



## DISPLACEMENT BASED SEISMIC DESIGN OF CONCRETE WALL FRAME STRUCTURES

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### ABSTRACT

Displacement-based method of seismic design is an important step in designing structures to meet specific performance objectives. In the methodology of displacement-based design presented here, simple empirical relations are used to estimate the ultimate and yield displacements, which determine the ductility demand in the structure. The ultimate displacement is calculated so that the damage to non-structural failure is controlled, the local ductility demands in the elements are less than their ductility capacities, and the structure remains stable under P- $\Delta$  effect. For a multi-storey building, the properties and the yield and ultimate displacements of an equivalent single-degree-freedom system are obtained on the basis of an assumed displacement shape. The seismic demand is represented by the site-dependent inelastic uniform hazard spectrum for the calculated ductility. The design base shear is determined from this spectrum and the ultimate displacement. The structure is designed for the effect of design base shear. Refined estimates of yield displacement, ultimate displacement, and base shear strength are now obtained by carrying out pushover analysis of the structure and moment-curvature analysis of the wall cross section. The process is repeated until convergence. Finally, a multi-mode pushover analysis is carried out to obtain better estimates of the design shear in the members. The method is applied to the design of several reinforced concrete wall frame buildings. Nonlinear time history analyses of the designed structures for their responses to a series of spectrum compatible ground motions show that the proposed method is promising and provides reasonable estimates of the demand roof displacement, inter-storey drifts and storey shears.

### Introduction

Experience of damage caused to structures, with the attendant loss of life and property, during past earthquakes has led to the recognition that a performance-based approach must be used in the seismic design of structures. In practical application of this approach, it is usual to specify a set of discrete performance levels, ranging from fully operational to near collapse, which the structure would be required to meet under specified levels of earthquake hazard. The earthquake hazard is determined from a probabilistic seismic hazard analysis and expressed in terms of the annual frequency of exceedance or the return period. Quantitative performance levels are defined through limiting values of measurable response parameters, such as storey drifts, floor velocities and accelerations, element deformation and ductility demands, and damage indices. For

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the present study, we focus on structural and nonstructural damage. Because element deformations and ductility demands can be related to storey displacements and drifts, it is evident that both the structural and nonstructural damage could be controlled by limiting the storey drifts and displacements.

A significant amount of research on displacement-based seismic design (DBSD) has been carried out over the past 15 years. The essential concepts of DBSD were developed by Freeman and others (Freeman et al, 1975) and later refined by Priestley and Calvi (1997). The method proposed by these authors is known as the capacity spectrum method or the method of equivalent linearization. Fajfar (1999) and Chopra and Goel (2001) presented an alternative to the equivalent linearization method in which the demand was expressed by an inelastic spectrum and the capacity by the realistic force displacement relationship. In this study, we adapt this alternative method for application in the design of new structures of regular and symmetric layout in which lateral resistance is provided by a combination of concrete shear walls and frames. The DBSD method presented here is combined with multi-mode pushover analyses (Chopra and Goel 2002) to account for the higher mode effects.

### Displacement Based Seismic Design

Humar (2008) has outlined the essential steps in the displacement based seismic design method presented in this paper. They are suitably modified here for wall frame structures. The method is based on obtaining approximate estimates of the displacement at yield and the acceptable ultimate displacement.

#### Yield Displacement

In a wall frame system the lateral displacement at first yield at the base of the wall can be obtained by assuming that the curvature as well as the moment in the wall varies in the shape of a triangle from zero at inflection height above base,  $h_e$ , to a maximum at the base, and that for  $h > h_e$  the moment can be taken as negligible. The displacement at level  $i$  corresponding to the first yield in the wall,  $\Delta_{yi}$ , is then given by (Sullivan et al 2006)

$$\Delta_{yi} = \frac{\phi_y h_i^2}{2} - \frac{\phi_y h_i^3}{6h_e} \text{ for } h \leq h_e \quad (1)$$

$$\Delta_{yi} = \frac{\phi_y h_e h_i}{2} - \frac{\phi_y h_e^2}{6} \text{ for } h > h_e$$

where  $\phi_y$  is the effective yield curvature of the wall, which can be obtained by using one of the empirical relations available in the literature, for example that by Paulay (2002).

$$\phi_y = 1.8\varepsilon_y / l_w \quad (2)$$

In Eq. 2  $\varepsilon_y$  is the yield strain of reinforcing steel and  $l_w$  is the length of the wall.

The inter-storey yield displacement of the frames,  $\theta_{fy}$  is obtained from (Priestley 2003)

$$\theta_{fy} = 0.5\varepsilon_y l_b / D_b \quad (3)$$

where  $l_b$  is the span and  $D_b$  the depth of the beam. Assuming that all storeys yield simultaneously, the corresponding displacement of the frame at roof is given by

$$\Delta_{fyr} = H \times \theta_{fy} \quad (4)$$

where  $H$  is the total height of the structure. The designer now has the freedom to assign reasonable proportions to the base shear carried by the walls and the frames. The global yield displacement is then calculated from

$$\Delta_y = \frac{V_b}{V_w / \Delta_{yr} + V_f / \Delta_{fyr}} \quad (5)$$

where  $V_b$  is the total base shear and  $V_w$ , and  $V_f$  are the wall and frame strengths, respectively.

### Ultimate Displacement

The acceptable ultimate displacement is the smallest of the following:

- 1) The roof displacement corresponding to inter-storey drift limit specified in the codes (assumed here as 0.025) to prevent excessive structural and non-structural damage. The total displacement at level  $i$ ,  $\Delta_{ui}$  is given by

$$\Delta_{ui} = \Delta_{yi} + (h_i - l_p / 2) \times (0.025 - 0.5\phi_y h_e) \quad (6)$$

where  $l_p$  is the plastic hinge length, which for walls varies between 0.3 to 0.8 times the wall length (Paulay and Priestley 1992). Here we will assume  $l_p$  as being half of the wall length. This would provide a conservative estimate of the acceptable ultimate displacement.

- 2) The roof displacement at which the ductility demand in any element of the structure reaches its capacity. As an example, the wall displacement at which its curvature demand reaches its capacity can be calculated as

$$\Delta_{ui} = \Delta_{yi} + (h_i - l_p / 2) \times (\phi_u - \phi_y) \times l_p \quad (7)$$

where  $\phi_u$  is the wall curvature capacity, which depends on whether or not the concrete is confined. It is often difficult and expensive to provide confinement to the longitudinal steel in a shear wall. Consequently, in practice, concrete is considered as being unconfined and the local ductility capacity is obtained from maximum acceptable concrete compression strain value for unconfined concrete. An advantage of performance based design is that the designer has the freedom to choose the acceptable strain; the acceptable ultimate displacement is then computed on the basis of the selected strain limit.

- 3) The roof displacement beyond which instability may be caused by P- $\Delta$  effect.

In preliminary design only the first of the above three limits may be available; however, for subsequent design iterations more accurate estimates of the ultimate displacement can be obtained considering both the ductility capacity and the P- $\Delta$  effect.

## Equivalent SDOF Model

A multi-degree-of-freedom (MDOF) must be modeled by an equivalent SDOF system. For that purpose, a deformed shape for the structure should be available. In preliminary design, this could be the ultimate displacement estimate at each level; however, for subsequent iterations, the first mode shape of structure may be used. Assuming vector  $\boldsymbol{\varphi}$  to be the deformed shape the dynamic parameters are calculated as follows:

$$\Gamma = \frac{\boldsymbol{\varphi}^T \mathbf{M} \mathbf{1}}{\boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi}}, \quad M^* = \frac{(\boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi})^2}{\boldsymbol{\varphi}^T \mathbf{M} \mathbf{1}} \quad (8)$$

where  $\mathbf{M}$  is the mass matrix,  $\mathbf{1}$  is a unit vector,  $\Gamma$  is the participation factor and  $M^*$  the effective mass. The ultimate and yield displacements of the SDOF system are obtained on dividing the ultimate and yield displacement of MDOF by  $\Gamma$ .

## Inelastic Demand Spectra

The seismic demand is represented by an inelastic spectrum corresponding to the ductility capacity  $\mu$  and is obtained from the elastic uniform hazard spectrum for the site. The demand spectrum is expressed in the form of spectral acceleration versus spectral displacement (ADRS). The peak seismic displacement demand imposed in the inelastic system is determined from

$$D = \frac{\mu}{R_y} \left( \frac{T_n}{2\pi} \right)^2 A \quad (9)$$

where  $D$  is the spectral displacement,  $A$  is the elastic spectral acceleration at the natural period  $T_n$ , and  $R_y$  is the yield reduction factor. The construction of ADRS spectrum requires the specification of a  $R_y$ - $\mu$ - $T_n$  relation. The following relationship proposed by Krawinklar and Nassar (1992) is used here, but other empirical relations available in the literature may be used.

$$c = \frac{T_n^a}{1 + T_n^a} + \frac{b}{T_n} \quad (10)$$
$$R_y = [c(\mu - 1) + 1]^{1/c}$$

where  $a$  and  $b$  are parameters that depend on the nature of force-displacement relationship. For an elasto-plastic systems  $a = 1$ , and  $b = 0.42$ .

## Capacity Diagram and Pushover Analysis

The spectral acceleration  $A_y$  corresponding to the acceptable ultimate displacement of the equivalent SDOF system is obtained from the inelastic acceleration displacement response spectrum. The design base shear is then given by  $V = M^* A_y$ .

This base shear is distributed over the height of building in proportion to  $\mathbf{M}\boldsymbol{\varphi}$  to find out the member forces and the members are designed for these forces. A modal analysis may now be carried out to obtain the mode shapes and frequencies. A nonlinear pushover analysis including the P- $\Delta$  effect is carried out next for lateral forces distributed in proportion to the vector  $\mathbf{M}\boldsymbol{\varphi}_1$ .

The resulting pushover curve is idealized by a bi-linear curve to estimate the yield displacement. The limit on the ultimate displacement imposed by the ductility capacity of the wall is determined from the moment-curvature relationships for the walls. The limit on ultimate displacement to prevent instability due to P- $\Delta$  effect can be determined by measuring from the pushover curve the displacement corresponding to a 5% to 10% decrease in the strength of the structure from its peak value. Based on the updated displacement estimates a new value of the ductility capacity is determined and a new inelastic spectrum is constructed. If the base shear obtained from the revised spectrum is close to that in the previous iteration, the design is considered satisfactory, if not, the process is repeated.

## Displacement Based Seismic Design Procedure

The steps in displacement-based design of structures can be summarized as follows:

1. For preliminary design of the wall frame structure, calculate the yield displacement and the displaced shape using Eqs. (1) through (5).
2. Calculate the acceptable ultimate roof displacement considering the three limiting criteria: inter-storey drift, member ductility capacity, and prevention of instability under P- $\Delta$ , using Eqs. (6) and (7) and pushover analysis (member ductility capacity and instability criteria may not be available at the preliminary design stage).
3. Obtain the properties of the equivalent SDOF system using an assumed displacement shape, such as that obtained in step 1, and calculate its yield and ultimate displacements.
4. Construct the inelastic ADRS for the ductility demand obtained from Step 3, determine the spectral acceleration corresponding to the ultimate displacement and calculate the base shear. Distribute the base shear in proportion to the assumed displaced shape and determine the member forces. Design the structure for such forces.
5. Carry out nonlinear pushover analysis for a force distribution proportional to the assumed displaced shape. Based on the results of pushover analysis and moment-curvature relationships of the designed sections, update the yield and ultimate displacement estimates.
6. Repeat steps 3 through 5 until the design base shear converges. At this stage the calculated base moment in the wall can be used as an acceptable estimate for the purpose of design; however, a multi-mode pushover analysis needs to be carried out to obtain better estimates of the shears in walls and frames.

## Case Studies

A series of 6, 12, and 20 storey hybrid wall frame buildings whose plan is shown in Figure 1 are designed based on the proposed DBSD procedure to examine the effectiveness of the method. The height of first storey is 4.85 m; the other stories are each 3.65 m in height. The lateral resistance in the North-South direction is provided by two shear walls and two frames as shown in Figure 1. Dimensions of the walls, columns and beams for all three buildings are shown in Table 1. The dead load on all floors and the roof is 5.8 kN/m<sup>2</sup>. The live load is 2.4 kN/m<sup>2</sup>. The snow load on roof is 2.2 kN/m<sup>2</sup>. The strength of concrete is 30 MPa and the steel yield strength is 400 MPa. The buildings are located in the city of Los Angeles and are designed for an earthquake with 10% chance of exceedance in 50 years represented by the target response spectrum shown in Figure 2. The structures are designed for earthquake forces in the N-S direction neglecting the accidental torsion effect.

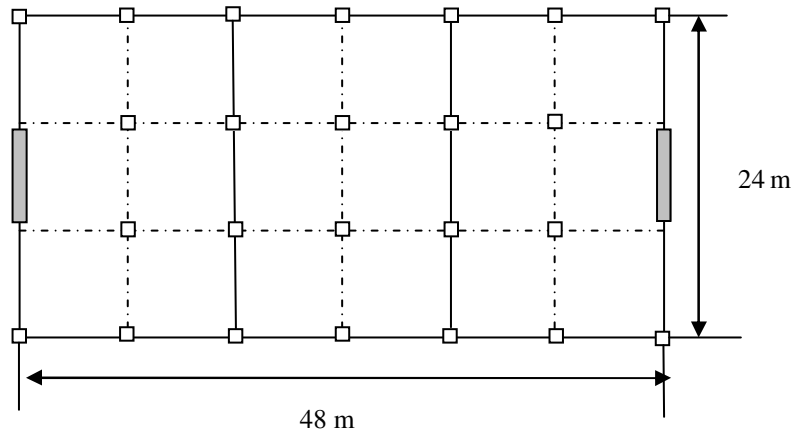


Figure 1: Typical plan view of the wall frame buildings

Table 1: Characteristics of the wall frame buildings

	<b>6-storey</b>	<b>12-storey</b>	<b>20-storey</b>
Beams width x depth (mm)	400 x 500	400 x 600	400 x 700
Interior columns (mm)	600 x 600	700 x 700	800 x 800
Exterior columns (mm)	600 x 600	700 x 700	800 x 800
Walls length (mm)	5000	6000	8000
Walls thickness (mm)	400	400	400
Interior columns gravity load DL+0.5LL (kN)	2846.4	5903.6	10258.2
Exterior columns gravity load DL+0.5LL (kN)	1531.7	3222.9	5695.4
Walls gravity load DL+0.5LL (kN)	3708.3	7856.4	14571.3

It is assumed that the concrete is unconfined and the ultimate capacity of a section is reached when concrete strain is 0.004. In the preliminary design, the ultimate displacement limit is assumed to be that corresponding to the inter-storey displacement of 0.025 times the storey height. It is observed that in the subsequent iterations the ductility capacity governs the acceptable ultimate displacement. In the first two iterations, the displaced shape is assumed to be that given by Eq. 1. As evident from the data presented in Table 2, for all three buildings, the design base shear converges within two iterations. The corresponding demand and capacity curves are shown in Figure 3. In the final iteration, the displaced shape is assumed to be that given by the first mode shape and estimates of the yield and ultimate displacements are obtained from a pushover analysis using a first mode force distribution and from a moment-curvature analysis of the wall section. In calculating the ultimate displacement, the limit corresponding to a 5% decrease in ultimate strength under the P-Delta effect is also taken into account. Such limits are shown in Figure 4. The idealized curve in Figure 4 is obtained by drawing a horizontal line through the 95% value of the maximum strength and then connecting it by a straight line to the origin such that the positive and negative areas between the idealized curve and the actual pushover curve are equal. It is found that the limit imposed by the ductility capacity of unconfined concrete wall section governs in each case, and the strength provided in the second iteration is found to be adequate.

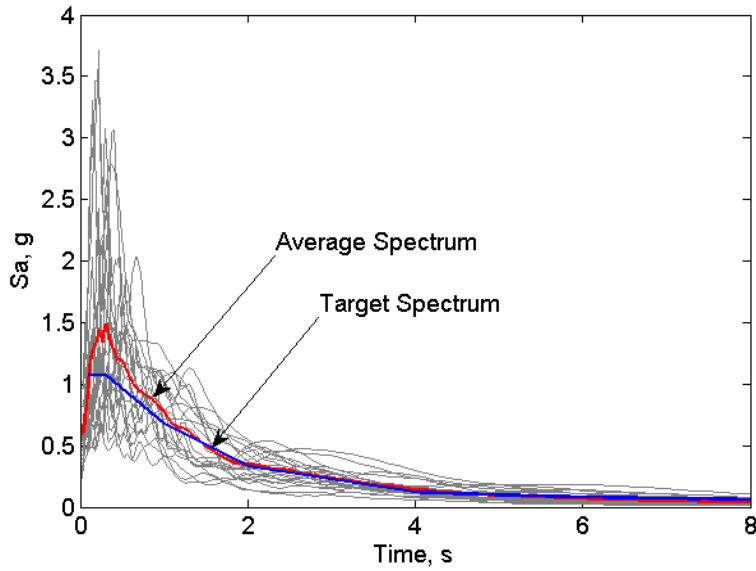


Figure 2: Design response spectrum and average of the spectra of 20 selected earthquakes

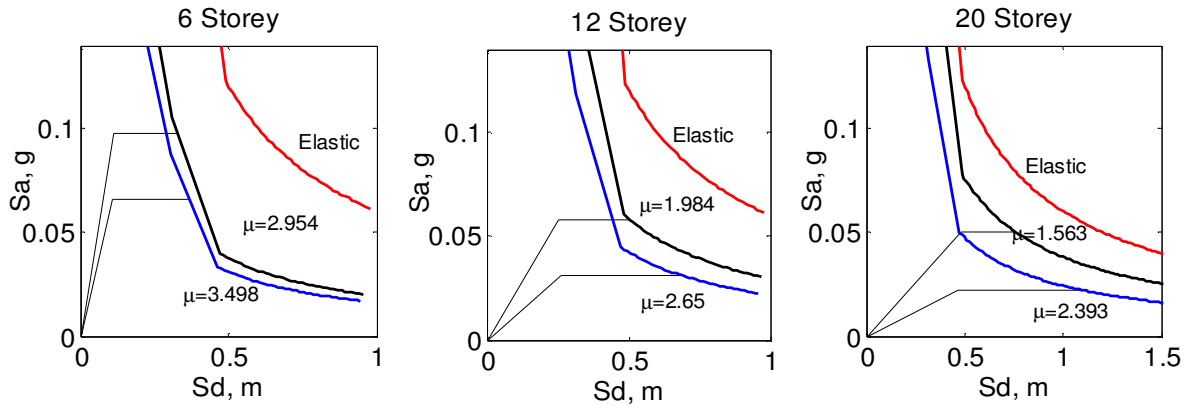


Figure 3: Demand and capacity curves for the preliminary design and the final iteration

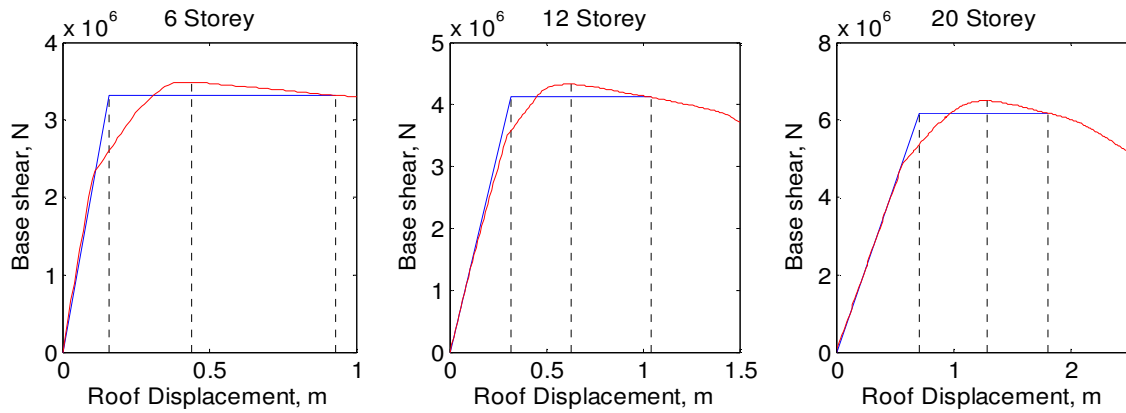


Figure 4: Pushover curves and idealized force-displacement relations

Table 2: DBSD iterations

		he (m)	$\Delta y$ (m)	$\Delta u$ (m)	$\mu$	Sa (g)	V (kN)
<b>6 Storey</b>	preliminary	18.10	0.147	0.515	3.498	0.0658	2632.2
	1st iteration	18.44	0.143	0.463	3.246	0.0863	3444.7
	2nd iteration	18.46	0.141	0.46	3.256	0.0869	3469.1
	Pushover	-	0.157	0.464	2.954	0.0977	3527.9
<b>12 Storey</b>	preliminary	30.40	0.381	1.009	2.65	0.0310	2469.6
	1st iteration	31.69	0.387	0.772	1.993	0.0552	4325.9
	2nd iteration	32.07	0.391	0.773	1.977	0.0557	4361.9
	Pushover	-	0.37	0.734	1.984	0.0581	4233.1
<b>20 Storey</b>	preliminary	48.00	0.691	1.654	2.393	0.0219	3012.2
	1st iteration	49.64	0.713	1.194	1.675	0.0442	5928.9
	2nd iteration	50.35	0.724	1.207	1.667	0.0439	5890.4
	Pushover	-	0.707	1.106	1.563	0.0506	6487.9

Table 3: Dynamic properties for multi-modal pushover analyses

		First Mode	Second Mode	Third Mode	Fourth Mode	SRSS 3 Modes	SRSS 4 Modes
6 Storey	Period (sec)	2.111	0.377	0.137	0.070		
	M*/M <sub>total</sub>	0.7082	0.1961	0.0625	0.0237		
	Target Disp. (m)	0.464	0.020	0.0012	0.00013		
	Base shear (kN)	3,410	6,827	2,988	256.6	8,195	8,199
	Roof Disp. (m)	0.464	0.0135	0.0012	0.00013	0.4602	0.4602
12 Storey	Period (sec)	4.150	0.825	0.310	0.160		
	M*/M <sub>total</sub>	0.6805	0.1768	0.0656	0.0331		
	Target Disp. (m)	0.734	0.090	0.0092	0.0015		
	Base shear (kN)	4,329	9,003	7,581	4,127	12,540	13,202
	Roof Disp. (m)	0.734	0.0536	0.0092	0.0015	0.7320	0.7320
20 Storey	Period (sec)	6.347	1.320	0.508	0.264		
	M*/M <sub>total</sub>	0.6726	0.1648	0.0636	0.0334		
	Target Disp.	1.1057	0.1786	0.0255	0.0051		
	Base shear (kN)	6,416	11,682	11,340	6,955	17,499	18,831
	Roof Disp. (m)	1.100	0.1106	0.0255	0.0051	1.106	1.106

To obtain internal forces in the members, multi-mode pushover analyses are carried out. Table 3 shows the modal periods and the target displacements calculated for MPA. It is observed that all three building structures become inelastic in the second mode pushover analysis. In addition, the roof displacement is observed to change direction as the structure is pushed under second mode load distribution. Therefore, in order to capture the inelastic response in second mode, target displacement is calculated at a mid-height floor level where displacement increases monotonically with the lateral load. The target roof displacements for second mode, shown in



Table 3, are obtained from the demand capacity diagrams using iterations to find the performance point.

Table 3 also shows the modal contributions to the base shear and roof displacement as well as the SRSS combinations of the modal values. It is seen that the higher modes make very substantial contributions to the base shear; however, the first mode pushover analysis can estimate the roof displacement quite accurately.

In order to assess the performance of the proposed method, nonlinear time history analyses are carried out on each building for a set of 20 ground motion records developed by Somerville et al (1997). These records were obtained by scaling 20 real recorded motions so that they closely matched the Los Angeles design spectrum for 10% probability of exceedance in 50 years. The scale factor for a record was determined such that the weighted squared error between the spectral values of the record and the design spectrum at periods of 0.3, 1.0, 2.0 and 4.0 second using the weights of 0.1, 0.3, 0.3 and 0.3, respectively, was a minimum. The ground motions and their scaling factors are available in the cited reference. Figure 2 shows the elastic spectra for the 20 ground motion records along with their average and the design spectrum.

The estimated storey shears for all three buildings are close to the average of the four nonlinear dynamic analyses. The displacements and drifts are estimated fairly accurately for 6-storey building; however they are overestimated for 12-storey and 20-storey structure (see Figure 5).

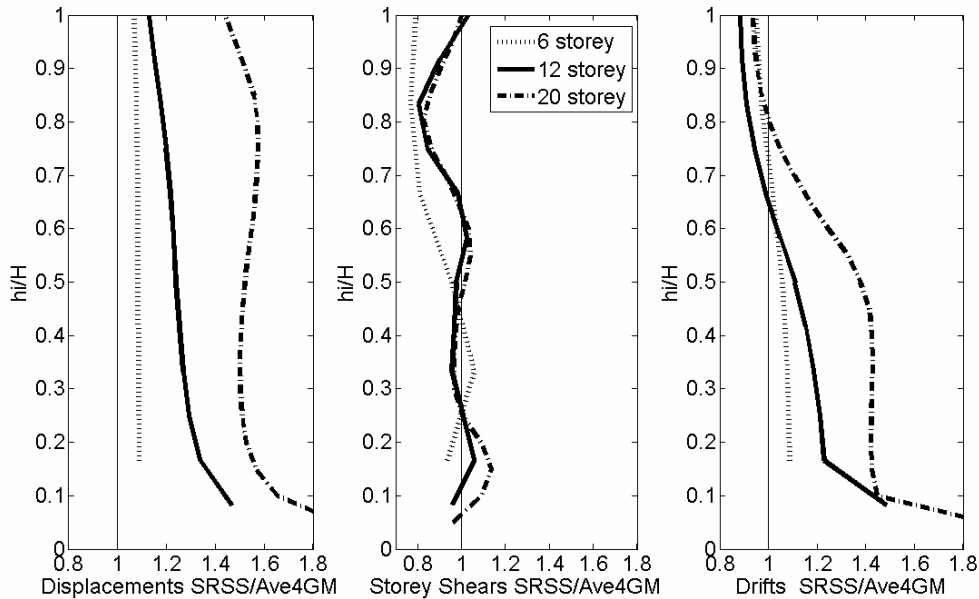


Figure 5: Comparison of SRSS of 4-mode pushover analyses with the average of nonlinear dynamic analyses

### Conclusions

The proposed design procedure performs appropriately in seismic design of hybrid wall frame structures. It is observed that the design base shear converges within a few iterations;

however, the multi-mode pushover analysis reveals that higher modes contribute significantly in the storey shears although they make fairly small contribution to inter-storey drifts and displacements. Nonlinear time history analyses of the designed structures for their responses to a series of spectrum compatible ground motions show that the proposed method is promising and provides reasonable estimates of the demand roof displacement, inter-storey drifts and storey shears.

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