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ENERGY-BASED CRITERION FOR THE SELECTION OF THE SEISMIC INPUT FOR INELASTIC DYNAMIC ANALYSES

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ABSTRACT

When non linear dynamics analysis is adopted, a set of ground motion accelerograms - recorded, generated or simulated - must be chosen to reproduce the seismic input. They are typically selected using a criterion of compatibility of their average response spectrum with an elastic acceleration spectrum representative of the site spectrum. It can be observed that the choice of a seismic input consisting of natural records so selected is not satisfactory for an effective evaluation of the nonlinear seismic response aimed at a performance based analysis. This aspect has been investigated considering the maximum responses of SDOF systems to linear and nonlinear analyses. A different criterion, based on the compatibility of constant ductility spectra is then taken into account for the selection of records. An energy-based criterion is also proposed, defining an energy parameter, called pseudo-energy, accounting for both the force and displacement in the post-elastic behavior. Finally the selection methods are assessed with reference to a MDOF framed system representative of a real case.

Introduction

In recent years the up-to-date earthquake-resistant design methods provide for adopting design rules based on the direct damage control of the resistant members and on the consequence on non structural elements, through the knowledge of the inelastic response of the structures computed using nonlinear dynamic analyses. At present, this kind of analysis represents the best tool for the evaluation of the structural seismic response. When non linear dynamics analysis is adopted, a set of ground motion accelerograms must be chosen to reproduce the seismic input. They, according to the European seismic code (Eurocode 8 2005), can be recorded, generated or simulated; in any case they should be selected or generated, within a population of accelerograms, according to a selection criterion. The usual criterion is based on the compatibility of the average response spectrum with an elastic acceleration spectrum representative of the site spectrum, but probably this criterion is not adequate in case of strong inelastic behavior of the structure. The present paper is aimed at investigating this critical aspect.

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The accelerograms of the PEER Strong Motion Database (PEER NGA Database 2009) are used as time histories archive for the following formulation. First of all, the standard selection criterion based on the elastic pseudo-acceleration is examined, then, to improve the selecting method, other methodologies have been used. At this aim new database information was necessary, like the pseudo-acceleration and displacement spectra for different fixed value of displacement ductility. The enhanced database was obtained through a number of analyses of EPP-SDOF systems carried out using the computer code Bispec (Bispec 2009). The definition of a new database was necessary because the available response spectra are usually expressed in terms of elastic pseudo-acceleration or displacement and inelastic spectra should be obtained from those through analytical formulas. On the contrary, in this way a complete database of inelastic spectra for assigned value of ductility are available and, for the generic time history, an approximate evaluation of inelastic spectra is not necessary, but it is directly computed. On the other hand, reference inelastic spectra are not provided by codes, therefore some formulations known in literature, for pseudo-acceleration and displacement spectra, are taken into account to overcome this problem. The aim of this study is to show that the selection of a set of accelerograms for dynamic analyses can be done according to a compatibility with different reference spectra and that the consequence of the choice of the criterion is relevant for the control of the analysis result and of the structural performances . The compatibility criteria examined are based on matching the conventional pseudo-acceleration elastic spectrum, the constant ductility acceleration spectrum, and the pseudo-energy , resulting by the introduction of a new energy-based parameter called pseudo-energy.

Definition of Reference Spectra

Design codes usually provide for a seismic action formulation in terms of pseudo-acceleration elastic response spectrum, while the design one can be obtained by way of dividing it by a reduction factor R. This parameter is defined as the ratio between the elastic and inelastic base shear of the system. Usually, this parameter depends on the typology of the resistant system (reinforced concrete frames, steel frames, masonry walls, etc.), number of stories and span, the class of ductility etc. When the R factor has been defined for the structure, consequently the design spectrum is computed. A preliminary design can be realized according to this inelastic parameter, which represents the non linear behavior of the system. In the present article it is used a different formulation of the seismic demand spectrum. In order to characterize the inelastic behavior of the model, it has been assumed the ductility as a reference parameter. The nonlinear parameters R and μ are linked by specific relations present in the literature. In the present study the formulations proposed in (Newmark and Hall 1982) have been used, in order to express the reduction factor of the force, R, depending on the ductility, μ. Once the ductility, μ, and the elastic pseudo-acceleration spectrum, PsA_e, have been determined, the associated inelastic spectrum, PsA_μ, can be easily computed as

$$PsA_\mu = \frac{PsA_e}{R(\mu)} \tag{1}$$

In addition, codes give another formulation for the calculation of the elastic demand spectrum in terms of displacement, D_e, which can be obtained by the one in terms of pseudo-acceleration as

$$D_e = PsA_e \cdot \left(\frac{T}{2\pi}\right)^2 \quad (2)$$

The relation proposed in (Miranda and Ruiz-Garcìa 2002) can be used to represent the elastic displacement spectrum for a prefixed value of ductility, μ. This can be obtained by means of the definition of the following parameter

$$R_\delta = \frac{D_\mu}{D_e} \quad (3)$$

where D_μ e D_e represent, respectively, the values of the inelastic and elastic displacement spectra for assigned values of T and μ. In Fig. 1 both the pseudo acceleration spectra and displacement spectra for different ductility values (μ=1, 2, 4, 6) are reported making reference to the formulations of the Italian codes (NTC 2008) which is similar to the European one (Eurocode 8 2005). The reported graphs, corresponding to a bedrock acceleration a_g = 0.25 g and to a medium-soft soil, type C, are characterized by the following parameters: PGA = 0.313 g, T_A = 0.05 s, T_B = 0.15 s, T_C = 0.50 s, T_D = 2.00 s.

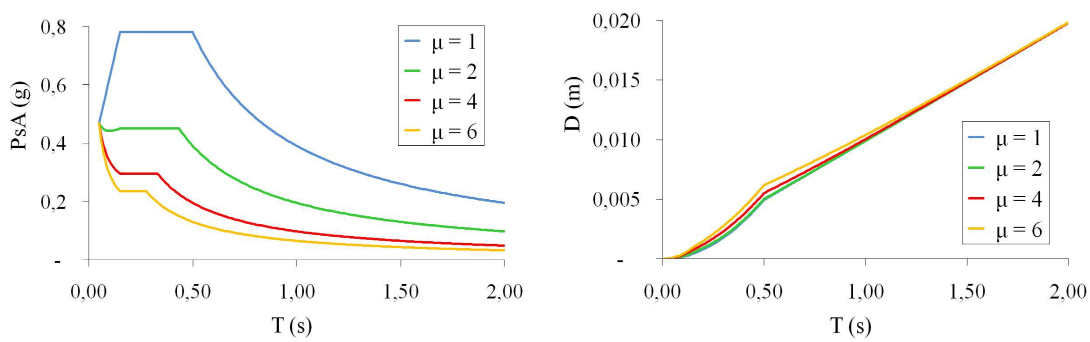


Figure 1. Pseudo-acceleration spectra (left) and displacement spectra (right) for different ductility values, obtained by employing the Newmark-Hall and Miranda relations.

Selection Criterion Based on Elastic Pseudo-Acceleration Spectrum

Once the accelerograms' database to be used is defined, a reliable method for the selection of an accelerograms' group to be used in the analysis is necessary. The first step consists of the assumption of a reference target spectrum. Initially, the pseudo-acceleration elastic spectrum has been considered, as suggested by common codes. The objective is to choose 7 accelerograms characterized by an average spectrum, in terms of pseudo-acceleration, according to the reference spectrum. This selection criterion is defined PsA_e criterion. An additional suitable condition consists of having a good agreement between the spectrum of the single record and the reference spectrum. This aspect has a fundamental purpose in the present application. Let us consider, in order to fix the ideas, to take 2 different accelerograms and to use these seismic inputs for the nonlinear analysis of a SDOF system. Without a preliminary selection it could happen what is shown in Fig. 2 (left), where one of the accelerogram gives a nearly elastic behavior (green diagram), while the other one is characterized by a strongly inelastic response (red diagram). On the contrary, with a suitable pre-selection, it can be obtained the responses shown in Fig. 2 (right) where both the accelerograms produce a comparable inelastic behavior of the considered SDOF system.

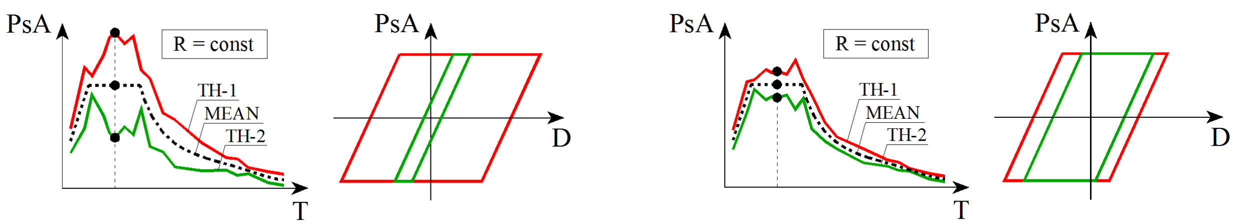


Figure 2. Effects of uncontrolled (left) or pre-selected (right) acceleration time histories

A preliminary operation has been made within the total population of time histories, identifying those ones characterized by an elastic response spectrum not too far from the reference one. This has been carried out by creating a directory of accelerograms ordered according to the standard deviation, δ, of their spectrum values. This parameter quantifies the agreement level that exists between a generic time-history spectrum and the reference one. The standard deviation is defined as follows

$$\delta = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} \left(\frac{P_i^{(T)} - P_i^{(R)}}{P_i^{(R)}} \right)} \tag{4}$$

where P_i is the spectrum value corresponding to the generic period T_i; the apexes T and R respectively refer the time history and the reference spectrum; N represents the number of considered periods within the control interval. In the present case the control interval is assumed to range from 0.05 to 2.00 s.

After that the directory has been built according to the standard deviation of the accelerograms, the best fitting records have been considered. Referring to these time histories it has been possible to combine 7 registrations each time, in order to obtain the average spectrum best approximating the reference one. The combination characterized by the smallest standard deviation value is assumed as set of ground motion records for the dynamic analyses.

Selection Criterion Based on Constant Ductility Pseudo-Acceleration Spectrum

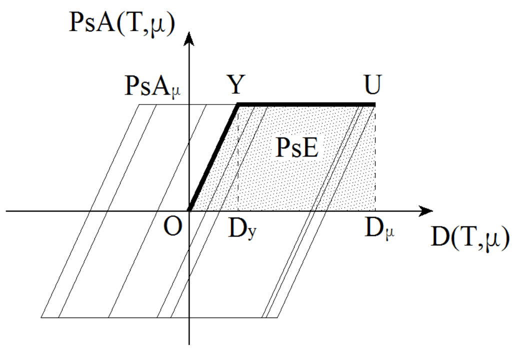
The second selection criterion (defined PsA_μ criterion) is based on the use of response inelastic spectra. With a process similar to the previous one, a new selection can be realized to choose a group of 7 records. This selection criterion is based on the compatibility of the average inelastic spectrum, in terms of pseudo-acceleration, of the 7 accelerograms with the elastic reference spectrum, reduced by the appropriate factor R. Generally, the codes suggest a reference value of the strength reduction factor R that can be related to the ductility value μ. The value of the ductility factor has to be chosen a priori. Consequently, the choice of the combination of the 7 accelerograms, characterized by the lowest value of standard deviation δ, can be easily done when the reduced reference spectrum is assigned. Also in this case a preliminary selection has been adopted, for the same reasons previously explained.

Selection Criterion Based on Pseudo-Energy Spectrum

The third selection criterion presented here - identified as PsE criterion - corresponds to a new procedure for the choice of the 7 registration to be assumed as seismic input based on an energy related approach. At this aim an energy-based parameter, named pseudo-energy (PsE), has been defined (Mezzi et al. 2006). Through it, both the force and displacement in the post-elastic behavior are taken into account. This parameter is defined as

$$PsE(T,\mu) = \frac{2\pi^2}{T^2} \cdot \left(\frac{2\mu - 1}{\mu^2}\right) \cdot D(T,\mu)^2 \qquad (5)$$

Fig. 3 gives a graphic representation of the pseudo-energy: it represents the area under the conventional response curve of the EPP-SDOF system. This parameter can be directly obtained by the displacement spectrum, when the period, T, and the ductility, μ, are fixed. The PsE can be computed by considering the conventional envelope curve obtained through the knowledge of the yield force and of both the yielding, D_y, and maximum, D_μ, displacement.



$$\mu = \frac{D_\mu}{D_y} = \frac{D(T,\mu)}{D_y(T,\mu)} \qquad R = \frac{PsA_e}{PsA_\mu} = \frac{F_e(T,\mu=1)}{F_\mu(T,\mu)}$$

$$PsA(T,\mu) = \frac{D(T,\mu)}{\mu} \cdot \left(\frac{2\pi}{T}\right)^2$$

$$PsE(T,\mu) = \frac{2\pi^2}{T^2} \cdot \left(\frac{2\mu - 1}{\mu^2}\right) \cdot D(T,\mu)^2$$

Figure 3. Graphic interpretation and evaluation formulas of the pseudo-energy.

A new selection criterion is available, once the definition of PsE has been introduced. As before, a pre-selection within the accelerograms' population has been realized. The use of pseudo-energy in pre-selecting the acceleration time histories did not bring good results, therefore it has been decided to maintain the pre-selection criteria based on the pseudo-acceleration. This fact can be explained indeed PsE is a parameter characterized by larger uncertainness with respect to the PsA: when the standard deviation δ is defined in terms of pseudo-acceleration, it is proportional to the spectrum displacement D; when, on the contrary, the standard deviation δ is defined in terms of pseudo-energy, it is proportional to the square of the displacement D. Through the pre-selection based on compatibility in the pseudo-acceleration, a list of records ordered by increasing standard deviation is compiled. It can then be found the best combination, consisting of 7 accelerograms, characterized by the lowest standard deviation δ, computed with reference to the pseudo-energy spectra.

Comparison of the Responses Computed on EPP-SDOF

After defining the different selection criteria, an assessment of the reliability of each one is required. Therefore, it was made the comparison between the responses of dynamic analyses of EPP-SDOF systems carried out by using the three groups of 7 accelerograms, derived from the three different selection criteria previously presented. Responses have been considered in terms of displacement, pseudo-acceleration, pseudo-energy. The results, expressed in terms of standard deviation of the response spectrum with respect to the reference one, are reported in Fig.4 for reference ductility values equal to 1, 2, 4 and 6. The detail of the computed standard deviation values are reported in Table 1.

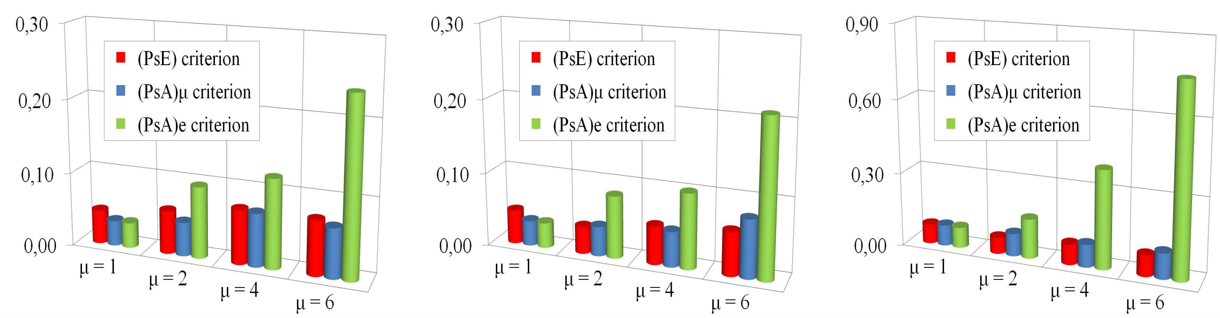


Figure 4. Standard deviation, δ, of pseudo-acceleration (left), displacement (middle), pseudo-energy (right) for different ductility μ and for the assumed selection criteria.

The results show that, when the selection criterion based on the fitting of the elastic pseudo-acceleration spectra is adopted, the scattering of the responses obtained in nonlinear analysis is very relevant and significantly grow while the considered ductility increases. On the contrary, when the PsA_{μ} criterion, based on the reduced inelastic spectra, is adopted, the average spectrum of nonlinear responses well approximates the reference spectrum in terms of acceleration. The standard deviation δ of the average acceleration spectrum of the selected accelerogram lowers from 0.097 to 0.046 passing from the PsA_e to the PsA_{μ} selection criterion, if a ductility level $\mu = 2$ is considered. The difference is more significant if higher ductility levels are considered: for $\mu = 6$ the standard deviation, δ, pass from 0.237 to 0.067, passing from the PsA_e to the PsA_{μ} criterion. The same effect can be observed in the case of the criterion based on the PsE, even if the values of δ resulting in this case are slightly larger.

Furthermore, the criteria PsA_{μ} and PsE also determine a best agreement of the average displacement spectra with the reference one, with respect to that offered by the criterion PsA_e. Both the criteria lead to comparable results and to contained values of the standard deviation of the response. For a ductility level $\mu = 4$, the standard deviation δ of the average displacement response spectrum of the selected accelerogram lowers from 0.097 to 0.046 passing from the PsA_e to the PsA_{μ} selection criterion. In the field of the higher ductility values, the standard deviation of the responses, in terms of displacement and pseudo-energy, results even smaller if using the PsE criterion.

Table 1. Standard deviation of the pseudo-acceleration, displacement, pseudo-energy for the different criteria adopted and for different values of ductility.

Parameter	**μ**	**PsA_e criterion**	**PsA_μ criterion**	**PsE criterion**
Pseudo-acceleration Standard Deviation	1	0.0343	0.0343	0.0462
	2	0.0969	0.0459	0.0585
	4	0.1204	0.0719	0.0737
	6	0.2374	0.0670	0.0738
Displacement Standard Deviation	1	0.0343	0.0343	0.0462
	2	0.0846	0.0402	0.0370
	4	0.1013	0.0477	0.0519
	6	0.2106	0.0782	0.0589
Pseudo-energy Standard Deviation	1	0.0840	0.0840	0.0794
	2	0.1604	0.0915	0.0664
	4	0.3949	0.0919	0.0834
	6	0.7610	0.1037	0.0847

An interesting comparison can be made in terms of PsE spectra. Let us consider an acceptable variation range of ± 20% with respect to the reference spectrum. Recalling the definition of the pseudo-energy, it can be observed that this parameter is proportional to the square of the displacement. It can also be noted that PsA is instead a linear function of the spectral displacement D. For the reasons set out above, the comparisons based on pseudo-energy, pseudo-acceleration or displacement, are similar.

The top left graph in Fig. 5 shows that the selection criteria based on PsA_e and PsA_μ are, of course, equivalent when it is assumed an elastic behavior, with slight differences in case of elastic PsE. Also when μ increases, the PsE and PsA_μ criteria lead to select groups of records characterized by an average spectrum of responses in good agreement with the reference one, as shown in Fig. 5. If using a criterion based only on elastic pseudo-acceleration (PsA_e), errors are significant, especially when the ductility increases.

The results clearly show that a selection criterion based exclusively on the compatibility of pseudo-acceleration elastic spectra is not adequate to properly select the seismic input to be employed in nonlinear dynamic analises. An improvement of the selection can be achieved by working with constant ductility inelastic response spectra, in terms of acceleration or energy. This leads to the choice of a set of records that provides for a low dispersion of the responses in nonlinear analysis.

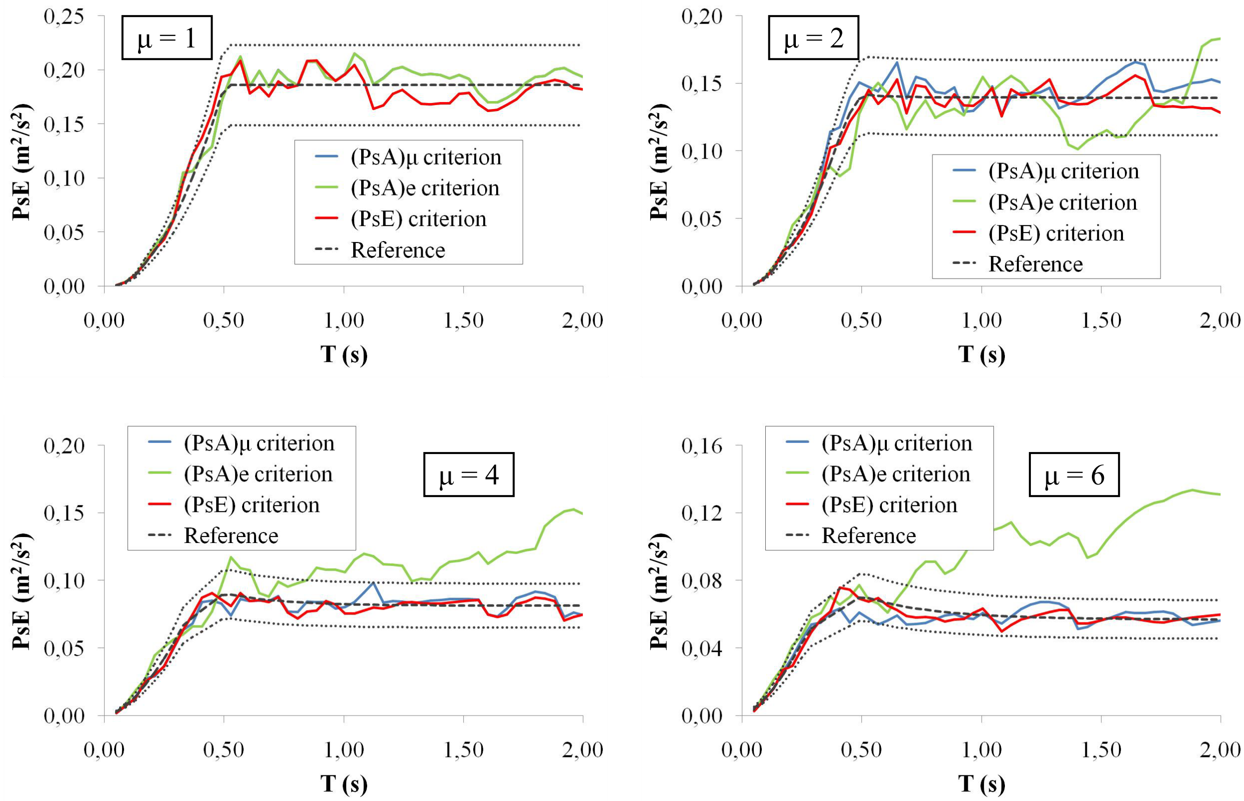


Figure 5. Comparison of pseudo-energy spectra for different values of ductility μ and the considered selection criteria.

Assessment of the Selection Methods through the Analysis of a Sample MDOF System

The response of a MDOF sample system is finally examined to assess the reliability of the proposed methods. A three-bays four-stories reinforced concrete plane frame is considered. The span of the bays is 4.50 m. The story height is 4.00 m for the first floor and 3.00 m for the other ones. 300x500 mm are the dimensions of the edge columns, 300x600 mm are those of the central ones. The beams have dimensions 300x500 mm at the first floor and 300x400 mm at the upper ones. The frame is designed, according to the specification of the Italian and European code for the low ductility class, using a reduction factor R = 2.76. The seismic design is performed using a modal response spectrum analysis. Dead loads of 5.0 kN/m^2 and live loads of 2.0 kN/m^2 are assumed at the intermediate floors. At the top floor, dead and live loads are 3.75 kN/m^2 and 1.25 kN/m^2, respectively. A concrete type C25/30 and a steel with yield strength of 450 MPa are considered. For both columns and beams the required different strength levels are obtained by varying the amount of reinforcement. Time-history nonlinear analyses are carried out using the code Ruaumoko (Ruaumoko 2005). The seismic input is represented by groups of 7 accelerograms selected through the three criteria previously listed. The structural responses corresponding to the three choices are then compared. The parameters used for the comparison are: the absolute storey displacements, the storey drift ratios, the base shear. Also the residual storey drift ratio have been considered for the assessment of structural damage.

The different inputs roughly lead to the same values of the base shear. Obviously this is related to the nonlinear behavior associated to the sizing of the strength of the structural elements in the global design. The strength reduction factor R is computed, from the dynamic analyses, as the ratio between the maximum base shear found in the elastic case and the one obtained in the nonlinear case. The analysis results show values of R approximately equal to 2. This value is comparable with the reduction factor, R=2.56, assumed in the design process. The first natural period of vibration of the structure is equal to $T_1 = 0.509$ s: in this range of periods R is related to the ductility by the expression $R = \mu$. In conclusion, the inelastic reference spectra to be used for the selection of the time histories, according to the PsA_μ and PsE criteria, are considered with reference to a value of the ductility $\mu = 2$.

As far as the absolute storey displacements are concerned, the response has been considered in terms of the average value of the results obtained from the 7 time histories and of the standard deviation associated with the input dispersion. Results similar to those found for the SDOF equivalent models are computed, as shown in Fig. 6. The average response values result comparable among them, independently on the selection criterion of the accelerograms, which does not appear so relevant. A different consideration can be made about the standard deviation. When criteria PsA_μ or PsE are used, based on inelastic parameters, ordinary dispersion values are found, with COV's up to 30%. On the contrary, when using the criterion PsA_e, the spreading of the results is significant, with COV values of about 50% and over. Similar considerations can be made on the storey drift ratios. Fig. 7 shows the results obtained for ductility $\mu = 2$.

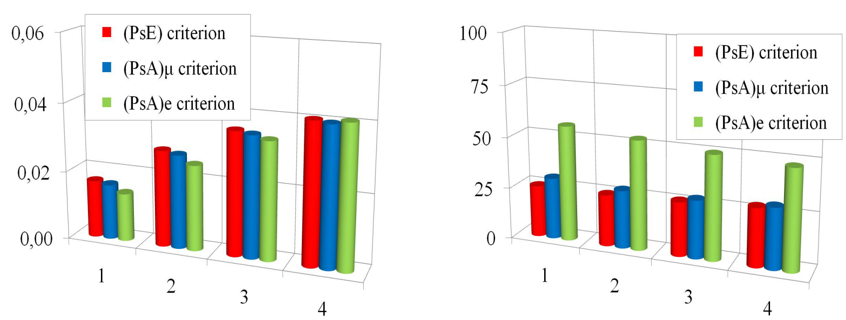


Figure 6. Comparison of the mean (left) and percent coefficient of variation (right) of the absolute storey displacement for the different criteria adopted and the case $\mu = 2$.

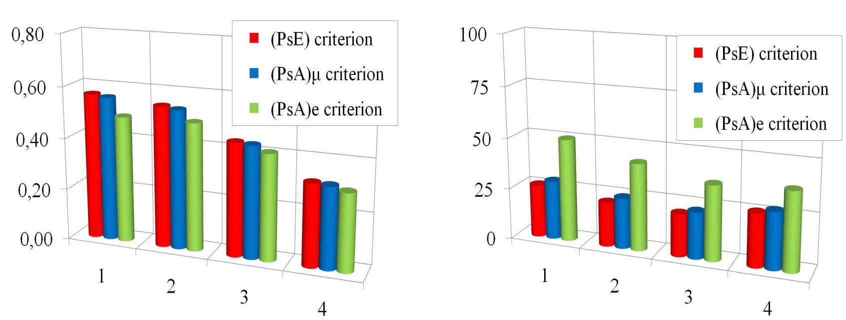


Figure 7. Comparison of the mean (left) and percent coefficient of variation (right) of the storey drift ratio for the different criteria adopted and the case $\mu = 2$.

Finally, some remarks on the residual storey drifts can be formulated. The analysis results, not reported here for the sake of brevity, show that comparable values were found in the case of the selection criteria PsE or PsA_μ, while a significant increase results when the selection criterion PsA_e is adopted. Furthermore, the standard deviation in the last case, PsA_e, is still greater than in the other two cases, PsE or PsA_μ.

Conclusions

The selection criterion proposed by many seismic codes for the selection of accelerograms to be used as seismic input in the dynamic analysis, based on matching the elastic response spectrum in pseudo-acceleration, is appropriate only when the structure presents a purely elastic behavior. The analysis of the response of EPP-SDOF systems for assigned ductility values shows that this selection criterion is not adequate for a correct estimation of the nonlinear response due to the resulting large scattering. A better criterion for the selection of accelerograms can be based on the compatibility with constant ductility inelastic spectra, obtained by reducing the elastic spectrum with the appropriate factor R. A further improvement of the selection has been proposed using an energy-based parameter, called pseudo-energy, accounting for both the plastic threshold and displacement. The results obtained for the SDOF systems have been assessed in the case of a MDOF framed system.

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