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COMPARSION OF ESTIMATED SEISMIC DEMAND THROUGH PROBABILISTIC AND INCREMENTAL DYNAMIC ANALYSIS

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ABSTRACT

In this paper, as a part of probabilistic seismic demand estimation within the concept of performance based earthquake engineering using a Bayesian regression analysis, the mean and standard deviation of demand model parameters are estimated based on two different strategies. First direct Nonlinear Dynamic Analysis (NDA) using 80 real ground motions selected based on bin strategy and second Incremental Dynamic Analysis (IDA) of structures subjected to the same suite of records. Also the IDA results are used in two different ways, first by limiting the maximum scaled first mode spectral acceleration and second by limiting the maximum inter-story drift of the data points which used to estimate the parameters. The difference in estimated parameters shows how the structures characteristics affect the demand model. This paper indicates that, although two methods result in different model parameters, the estimated standard deviation are similar in many cases. Also only in the region of linear response of low-rise frame both PSDA and IDA would end up to the same estimation for seismic demand and for mid and high-rise frames, large differences between two methods are observed.

Introduction

A reliable estimation of seismic demand is an essential part of recently developed performance based earthquake engineering. In this estimation the randomness and uncertainty in the ground motion and nonlinear structural response must be considered. Because of these uncertainties, a probabilistic methodology, which is called Probabilistic Seismic Demand Analysis (PSDA), is necessary to estimate the seismic demand of a structure at a designed site. PSDA is an approach to compute the mean annual probability of exceeding a specified seismic demand for a given structure at a designed site (Cornell 1996). Although different methods have been proposed to PSDA until now (Han and Wen 1997), the suggested method by Cornell and co-workers (e.g., Bazzurro 1998) is the most applicable method to estimate seismic demand.

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In short, this new method combines a ground motion Intensity Measure (IM) parameter (e.g., first mod spectral acceleration) hazard curve for the designed site, typically computed trough probabilistic seismic hazard analysis, with the demand (e.g., maximum inter-story drift) resulted from Nonlinear Dynamic Analysis (NDA) of the given structure under a suite of earthquake ground motion records (Luco 2002). In order to estimate the maximum inter-story drift of a specific structure, denoting by *DR*, based on first mod spectral acceleration as IM parameter, denoted by *Sa*, the PSDA can be expressed mathematically as follows:

$$P[DR > x] = \int_{0}^{\infty} P[DR > x \mid Sa = y].\mid dH_{Sa}(y) \mid \qquad (1)$$

In this equation $P[DR>x]$ means annual frequency of *DR* exceeding the value *x*, $H_{Sa}(y)$ spectral acceleration hazard at $Sa=y$, means annual frequency that *Sa* at a given site will equal or exceed value *y* and notation |d…| means its differential with respect to *Sa*, also evaluated at *y*.

The term $P[DR>x|Sa=y]$ means the probability of DR exceeding the value *x* given (i.e., conditioned on knowing) that *Sa* equals y. By assuming a normal distribution, for dispersion of naperian logarithm of drift along its mean, this probability can be calculated as:

$$P[DR > x \mid Sa = y] = 1 - \Phi\left(\frac{\mu_{\ln(DR)}(Sa = y) - \ln(x)}{\sigma}\right) \qquad (2)$$

In this equation μ and σ are the mean and standard deviation of selected demand model. A demand model is a mathematical expression relating the ground motion IM (here *Sa*) to structure specific demand measure (here *DR*). Although selecting an applicable, sufficient, efficient and effective demand model is a matter of concern until now (Mackie and Stojadinovic 2001), in this article, the following model, proposed by Cornell et al. (2002) based on extensive regression analysis of response of steel structures, is selected.

$$\ln(DR) = a.\ln(Sa) + w + \sigma.\varepsilon \qquad (3)$$

Where ε is a standard normal random variable with zero mean and unit standard deviation, σ is the unknown standard deviation of model and *a* and *w* are the unknown model parameters. All of the unknown parameters must be estimated based on the results of NDA of structure.

The purpose of this research is the study of the effects of NDA results on estimated model parameters and standard deviation and effects of different estimated parameters on estimated demand. In this article, the required data to estimate the unknown parameters are gathered from two different sources, first the defined structures, which are generic Steel Moment-Resisting Frames (SMRFs), are analyzed subjected to a set of 80 un-scaled ground motions, selected based on a bin strategy, and the calculated NDA results are used to estimated the mentioned parameters.

At the second step, in order to create a widespread database, the 80 records are used in an Incremental Dynamic Analysis (IDA), the dynamic equivalent to familiar static pushover analysis. Given a structure and a ground motion, IDA is done by conducting a series of NDA. In this process the IM of ground motion is incrementally increase and *DR* is monitored during each analysis (Vamvatsikos and Cornell 2002). The extreme values of *DR* are plotted against the corresponding value of the IM for each level to produce a database, and then the unknown model parameters are estimated based on this database. In this article the IDA is carried out by two different methods, by limiting the value of *Sa* and by limiting the value of drift.

In order to study the effects of different estimated model parameters on estimated seismic demand, by using related spectral acceleration hazard curves, a PSDA is achieved for SMRFs. To achieve a valid comparison between resulted demand, calculated based on different estimated parameters, collapse cases are not included in IDA, because the unscaled records are not strong enough to cause such a behavior in modeled SMRFs. Also in order to consider both inherent randomness and uncertainty, associated with seismic events and response of structures, Bayesian regression analysis, which is a powerful tool to simultaneous modeling of randomness and uncertainty is used to estimate all unknown parameters.

Definition of Used Generic Steel Moment-Resisting Frames

In this article, NDA is carried out using a family of two-dimensional single-bay generic SMRFs with number of stories equal to 3, 6, 9, 12 and 15, and first mode periods equal to 0.3, 0.6, 0.9, 1.2 and 1.5 second respectively. Some main characteristics of this family of frames are as follows, more details can be found in (Medina and Krawinkler 2005):

- The same mass is used at all floor level
- Relative stiffness is tuned so that the first mode is straight line
- Plasticization just occurs at the end of the beams and the bottom of the first story columns
- Frames are designed so that simultaneous yielding at all plastic hinge locations is attained under a parabolic (NEHRP, k=2) load pattern.
- Moment-rotation hysteretic is modeled by using rotational springs with peak-oriented hysteretic rules and cyclic deterioration parameter equal to 30 and 3% strain hardening.

These frames are modeled using the opens system for earthquake engineering simulation, developed by Pacific Earthquake Engineering Research Center (OpenSees 2009).

Selection of Ground Motion Records

An appropriate estimation of seismic demand through NDA requires a suitable selection of ground motion records which must represent the seismic hazard condition of target territory at different return periods. In this article, using a bin strategy, 80 records are selected from the PEER Center Ground Motion Database (http://peer.berkeley.edu/smcat/) and are classified into four magnitude-distance bins for the purpose of time history analysis of SMRFs (Medina and Krawinkler 2003). The record bins are designated as follows:

- Large Magnitude-Short Distance Bin, LMSR, ($6.5 < Mw < 7.0$, 13 km < R < 30 km),
- Large Magnitude-Long Distance Bin, LMLR, ($6.5 < Mw <7.0$, 30 km < R < 60 km),
- Small Magnitude-Short Distance Bin, SMSR, ($5.8 < Mw < 6.5$, 13 km < R < 30 km), and
- Small Magnitude-Long Distance Bin, SMLR, ($5.8 < Mw < 6.5$, 30 km < R < 60 km).

Using NDA of Unscaled Ground Motions Results to Estimate the PSDM Parameters

In this section, the defined SMRFs are subjected to 80 selected ground motion records and a database consists of the results of achieved NDA is created. In this database for each model, 80 pairs of calculated data (*DR*, *Sa*) are available. These data are applied in a Bayesian regression analysis to estimate the parameter and standard deviation of selected PSDM, shown in Table 1. As seen in the Table 1, the model parameters and specially the standard deviation of models differ a lot in various SMRFs, because of their different behavior subject to the selected ground motion records. Fig.1 shows that the 3-story model behaves completely linear but there is large dispersion in resulted data of 15-story model, which could explain the larger standard deviation in this model. Hereafter the estimated parameters and standard deviation by using NDA of un-scaled ground motions results are called Case1 for short.

Table 1. Estimated model parameters and standard deviation, using Bayesian regression analysis

	Number of stories				
Estimated Parameter	3	6	9	12	15
Estimated Mean of a:	0.9962	0.9107	0.8630	0.7160	0.7307
Estimated Standard Deviation of a:	0.0663	0.1556	0.2076	0.2304	0.2546
Estimated Mean of w:	-5.9138	-5.1127	-4.5748	-4.3853	-3.9935
Estimated Standard Deviation of w:	0.0781	0.1985	0.2910	0.3532	0.4105
Estimated Mean of σ:	0.0248	0.1557	0.2588	0.3228	0.4438
Estimated Standard Deviation of σ:	0.0100	0.0624	0.1040	0.1319	0.1801

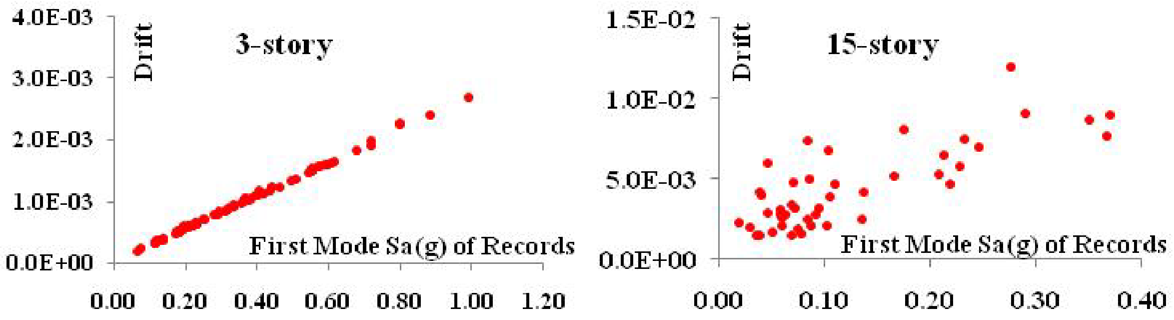

Figure 1. Results of NDA of 3 and 15-story SMRFs subjected to 80 ground motion records

Incremental Dynamic Analysis of SMRFs

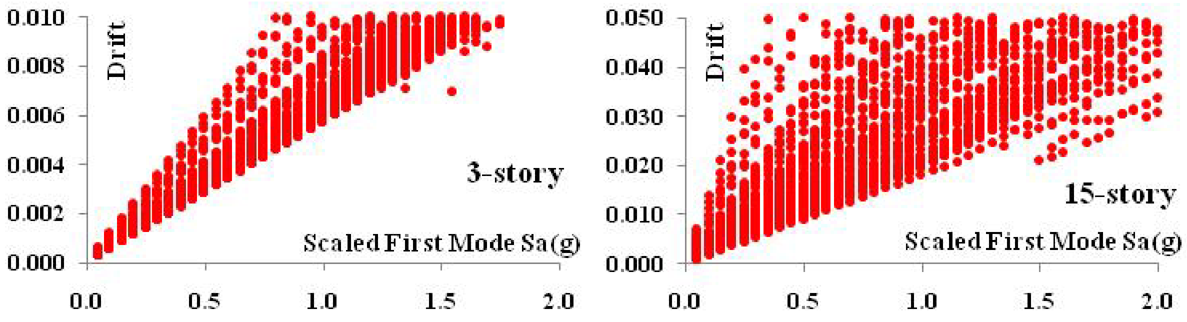

Figure 2. Results of IDA of 3 and 15-story SMRFs subjected to 80 ground motion record

Similar to Fig. 1, Fig. 2 shows the results of IDA of 3 and 15-story frames, subjected to the 80 selected ground motion records. In this paper, the IDA is carried out by scaling the first mode spectral acceleration of records from 0.05g to a final value, which leads to collapse of frames, with 0.05g steps. At each step, a NDA is achieved by using these scaled records and the resulted maximum inter-story drifts along with the value of scaled *Sa* are gathered in a database. This database is used to estimate the unknown parameters of demand model in two different manners, first by limiting the maximum scaled first mode spectral acceleration (*Sa*) and second by limiting the maximum inter-story drift (*DR*) of the data points which are used to estimate the unknown parameters.

Using IDA Results with Limited Maximum *Sa* to Estimate the PSDM Parameters

Fig.3 shows how changing the limited maximum value of *Sa* of used data points in Bayesian regression affects the estimated unknown parameters. The horizontal axis of these charts shows the maximum value of *Sa*, which is used to estimation. For comparison in following parts this results are named Case2. In Fig. 3 the mean of model parameters are illustrated for all range of maximum used *Sa* along with the estimated parameters extracted from direct PSDM with non-scaled ground motion shown with solid blue line.

Generally the following hints can be concluded form this figure:

- In 3-story building, the outcome of Case1 resembles to Case2. With increasing the number of stories, however, Case2 is tangibly deviates from Case1 in an increasing manner.
- For all parameters there are limits of *Sa* after which all parameters are aseptically approaching to a constant level.
- In case of standard deviation of model, which is a representative parameter of epistemic and aleatory uncertainties, for 3-story building, up to *Sa*=1.5g which could be an upper limit for a linear response the result of Case1 is the same as Case2. For *Sa* above 1.5g, the range includes nonlinear responses and more uncertainty, the estimated standard deviation of Case2 is much higher than Case1. The increasing of standard deviation stopped at certain *Sa* level, when the model shows nonlinear behavior to all scaled records.
- In 6, 9 & 12-story buildings the σ in Case1 and Case2 are fairly the same.
- In 15-story building Case2 where the behavior of structure remains linear with increasing the *Sa* a reducing tend for σ even less than Case1 is observed. It can be generally concluded that discrepancies between Case1 and Case2 can be attributed to the linear or nonlinear response of structures. The more nonlinearity in response, the larger difference between Case1 and Case2 would be expected.

Using IDA Results with Limited Maximum *DR* to Estimate the PSDM Parameters

In order to investigate the effects of maximum inter-story drift, which is used in Bayesian regression, on estimated unknown parameters, the charts of Fig. 4 are plotted. The horizontal axis of these charts shows the maximum value of drift, which is used to estimation. This case is called Case3 for further reference in comparison with Case1 and Case2. The estimated parameters of drift-limited IDA oriented PSDM (Case3) are compared with direct method (Case1) in this Figure. Generally the following hints can be concluded form this figure:

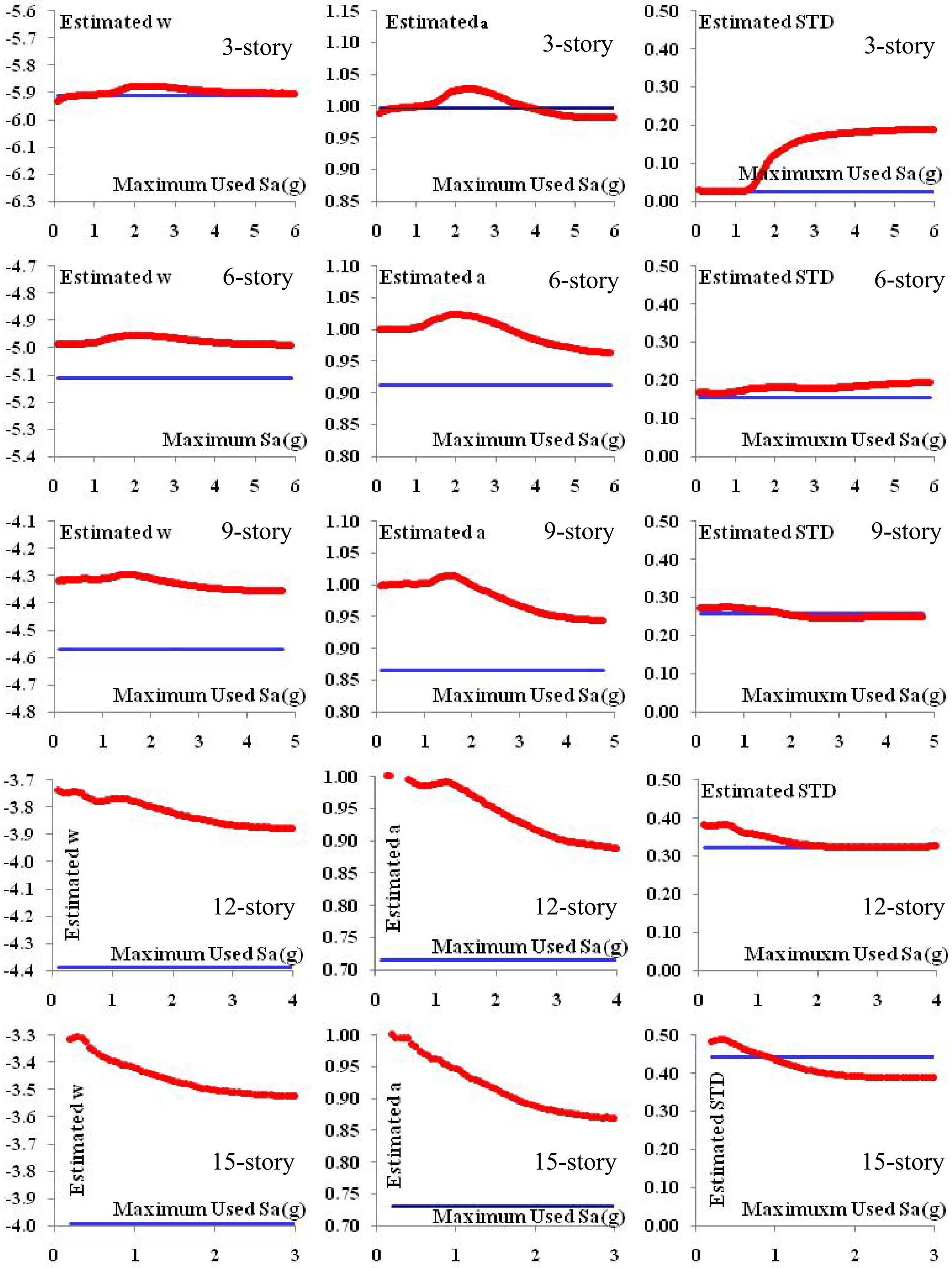

Figure 3. Estimated model parameters (*a* and *w*) and standard deviation (STD) in Case2 for different number of stories. Blue line shows the estimated parameter in Case1.

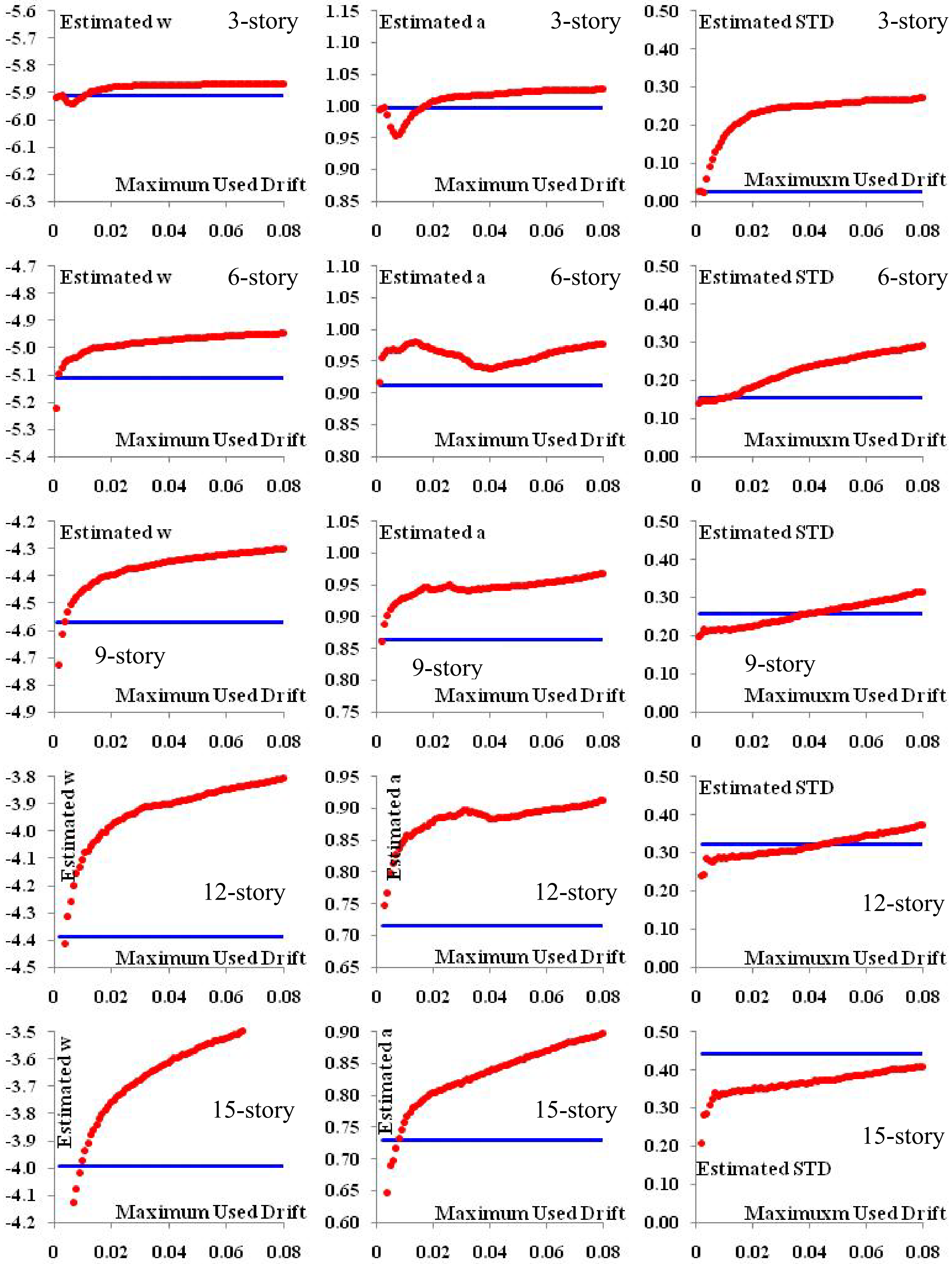

Figure 4. Estimated model parameters (*a* and *w*) and standard deviation (STD) in Case3 for different number of stories. Blue line shows the estimated parameter in Case1.

- In this case the trend of PSDM parameters is not asymptotically approached to a saturated level and they increase by increasing the value of maximum used drift. Also the variation of parameters is more irregular in comparison with the case2.
- Similar to Case2, in Case3 it is shown that the parameters of the low-rise model (3-story) is the same as Case1, while with increasing the height of frames, an additive diversion between two cases is observed. Such results show that developing a closed-form framework for probabilistic-based design is a function of structural behavior.
- There is a drift limit for each building in which the corresponding σ of Case1 and Case3 are the same. This results show that providing selection an appropriate maximum used drift, the PSDA and IDA methods can be applied interchangeably.
- Generally, the difference between this case and Case1 is completely considerable. In the other word, in order to estimate similar parameter from PSDA and IDA, the first mode spectral acceleration of records must be selected as controlling parameter, and the drift is not the suitable one.

Comparison of estimated demand through probabilistic and incremental dynamic analysis

In order to investigate the effect of employing the outcome of a PSDM parameters based on using un-scaled records and IDA results, in this part the drift hazard demand curve of all generic frames are produced for a typical seismic hazard zone of Tehran the Capital of Iran. The seismic hazard curves of Tehran in terms of spectral acceleration at first natural period, based on their first mode spectral acceleration, can be approximated by the below power equation. The value of *k* and *t* for all modeled generic frames are shown in Table 2.

Table 2. Equation and its parameters used to calculate seismic hazard of *Sa* in Tehran city

Equation and its parameters		Number of stories [*Sa*(first mode period)]				
		3[*Sa*(0.3)]	6[*Sa*(0.6)]	9[*Sa*(0.9)]	12[*Sa*(1.2)]	15[*Sa*(1.5)]
$H(Sa) = k.(Sa)^t$	*k*	8.422 E-4	2.661 E-4	8.947 E-5	3.444 E-5	2.473 E-5
	t	-2.683	-2.191	-2.105	-2.140	-2.140

Later, incorporating the seismic hazard curve into PSDM through a numerical integration of Eq. 1 for Case1 and Case2, probabilistic seismic drift hazard for the selected region of Tehran are produced and shown in Fig. 5. For Case2, the upper limit of *Sa* used in IDA, is the first mode spectral acceleration with mean annul probability of exceeding lower than 0.0001. These values for generic models of current research with first period of 0.3, 0.6, 0.6, 0.9, 1.2 and 1.5 second are 2.1g, 1.35g, 0.95g, 0.6g and 0.55g respectively in Tehran territory. Fig. 5 is an evidence of different seismic drift demand estimated by direct PSDA (Case1) and IDA (Case2). However, in case of 3-story frame which is a representative of stiff building with small period, the difference is negligible except for very low probability. With increasing the number of stories, the difference between two methods becomes wider and in the case of 12-story and 15-story frames covers almost all range of drift. Hence, it can be generally concluded that only in the region of linear response of very stiff frames (low-rise buildings) both PSDA and IDA would end up to the same estimation for probabilistic seismic demand. For mid-rise and high-rise buildings, large difference between two methods is an indication of big source of parametric uncertainty whose proper consideration must attain a great deal of attention in codified methods.

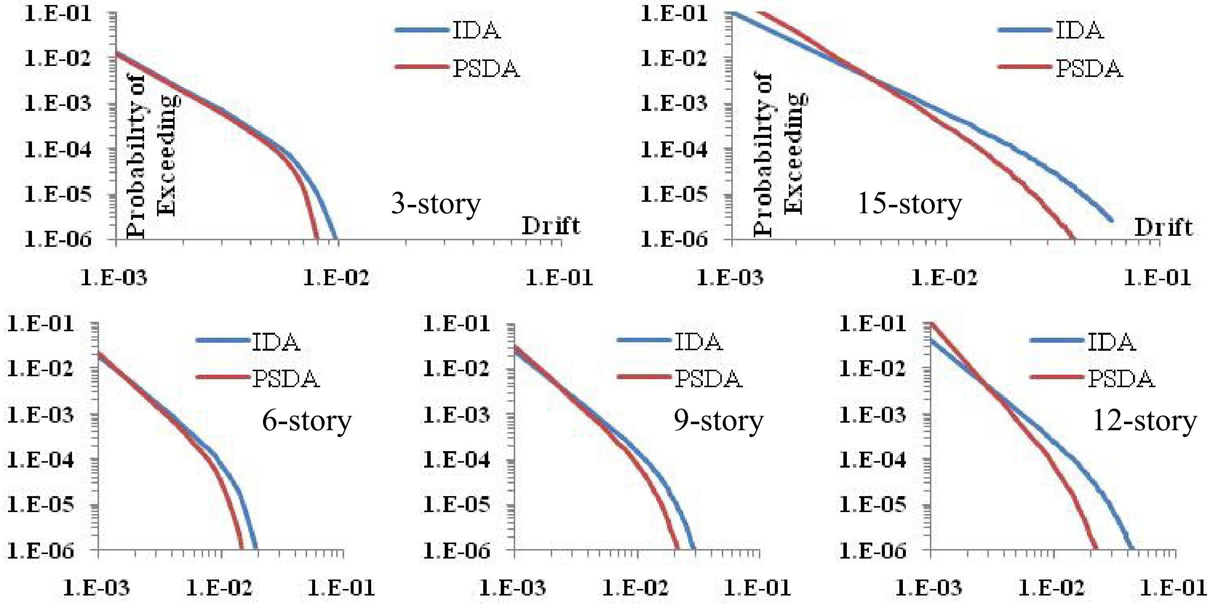

Figure 5. Comparison of estimated demand through PSDA and IDA Case2 for different models

Conclusions

Probabilistic seismic drift estimation for generic SMRFs with 3, 6, 9, 12 and 15 story are carried out using two methods, direct PSDA with using un-scaled ground motion time histories and IDA with the same suite of ground motions. The method of IDA is adopted in two different ways itself, first by limiting the maximum scaled first mode spectral acceleration and second by limiting the maximum inter-story drift of the data points which are used to estimate the unknown parameters. In each method full probabilistic model of all parameters including standard deviation representing aleatory and model uncertainty are estimated using a Bayesian regression analysis. Although the results of 3-story model are resemble for weak to average earthquake, for higher level of earthquake intensity in 3-story model and for all range of seismic intensity for mid to high rise model the difference between PSDA and IDA is considerable. As an interesting phenomenon it has been observed that the estimated standard deviations in selected demand model, by both methods are approaching the same value while other parameters are different. Talking more specifically, it seems that standard deviation estimated by direct PSDA is the same as one estimated by IDA in nonlinear region of drift which in used models is where the secant stiffness between 2 adjacent drifts is reduced around 80%.

With no doubt, the IDA results are more accurate to estimate the PSDM parameters than PSDA results, but this method is difficult and time-consuming. So it is useful to replace the IDA by PSDA, however based on the results of current study, this situation is valid only for certain model parameters and structures. Generally, the results of this research show that although two methods result in different model parameters, the estimated standard deviation are similar in many cases. Also only in the region of linear response of very stiff frames (low-rise buildings) both PSDA and IDA would lead to the same estimation of seismic demand. For mid-rise and high-rise buildings, large difference between two methods is an indication of big source of parametric uncertainty whose proper consideration must attain a great deal of attention in codified methods.

At the end of this article, it must be noted that all of the introduced results are obtained from analyzing of generic steel moment frames. Although these frames are capable to demonstrate several basic performances of real frames, because of their special design (such as regularity in height and mass, not accounting the potential weak column behavior and associated story mechanisms and etc.) the results may not valid for all types of frames. Further investigation is necessary in these cases.

References

Bazzurro, P., C. A. Cornell, N. Shome, and J. E. Carballo, 1998. Three proposals for characterizing MDOF nonlinear seismic response, *Journal of Structural Engineering* 124 (11), 1281-1289.

Cornell, C.A., 1996. Calculating building seismic performance reliability: a basis for multi-level design norms, *Proceedings of the 11th World Conf. on Earthquake Eng.*, *Paper No. 2122*, Acapulco, Mexico

Cornell, C. A., F. Jalayer, R. O. Hamburger, and D. A. Foutch, 2002. Probabilistic basis for the 2000 SAC FEMA steel moment frame. *Journal of Structural Engineering* 128(4), 526-533

Han, S. W., and Y. K. Wen, 1997. Method of reliability-based seismic design. I: Equivalent nonlinear systems, *Journal of Structural Engineering* 123(3), 256-263.

Luco, N., 2002. Probabilistic seismic demand analysis, SMRF connection fractures, and near source effects, *Ph.D. Thesis*, Dept. of Civil and Environmental Eng., Stanford University, Stanford.

Mackie, K., and B. Stojadinovic, (2001). Probabilistic seismic demand model for California highway bridges. *Journal of Bridge Engineering* 6(6), 468-481.

Medina, R.A., and H. Krawinkler, 2003. Seismic demands for nondeteriorating frame structures and their dependence on ground motions, *Report No. 144*, Stanford University, Stanford.

Medina, R.A., and H. Krawinkler, 2005. Evaluation of drift demands for the seismic performance assessment of frames, *Journal of Structural Engineering* 131 (7), 1003-1013.

OpenSees, 2009. Open System for Earthquake Engineering Simulation, *Pacific Earthquake Engineering Research Center*, http://peer.berkeley.edu/

Vamvatsikos, D., and C. A. Cornell, 2002. Incremental dynamic analysis, *Earthquake Engineering & Structural Dynamics* 31 (3), 491-514.