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POLE ASSIGNMENT USING PSO-SA HYBRID ALGORITHM FOR SLIDING MODE CONTROL ON ISOLATED BRIDGES WITH COLUMNS OF IRREGULAR HEIGHT

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ABSTRACT

Bridge structures may be complicated due to various terrain, route alignment, ramps, interchanges, etc. If columns of a bridge are of irregular height but of identical cross section, the induced seismic forces and responses of each column are varied because of the different stiffness of each column. The isolation system has been extensively adopted in bridges to mitigate the induced seismic force. However, the deck displacements of isolated bridges maybe become excessively large under extreme ground motions. Structural control has been shown to effectively mitigate the seismic responses of isolated bridges in the past studies. So far there is no study on the structural control of isolated bridges with columns of irregular height. This research is aimed to study the effectiveness of the sliding mode control for the isolated bridge with the columns of irregular height. It is noted that the sliding surface of the sliding mode control dominates the dynamic behavior of structures. A four-span isolated bridge with columns of irregular height, which is designed based on the balancing-member-stiffness method, is analyzed in this study. Since the target bridge has to be idealized as at least a four-degree-of-freedom system for sufficiently understanding the dynamic behavior, it is needed to determine at least five poles of the sliding surface. Although it is possible to find out an optimal sliding surface by parametric study, the work is tedious and troublesome. The PSO-SA hybrid searching algorithm is adopted to explore the optimum sliding surface of the sliding mode control in this study. Through numerical simulation, the results demonstrate that the sliding mode control with the optimum sliding surface obtained by using the PSO-SA hybrid search method can more effectively decrease the seismic responses of the target bridge than by using the parametric study method.

Introduction

The deck displacements of isolated bridges may become excessively large under extreme ground motions. Structural control has been shown to effectively mitigate the seismic responses of isolated bridges in the past studies (Kawashima and Ruangrassamee 2001, Lee and Kawashima

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2006, 2007). Lee (2005) presented an alternative design method of sliding mode control. The numerical simulation results have indicated that the sliding mode control algorithm presents better control performance than LQR control algorithm in active control of typical isolated bridges which exhibit nonlinear behavior under near-field ground motions. However, there is no study on the structural control of isolated bridges with columns of irregular height so far. Bridge structures may be complicated due to various terrain, route alignment, ramps, interchanges, etc. If columns of a bridge are of irregular height but of identical cross section, the induced seismic forces and responses of each column are varied because of the different stiffness of each column.

A four-span isolated bridge with columns of irregular height, which is designed based on the balancing-member-stiffness method, is analyzed in this study. The sliding mode control is applied to this target bridge for mitigating the deck displacement. The sliding surface which dominates the performance of the sliding mode control can be determined by the LQR method or the pole assignment method. However, since the target bridge has to be idealized as at least a four-degree-of-freedom system for sufficiently understanding the dynamic behavior, it is needed to determine several parameters of the sliding surface.

Particle swarm optimization (PSO) is an evolutionary computation technology based on simulation of simplified social model, such as bird flocking (Eberhart 1995 and Kennedy 1995). Since PSO is implemented easily and efficiently, it has been applied to a number of fields (Kowk et. al 2006 and Leung et. al 2008). However PSO probably converge to a local minimum. Juang and Chuang (2007) proposed a hybrid searching algorithm combining particle swarm optimization and simulated annealing (PSO-SA) to explore optimal solutions for avoiding premature convergence. Simulated annealing (SA) is a local searching algorithm (Kirkpatrick et. al 1983). It can not only accept a near local optimum but also accept a worse solution, so it has the possibility to leave away the local optimum where the solution is entrapped. The performance and reliability can be improved by using a jumping function. In this study, the PSO-SA hybrid searching algorithm is adopted to explore the optimum sliding surface of the sliding mode control.

Through numerical simulation, the results demonstrate that the sliding mode control with the optimum sliding surface obtained by using the PSO-SA hybrid searching algorithm can more effectively decrease the seismic responses of the target bridge than by using the parametric study method. Also the PSO-SA hybrid searching algorithm is more suitable for practical application.

Target Bridge and Analytical Model

A four-span isolated bridge with three columns of irregular height and two abutments (Priestley et al. 1996) as shown in Fig. 1 is studied in this research. The superstructure consists of prestressed concrete box girders and reinforced concrete decks with a total length of 4@50 m=200 m. It is supported by three reinforced concrete columns having identical cross sections. Three columns are 14 m, 7m and 21 m, respectively, in length. The isolators on the top of the columns are designed by using the balancing-member-stiffness method. The balancing-member-stiffness means that the total stiffness of the column and the isolator at each pier is identical. Control devices are installed between the deck and the column to exert control forces. Assume that the deck of the isolated bridge is rigid in the longitudinal direction. The bridge can be idealized as a four degree-of-freedom lump-mass system shown in Fig. 2. The masses of the deck and three columns are 3058 T, 74 T, 34 T and 111 T, respectively. In the past study, isolated bridges to earthquake excitation have been shown to exhibit nonlinear behavior even under control (Lee and Kawashima 2006, 2007). Thus, the isolators and columns are both

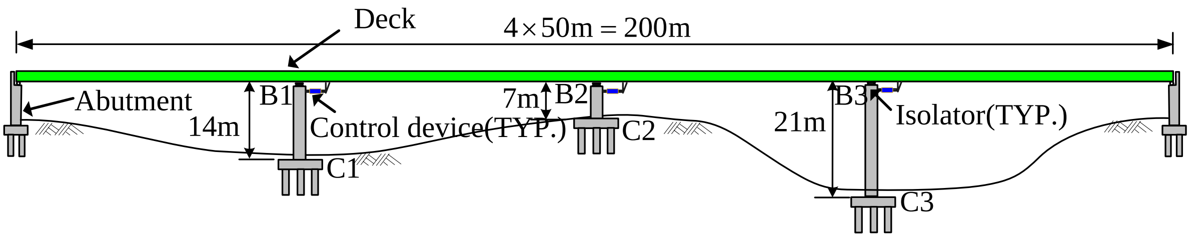

Figure 1. A four-span continuous isolated bridge with columns of irregular height

assumed to be perfect elastic-plastic. The yielding displacement and stiffness of columns and isolators are shown in Table 1. The damping ratios of the first two modes are assumed to be 5%.

Considering that the isolated bridge is subjected to one-dimensional ground acceleration $\ddot{x}_g$, the equations of motion are given by

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{F}[\mathbf{x}(t)] \;=\; \mathbf{H}\mathbf{U}(t)+\boldsymbol{\eta}\ddot{x}_g(t) \tag{1}$$

in which $\mathbf{x}(t)=\left[x_{c1} \quad x_{c2} \quad x_{c3} \quad x_d\right]^T$ is a vector with relative displacements of the columns and deck; $\mathbf{M}$ and $\mathbf{C}$ are mass and damping matrices, respectively; $\mathbf{F}[\mathbf{x}(t)]$ is a vector denoting the nonlinear restoring force as a function of $\mathbf{x}(t)$; $\mathbf{H}$ and $\boldsymbol{\eta}$ are the location matrices of the control devices and ground excitation, respectively.

Control algorithm

Based on a state space formulation, the equations of motion by Eq. 1 can be written as follows:

$$\dot{\mathbf{Z}}(t) \;=\; \mathbf{g}[\mathbf{Z}(t)]+\mathbf{B}\mathbf{U}(t)+\mathbf{E}(t) \tag{2}$$

where $\mathbf{Z}(t)=\left[\mathbf{x}(t) \quad \dot{\mathbf{x}}(t)\right]^T$ is a state vector; $\mathbf{g}\left[\mathbf{Z}(t)\right]$, $\mathbf{B}$ and $\mathbf{E}(t)$ are defined as

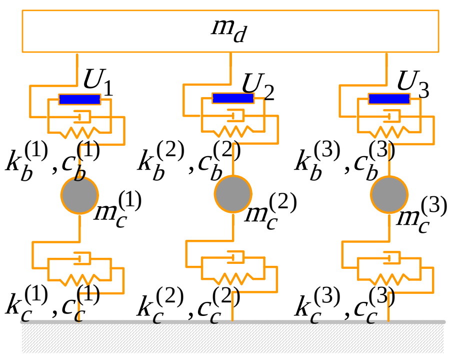

Figure 2. Analytical model

Table 1. The stiffnesses and yield displacements of the isolators and columns

Stiffness (kN/m)		Yield displacement(m)	
$k_b^{(1)}$	16100	$x_{yb}^{(1)}$	0.1304
$k_b^{(2)}$	11500	$x_{yb}^{(2)}$	0.1826
$k_b^{(3)}$	47000	$x_{yb}^{(3)}$	0.0447
$k_c^{(1)}$	30700	$x_{yc}^{(1)}$	0.0814
$k_c^{(2)}$	124400	$x_{yc}^{(2)}$	0.0201
$k_c^{(3)}$	13650	$x_{yc}^{(3)}$	0.1832

$$\mathbf{g}[\mathbf{Z}(t)] = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ -\mathbf{M}^{-1}[\mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{F}[\mathbf{x}(t)]] \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{H} \end{bmatrix};\ \mathbf{E}(t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\boldsymbol{\eta} \end{bmatrix} \ddot{x}_g(t) \tag{3}$$

The control force in sliding mode control is to drive the response trajectory to reach the designated sliding surface and then to maintain it there. It is noted that the sliding surface defined by $\mathbf{S} = \mathbf{0}$ must be stable. Define the sliding surface as a linear function of state vector $\mathbf{Z}$ such that

$$\mathbf{S} = \mathbf{P}\mathbf{Z}(t) \tag{4}$$

The control force can be designed to satisfy a reaching condition or to maintain the response on the sliding surface based on Lyapunov stability criteria

$$\mathbf{S}^T\dot{\mathbf{S}} \le 0 \tag{5}$$

An alternate design method of sliding mode control underlying Eq. (5) proposed by Lee (2005) is used herein. An estimated recursive controller is written as

$$\mathbf{U}(t+\psi) = \mathbf{U}(t) - (\mathbf{PB})^{-1}\mathbf{P}\dot{\mathbf{Z}}(t) - \boldsymbol{\delta}\boldsymbol{\lambda}^T(t) \tag{6}$$

where ψ is sampling time; $\boldsymbol{\lambda} = \mathbf{S}^T\mathbf{PB}$; $\boldsymbol{\delta}$ is a ($r \times r$) diagonal positive-definite matrix with diagonal entries $\delta_1, \delta_2, ..., \delta_r$. When an earthquake excitation is strong, the required control force may be extremely large. Therefore, the control force by Eq. 6 is modulated as

$$U_i^*(t+\psi) = \alpha_i^*[\mathbf{U}(t) - (\mathbf{PB})^{-1}\mathbf{P}\dot{\mathbf{Z}}(t)]_i - \delta_i\lambda_i \tag{7}$$

where α_i^* is a reduced factor of the control force in $0 \le \alpha_i^* \le 1$. In addition, if the estimated control force U_i exceeds the capacity of the control device or may cause large post-yielding displacement, it is necessary to decrease the control force by the factor α_i^* .

PSO-SA hybrid searching algorithm

A constrained optimization method is utilized in this study to search the optimum sliding surface of the sliding mode control for the target bridge under extreme earthquakes. The objective of the structural control is to decrease the deck displacement and the column ductility such that the objective function is designated as

$$f(\mathbf{X}) = W_1 \times \max(|x_d(t)|) + W_2 \times \sum_{\ell=1}^{m} \max(|x_c^{(\ell)}(t)|) \tag{8}$$

where $\mathbf{X} = [X_1, X_2, \ldots, X_n]^T$ consists of n design variables. m is the total number of the columns ; W_1 and W_2 are weighting factors. Constrained functions and the bounds for the design variables are given as

$$g_j(\mathbf{X}) = \frac{\max\left|U_j(t)\right|}{0.4\times(\text{deck weight})} - 1 \le 0, \quad j = 1 \sim m \tag{9}$$

$$g_{m+j}(\mathbf{X}) = \frac{\max(\left|x_c^{(j)}(t)\right|) + \max(\left|x_b^{(j)}(t)\right|)}{\hat{x}_c^{(j)}(t) + \hat{x}_b^{(j)}(t)} - 1 \le 0, \quad j = 1 \sim m \tag{10}$$

$$(X_i)_L \le X_i \le (X_i)_U, \; i = 1 \sim n \tag{11}$$

in which $\hat{x}_c^{(j)}$ and $\hat{x}_b^{(j)}$ denotes the peak column and isolator deformations, respectively, at the jth pier in a uncontrolled system for the basis of comparison; $(X_i)_L$ and $(X_i)_U$ are the lower and upper bonds of the design variable X_i, respectively.

The combination of the objective function and the constrained functions multiplied by penalty parameters leads the fitness function $\tilde{f}(\mathbf{X})$ as

$$\tilde{f}(\mathbf{X}) = f(\mathbf{X}) + \lambda_1 \sum_{j=1}^{m} \max(0, g_j(\mathbf{X})) + \lambda_2 \sum_{j=1}^{m} \max(0, g_{m+j}(\mathbf{X})) \tag{12}$$

where λ_1 and λ_2 are penalty parameters. The optimum solution can be obtained by minimizing the fitness function $\tilde{f}(\mathbf{X})$.

Particle swarm optimization is motivated from the optimization of social behavior of animals, such as bird flocking. In implementation of PSO, a solution of the studied problem is called a particle. The particles exchange information to one another for searching the optimum solution. The population of particles is initialized with random positions and velocities. A flock of p particles is considered in n-dimensional search space. The position and velocity of ith particle are updated according to the following:

$$\mathbf{v}_{k+1}^i = w_a \mathbf{v}_k^i + c_1 r_1 (\mathbf{p}_k^i - \mathbf{X}_k^i) + c_2 r_2 (\mathbf{p}_k^g - \mathbf{X}_k^i); \; \mathbf{X}_{k+1}^i = \mathbf{X}_k^i + \mathbf{v}_{k+1}^i \tag{13}$$

where the subscript k indicates the generation number and the superscript i indicates the ith particle; $\mathbf{v}$ and $\mathbf{X}$ are the velocity and position vectors of the particle, respectively; $\mathbf{p}_k^i$ is the best position vector of the ith particle prior to the (k+1)th generation while $\mathbf{p}_k^g$ is the global best position vector prior to the (k+1)th generation in the swarm; r_1 and r_2 are uniform random factors varying in the range of [0,1]; c_1 and c_2 are positive acceleration constants. w_a is an inertia weight presented by Eberhart and Shi (1998).

Fourie and Groenwold (2002) proposed a rule to dynamically decrease the inertia weight and maximum velocity if there is no more improved solution after running h consecutive generations. The rule is written as

$$\text{if } \; \tilde{f}(p_k^g) \ge \tilde{f}(p_{k-h}^g), \text{ then } w_{k+1} = \alpha w_k \text{ and } \mathbf{v}_{k+1}^{\max} = \beta \mathbf{v}_k^{\max} \tag{14}$$

where α and β are the decreased factors between 0 and 1. The initial maximum velocity $\mathbf{v}_0^{\max}$ is defined as

$$\mathbf{v}_0^{\max} = \gamma(\mathbf{X}_U - \mathbf{X}_L) \tag{15}$$

in which $\mathbf{X}_U$ and $\mathbf{X}_L$ are the upper and lower bond vectors of a particle, respectively; γ is a fraction to decrease the initial search space. Moreover, Juang and Chuang (2007) proposed a re-initialized rule to avoid insignificant maximum-velocity $\mathbf{v}^{\max}$ as

$$\text{if } \mathbf{v}_k^{\max} \le 0.001, \text{ set } \mathbf{v}_{k+1}^{\max} = \mathbf{v}_0^{\max} \tag{16}$$

The global best solution is of importance to guide the search direction. If the global best solution is entrapped in a local optimum solution, the searching efficiency will decrease. In order to prevent the solution of the PSO from converging to a local minimum, Juang and Chuang (2007) developed the PSO-SA hybrid searching algorithm combining the particle swarm optimization and simulated annealing. The SA provides a possibility to jump and leave away from a local minimum.

If the global best solution is not improved after h consecutive generations, a new solution $\hat{\mathbf{p}}_k^g$ near the global best solution is calculated by using two uniform random numbers R_1 and R_2 in the range [0,1] as

$$\text{if } R_1 > 0.5\,,\ \hat{\mathbf{p}}_k^g = \mathbf{p}_k^g + R_2(\mathbf{X}_U - \mathbf{p}_k^g);\ \text{otherwise, } \hat{\mathbf{p}}_k^g = \mathbf{p}_k^g - R_2(\mathbf{p}_k^g - \mathbf{X}_L) \tag{17}$$

Whether the new solution can be accepted or not is determined by the Boltzmann probability factor as

$$P = \exp(-\Delta\tilde{f}/KT) \tag{18}$$

where $\Delta\tilde{f}$ is the difference of the fitness value between the new solution $\hat{\mathbf{p}}_k^g$ and the current global best solution $\mathbf{p}_k^g$; K is Boltzmann's constant and T is the temperature.

If $\Delta\tilde{f}$ is smaller than or equal to zero, the new solution $\hat{\mathbf{p}}_k^g$ is accepted to be the new global best solution. If $\Delta\tilde{f}$ is larger than zero, generate a uniform random number between 0 and 1 and compare it with the Boltzmann probability factor P. If the generated random number is less than P, the new solution is accepted to be the new global best solution, or the current global best solution remains.

In processing SA, the initial temperature T_{start} and the final temperature T_{end} are given in advance. At each search, the temperature T_i is updated by multiplying the previous temperature T_{i-1} by a reduction factor T_{red}. The search continues until the temperature reaches the final temperature. Then the final solution is regarded as a new global best solution.

Numerical simulation and results

The PSO-SA hybrid searching algorithm is adopted herein to explore the optimum sliding surface of the sliding mode control for the target bridge. The bridge is subjected to a near-field ground motion recorded at JMA Kobe Observatory in the 1995 Kobe, Japan earthquake. The poles of the sliding surface determine the characteristics of structural dynamic behavior. In this study, totally five poles of the sliding surface for the target bridge are assigned directly, which is called the pole assignment method.

Assume that the poles are two conjugate complex numbers and three real numbers: $-\xi\omega \pm i\omega_d$, $-s_1$, $-s_2$ and $-s_3$ where ω and $\omega_d = \omega\sqrt{1-\xi^2}$ are undamped and damped frequencies, respectively, and ξ is the modal damping ratio. Therefore five parameters of the poles are the design variables, including $\omega, \xi, s_1, s_2, s_3$, in PSO-SA hybrid searching algorithm. In addition, α_i^* by Eq.7 is also considered as a design variable. The bounds for the design variables are given as follows:

$$0.1 \le f = 2\pi/\omega \le 10 \text{ ; } 0.05 \le \xi \le 0.99; \ 1 \le s_1,\ s_2,\ s_3 \le 1000 \text{ ; } 0.01 \le \alpha_i^* \le 1 \tag{19}$$

The design parameters of the PSA-SA hybrid searching algorithm are shown in Table 2. The weighting factors of the objective function are W_1 = 2 and W_2 = 0.5. The penalty parameters of the fitness function are $\lambda_1 = 10^3$ and $\lambda_2 = 10^9$.

Figure 3 shows the global best fitness values of all particles for ten searches. It is observed that the fitness value decreases as the number of generation increases and then converges before 500 generations. In the case of the best solution, the poles are $-1.66 \pm 6.68i$, -297, -303 and -74, and the restricted parameter α_i^* of control force is 0.5153.

Figure 4 shows the forces exerted by three control devices in the best case of the controlled system with the PSO-SA hybrid searching algorithm. Figure 4 also compares the deck displacement in the uncontrolled system and the controlled system with the PSO-SA search. As observed in Fig.4, the deck displacement decreases significantly under control. The hysteretic loops of the isolators and columns in the best controlled case and in the uncontrolled case are presented in Fig.5. It is found that all isolators and columns except for column 2 exhibit nonlinear behavior in the uncontrolled system while isolator 3 and column 2 clearly undergo nonlinear behavior in the controlled system.

The best and worst results in ten searches are compared with the results by the parametric study method (Lee and Chen 2008). Table 3 shows the fitness values, peak forces and peak responses in the uncontrolled system, the active controlled systems with the parametric study and the PSO-SA search. The fitness value, deck displacement and isolator deformations decrease dramatically in all controlled systems. The best case of the PSO-SA presents the best control

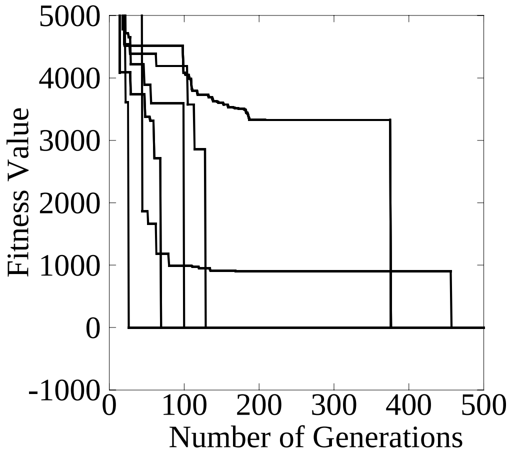

Figure 3. The fitness values

Table 2. Parameters of PSO-SA hybrid optimization method

parameters	value	parameters	value
k_{max}	500	h	3
N (population size)	20	w_{max}	1.4
c_1	0.5	w_{min}	0.8
c_2	1.6	T_{start}	450
γ	1	T_{red}	0.97
α	0.99	T_{end}	300
β	0.95		

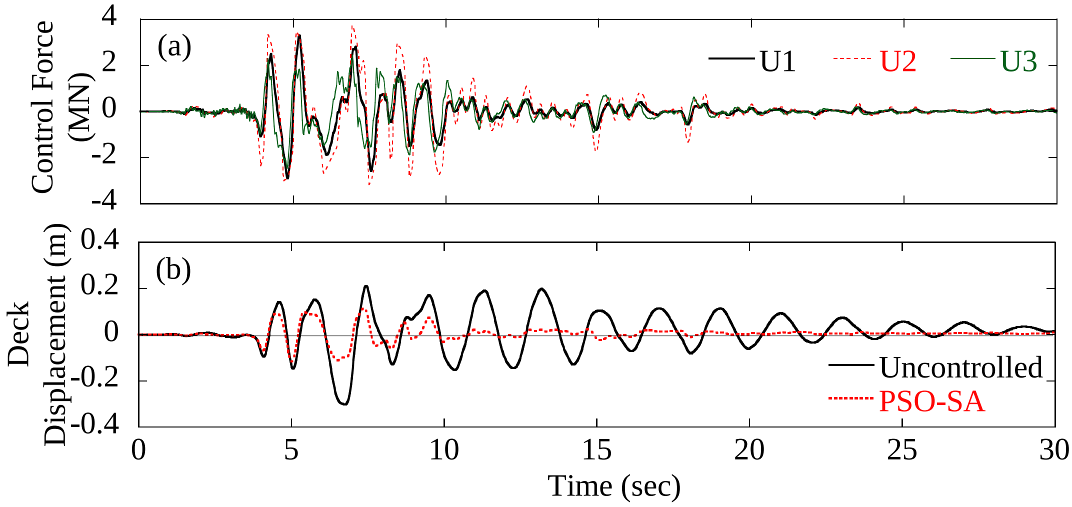

Figure 4. (a) Control force and (b) comparison of the deck displacement with and without control subjected to JMA Kobe ground motion

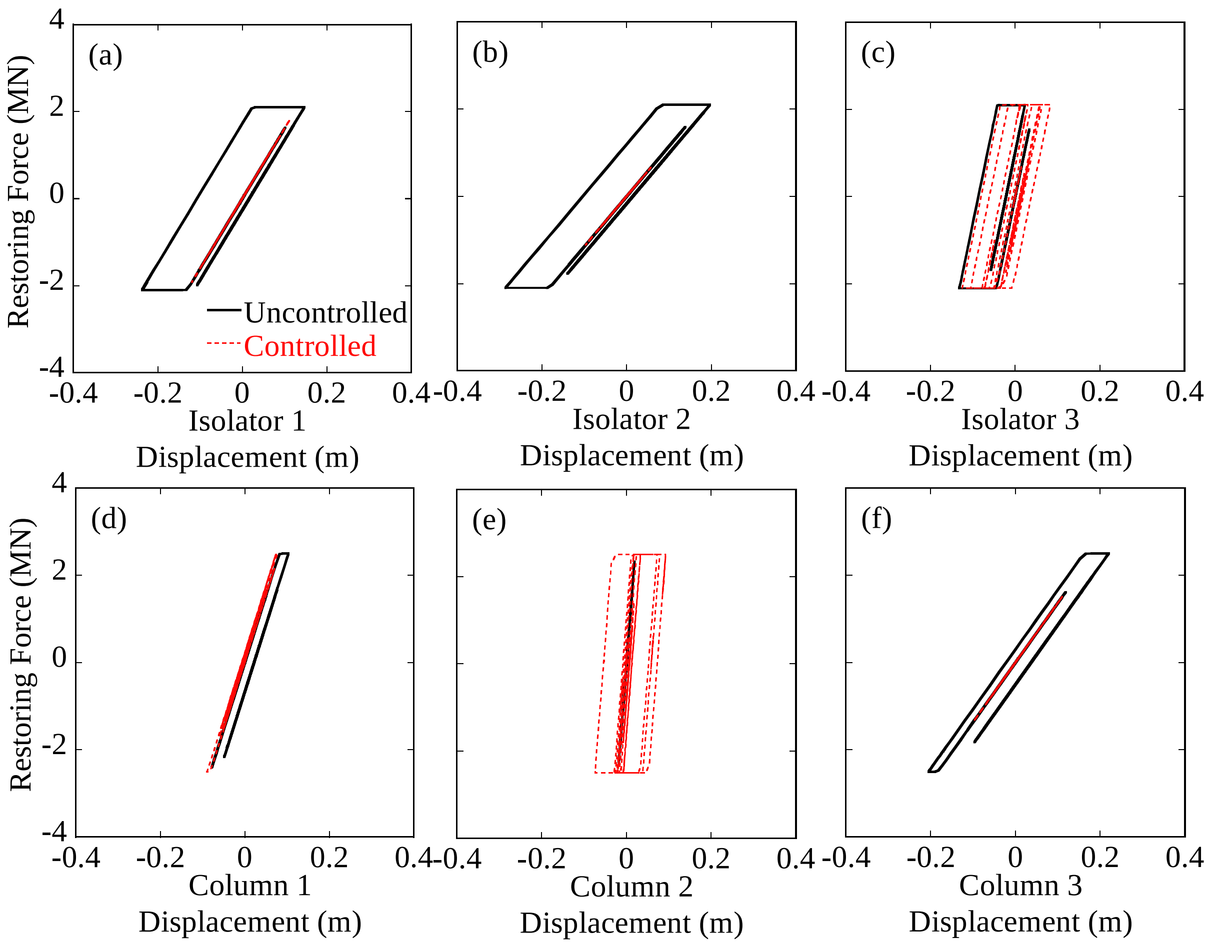

Figure 5. Hysteresis loop of (a) isolator 1 (b) isolator 2 (c) isolator 3 (d) column 1 (e) column2 (f) column 3

Table 3. Summary of the peak control forces and peak responses

Fitness value, peak force (kN) and responses (m) (1)	Uncontrolled (2)	Parametric study (3)	Best solution (PSO-SA method) (4)	Worst solution (PSO-SA method) (5)
Fitness value (m)	0.777	0.469	0.382	0.411
Control force U1		2339	3322	3069
Control force U2		3135	3745	4374
Control force U3		1130	2495	2813
Total control force		6222	8841	8944
Deck displacement	0.303	0.148	0.117	0.147
Isolator I1 deformation	0.237	0.098	0.119	0.146
Isolator I2 deformation	0.286	0.059	0.097	0.183
Isolator I3 deformation	0.132	0.027	0.125	0.200
Pier P1 deformation	0.103	0.094	0.089	0.058
Pier P2 deformation	0.019	0.123	0.093	0.070
Pier P3 deformation	0.219	0.128	0.112	0.107

performance on the deck displacement. The peak column displacements in both cases by the PSO-SA search are smaller than those in the case by the parametric study. The results reveal that the PSO-SA hybrid searching algorithm is superior to explore the optimum sliding surface of the sliding mode control such that the seismic responses of the isolated bridge can be significantly mitigated under the structural control.

Conclusions

The effectiveness of the sliding mode control for the isolated bridge with the columns of irregular height is studied. A four-span isolated bridge with three columns of irregular height and two abutments is analyzed. The PSO-SA hybrid searching algorithm is adopted to explore the optimum poles of the sliding surface, which dominates the dynamic behavior of structures, of the sliding mode control. The objective of the structural control is to mitigate the deck displacement and column deformation when the target bridge is subjected to strong earthquakes. Numerical simulations are carried out to understand the control performance of the sliding mode control with the sliding surface whose poles are determined by the PSO-SA hybrid searching algorithm. The analytical results are also compared to those with the sliding surface whose poles are determined by the parametric study method. The simulation results reveal that the sliding mode control can significantly decrease the seismic responses of the isolated bridge with columns of irregular height. The sliding mode control with the optimum sliding surface obtained by using the PSO-SA hybrid searching algorithm can more effectively decrease the seismic responses of the target bridge than by using the parametric study method. Also the PSO-SA hybrid searching algorithm is simpler and more suitable for practical application.

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