilde{\xi} = \xi - \alpha > 0$ and the failure loads for the first $n_1(\xi)$ rods are $\tilde{r}_i = r_i - \alpha > 0$, $i = 1, \ldots, n_1(\xi)$, where $r_i < \xi$ denote the failure loads of the first $n_1(\xi)$ specimens. Suppose the prior information on λ can be quantified by a Gamma distribution with parameters (s, μ), that is,

$$
f'(\lambda) \propto \lambda^{s-1}\, e^{-\mu \lambda}, \quad \lambda > 0. \tag{2}
$$

The likelihood function of λ corresponding to $n_1(\xi)$ failure loads and $n - n_1(\xi)$ censored data is

$$
\begin{aligned}
\ell(\lambda \mid \text{data}) &\propto \prod_{i=1}^{n_1(\xi)} \left(\lambda\, e^{-\lambda \tilde{r}_i}\right) \left(e^{-\lambda \tilde{\xi}}\right)^{n - n_1(\xi)} \\
&\propto \lambda^{n_1(\xi)} \exp\left[-\lambda \left(\sum_{i=1}^{n_1(\xi)} \tilde{r}_i + \tilde{\xi}\left(n - n_1(\xi)\right) \right) \right]
\end{aligned} \tag{3}
$$

since the strength of the last $n - n_1(\xi)$ rods is not known. We only know that the strength of these rods is larger than ξ, and the probability of this event is $1 - F_R(\tilde{\xi}) = \exp(-\lambda \tilde{\xi})$. The posterior density of λ can be obtained by multiplying the prior density in Eq. 2 and the likelihood function in Eq. 3, and has the expression

$$
f''(\lambda) \propto \lambda^{s_p(\xi)}\, e^{-\mu_p(\xi)\, \lambda}, \quad \lambda > 0, \tag{4}
$$

where

$$
\begin{aligned}
&s_p(\xi) = s + n_1(\xi) \\
&\mu_p(\xi) = \mu + \sum_{i=1}^{n_1(\xi)} \tilde{r}_i + \tilde{\xi}\left(n - n_1(\xi)\right).
\end{aligned} \tag{5}
$$

We note that the posterior distribution of λ is a Gamma distribution with parameters $\big(s_p(\xi), \mu_p(\xi)\big)$, that is, a distribution of the same type as the prior. If the prior and posterior distributions are of the same type, the prior is said to be a conjugate prior.

The results in the following two figures are for $s = 2$, $\mu = 1$, $n = 10$, $\alpha = 1$. Figure 1 shows the prior and posterior densities of λ in Eqs. 2 and 4 for $(n_1(\xi) = 3,\ \xi = 2)$ and

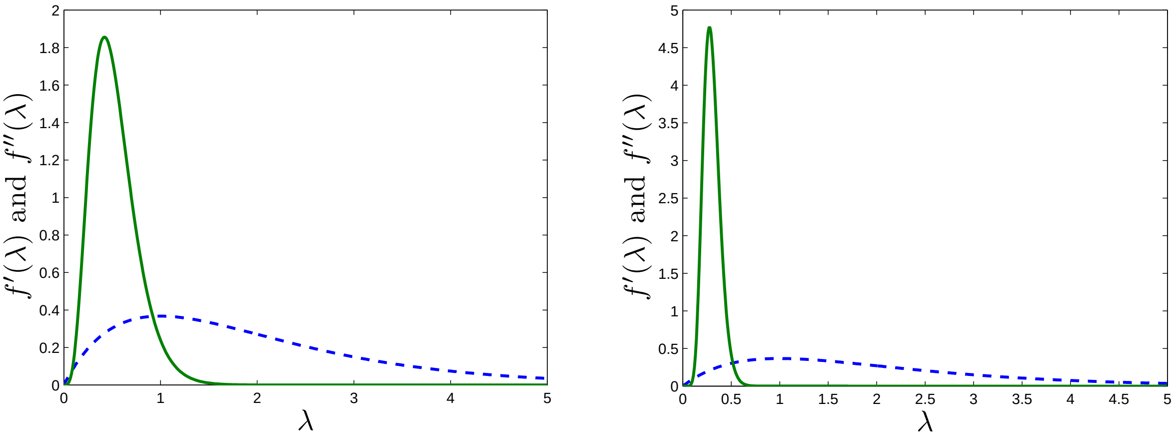

Figure 1: Prior and posterior densities $f'(\lambda)$ (dotted lines) and $f''(\lambda)$ (solid lines) for $(n_1(\xi) = 3,\ \xi = 2)$ (left panel) and $(n_1(\xi) = 10,\ \xi = 10)$ (right panel)

$(n_1(\xi) = 10,\ \xi = 10)$ in the left and right panels, respectively. The failure loads of the $n_1(\xi) = 3$ specimens under $\xi = 2$ are assumed to be $r_i = 1.1$, 1.5, and 1.9. The failure loads of the $n_1(\xi) = 10$ specimens under $\xi = 10$ are $r_i = 1.1$, 1.5, 1.9, 2.7, 3.9, 5.7, 6.7, 7.1, 8.9, and 9.5. The prior and posterior densities are drawn with dotted and solid lines, respectively. Since it is assumed that the prior information is rather weak, the parameters (s, μ) have been selected such that the prior density f' assigns similar probabilities to most values of λ. Accordingly, the change from the prior to the posterior density is essentially caused by data.

The posterior distributions $f''(\lambda)$ can be used to obtain a point estimate $\hat{\lambda}$ of λ, for example, the mean or the mode of $f''(\lambda)$, and determine the uncertainty in this estimate, for example, the uncertainty in the point estimate set equal to the mean of $f''(\lambda)$ is the variance of this density. We also note that $f''(\lambda)$ provides a full characterization of our state of knowledge regarding the uncertain parameter λ.

The calibration approach in Eqs. 1 to 5 extends directly to mechanical models describing the behavior of nonstructural systems. For example, mechanical models for gypsum walls are available in OpenSees, an open-source simulation and modeling software platform developed by the Pacific Earthquake Engineering Research (PEER) Center. Some of these models depend on up to 24 parameters, that is, the dimension of vector θ is 24, that need to be calibrated to data. Their ability of describing complex time histories is remarkable, as illustrated in Fig. 2 showing experimental and model generated force displacement time histories.

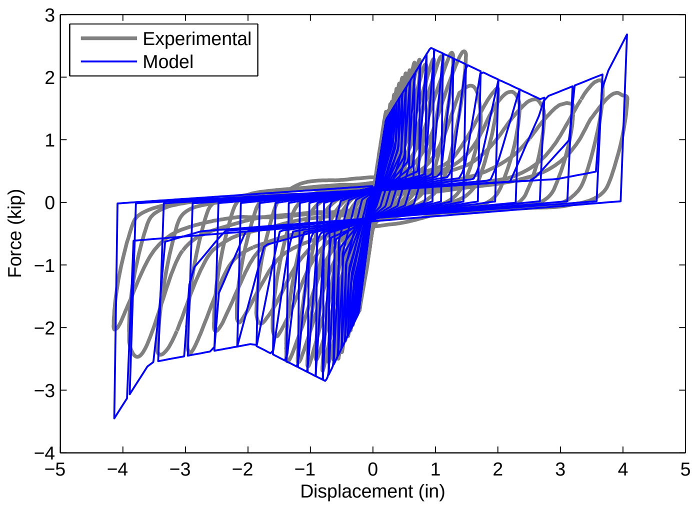

Figure 2: Force displacement response for gypsum walls

Fragility analysis

We construct fragility curves under two assumptions. First, only experimental results are used to obtain fragilities. Second, both experimental evidence and mathematical models are employed to calculate fragilities.

The probability $P_d(x)$ that a structural component or system subjected to a seismic load of intensity x reaches a damage state d is referred to as the component or system *fragility* for load intensity x and damage state d. The functions $P_d(x)$ of x indexed by d are called *fragility curves*. This definition of fragility can be generalized by expressing fragilities as functions of other indicators of seismic hazard, for example, earthquake moment magnitude and site-to-source distance (Kafali and Grigoriu 2007).

Fragility curves can be estimated from field and/or laboratory data or can be derived analytically. We discuss two methods for constructing fragility curves. The first method postulates a functional for fragility $P_d(x; \theta)$ depending on some unknown parameters θ, and is used in ATC-58. The second method, proposed in this study, develops a mechanical model for the structural system under consideration and uses this model to calculate fragilities. The proposed formulation (1) has the potential of reducing the uncertainty in the resulting fragility curves and (2) provides a framework for quantifying the uncertainty in these curves.

ATC-58 fragility

Fragilities proposed in ATC-58 are derived from laboratory test data, earthquake experience data, and/or other sources. Simplicity is the main advantage of ATC-58 fragilities. Since these fragilities can be calculated efficiently, they are ideal for codified design.

The derivation of the ATC-58 fragilities is based on two assumptions. First, the probability $P_d(x)$, that is, the probability that a system enters a damage state d under a load of intensity x, is assumed to be a lognormal distribution if viewed as a function of x for any damage state d. Second, the available information is assumed to be sufficient to estimate accurately the parameters of the lognormal model used to define $P_d(x)$. Developments in

ATC-58 do not use mechanical models to calculate fragilities.

Both assumptions used for fragility analysis in ATC-58 are questionable. The selection of the lognormal model seems to be somewhat arbitrary. The calibration of the lognormal fragility model to data without accounting for the statistical uncertainty in the estimated parameters of this model may result in unsatisfactory predications of system performance.

NEES fragility

Limitations of current methods for constructing fragilities motivate our effort to develop alternative techniques for fragility analysis. Since records on a system performance under actual and/or laboratory generated seismic actions are used to construct fragilities, the resulting fragilities cannot be used to assess the seismic performance of the system under loads other than those used in testing. Also, the hypothesis that fragility curves follow lognormal distributions can be too restrictive.

To overcome limitations of current methods for fragility analysis, we develop a Bayesian framework for constructing fragilities, that can account for damage data and information other than data. In the proposed framework, the construction of fragility curves involves the following three steps.

- *Step 1. Mechanical models.* Develop mechanical models of increasing complexity for gypsum partition walls or any other structural systems. The mechanical models have specified functional form but depend on vectors $\theta = (\theta_1, \dots, \theta_q)$ of uncertain parameters.

- *Step 2. Model calibration.* Use available field and/or laboratory data to calibrate the mechanical models developed in Step 1, that is, to find estimates $\hat{\theta}$ or posterior densities of θ.

- *Step 3. Fragility analysis.* Use the calibrated models to calculate fragility curves, that is, probabilities that a structural system enters a specified damage state under an arbitrary seismic action.

The resulting fragilities are satisfactory if the mechanical model used in analysis can capture essential features of a system behavior, there is sufficient information to accurately estimate the model parameters, and the relationship between system response and damage is well understood.

The probability of failure of a rod under a load of intensity x is $P(R \leq x \mid \lambda) = \exp(-\lambda\,(x-\alpha))$, conditional on the value of λ. The posterior fragility can be obtained from this relationship weighted with the posterior density $f''(\lambda)$ of λ, that is,

$$\begin{aligned} P(R \leq x) &= 1(x \geq 0) \int_0^\infty P(R \leq x \mid \lambda)\, f''(\lambda)\, d\lambda \\ &= \left[1 - \left(1 + (x-\alpha)/\mu_p(\xi)\right)^{-s_p(\xi)}\right] 1(x \geq \alpha). \end{aligned} \tag{6}$$

The fragility $P(R \leq x)$ is 0 for $x = \alpha$ and approaches 1 as $x \to \infty$. The plots in the left and right panels of Fig. 3 are *posterior fragilities* corresponding to the prior/posterior

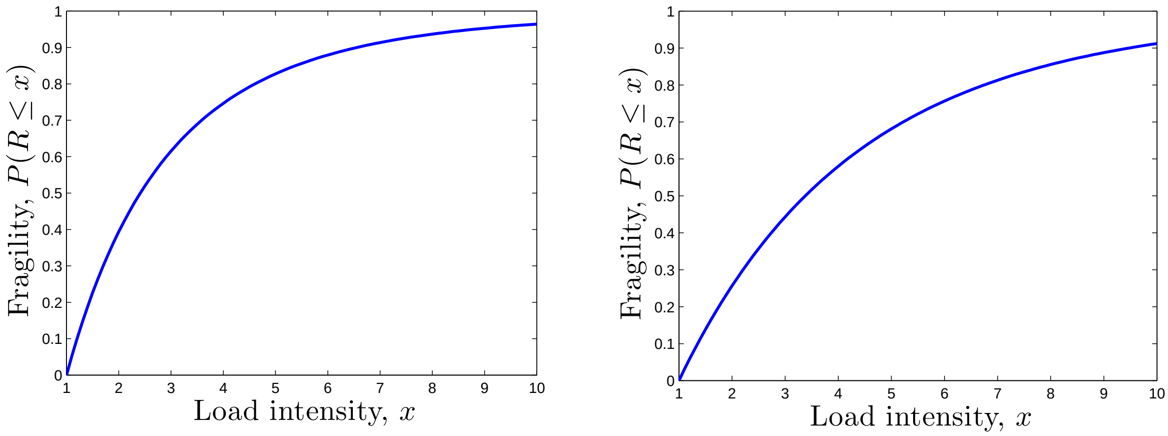

Figure 3: Posterior fragilities $P(R \leq x)$ for the prior/postorior densities in Fig. 1

densities of λ in the corresponding panels of Fig. 1. The additional information provided by the failure loads of the last 7 specimens used to construct the posterior density in the right panel of Fig. 1 reduces the posterior fragility, as can be seen by comparing the posterior fragilities in the two panels of Fig. 3. We also note that confidence intervals on fragility can be constructed simply. For example, the central 90% confidence interval on $P(R \leq x \mid \lambda)$ is $[(F''_h)^{-1}(0.05), (F''_h)^{-1}(0.95)]$, where F''_h denotes the distribution of $P(R \leq x \mid \lambda)$ viewed as a function of λ, a random variable with density f''.

Future work

It is common to construct fragilities by (1) selecting a collection of seismic ground accelerations $\{a_k(t), k = 1, \ldots, n\}$ recorded at the site of interest, (2) scaling the records so that their scaled versions $\{\tilde{a}_k(t), k = 1, \ldots, n\}$ have the same maximum ground acceleration, relevant spectral ordinate, or other ground motion intensity metric, and (3) calculating system damage probabilities, that is, system fragilities, as a function of a scale parameter ξ with respect to the accelerations $\{\xi\, \tilde{a}_k(t), k = 1, \ldots, n\}$.

There are two potential problems with this approach. First, scaling is inconsistent with probability theory in the sense that it changes in a rather arbitrary manned the probability law of the underlying random process describing site seismicity. Second, scaling does not change the frequency content of the records so that resulting fragilities are conditional on ground motions with a rather limited frequency content, that may or may not provide an adequate test for the seismic performance of a particular system.

We consider a hybrid method for constructing system fragilities. The method augments the collection of seismic ground accelerations $\{a_k(t), k = 1, \ldots, n\}$ recorded at a site with site specific simulated ground accelerations generated from a seismological model proposed in (Papageorgiou 1988). It is anticipated that the method will provide useful information on the seismic performance of structural and nonstructural systems since it will test seismic performance under ground motions with a broad range of moment magnitude and frequency content.

Conclusion

A general method has been proposed for constructing fragilities for structural and nonstructural systems subject to earthquakes. The method is based on (1) mechanical models describing system behavior under seismic loads, (2) algorithms for calibrating model parameters to data and prior information developed within the Bayesian framework, and (3) descriptions of site seismicity based on actual and simulated ground motion accelerations. The Bayesian formulation delivers fragility estimates and measures of the uncertainty in these estimates. Calibrated mechanical models provide a rational approach for predicting system behavior under loading conditions different from those use in laboratory experiments or observed in the field

Simple examples have been used to illustrate the implementation of the proposed methodology for model calibration and fragility analysis. The extension of the method to realistic mechanical models for nonstructural systems has been discussed using a mechanical model for gypsum walls. Future research directions have also been summarized.

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