



## NON-LINEAR BRIDGE RESPONSE TO DIFFERENTIAL SUPPORT MOTIONS

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### ABSTRACT

Three different methods are compared for evaluating the peak values of the pier drifts for an existing bridge model subjected to spatially varying support motions: linear and non-linear Response History Analysis (RHA) and the Multiple Support Response Spectrum (MSRS) method. The RHAs are performed for sets of support motions generated with the method of conditional simulation for a prescribed spatial variability model. In order to make consistent comparisons, the input excitations in the MSRS analysis are described by the same spatial variability model and the mean response spectrum of the simulated motions. The MSRS estimates are compared with the RHA results in order to evaluate the ability of the MSRS method to provide: (i) reliable estimates of the peak linear response and (ii) approximate estimates of the non-linear response employing the equivalent displacement rule.

### Introduction

Proper seismic design of bridges requires response analyses for support excitations that account for the spatial variability of the ground motions. In the random vibration approach, the seismic input is described in the frequency domain through the Power Spectral Densities (PSDs) of the support excitations and a coherency function, which models the spatial variability of the ground motion. This approach provides a statistical characterization of the response under the inherent assumption of stationarity. The Multiple Support Response Spectrum (MSRS) method (Der Kiureghian and Neuenhofer 1992) partially overcomes the assumption of stationarity by specifying the support motions through response spectra. This method is particularly appealing from a design viewpoint, since most seismic codes specify the earthquake input in terms of response spectra. However, it cannot be used for analysis of the structural response in the non-linear domain, which is required in performance-based design. The non-linear response of bridges is typically studied through Response History Analyses (RHA) with simulated ground motions, since closely spaced recorded motions are rare. These analyses, apart from being computationally costly, are only specific to the particular sets of ground motion considered. An approximate alternative employed in practice is to use the peak values of the pier drifts obtained from linear analyses to estimate seismic demands for inelastic capacity design (Chopra 2000). In this paper, we investigate the ability of the MSRS method to provide estimates of the non-linear response for a case-study of an existing bridge model subjected to support motions that have the characteristics of an actual earthquake record and vary according to a prescribed spatial variability model.

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## Simulation of spatially varying ground motions

The sets of support motions used in this study are generated with the Conditional Probability Density Function (CPDF) method (Kameda and Morikawa 1992). The CPDF method can be used to simulate a random earthquake ground motion at a specified location compatible with predefined time histories at separate stations and a coherency function. In this method, the Fourier coefficients of the predefined ground motions are used to derive the conditional probability density functions of the Fourier coefficients for the ground motion at the target point. Assuming zero-mean stationary Gaussian processes, closed form solutions for these functions are derived in terms of the auto- and cross-PSDs of the support motions and the Fourier coefficients of the given time histories.

In the subsequent example application, spatially varying support motions are generated by conditioning on an actual earthquake record. Assuming uniform soil conditions, the smoothed periodogram of the recorded accelerogram is used as the estimate of the auto-PSD of the support motions. The cross-PSDs are determined in terms of the auto-PSD and a specified coherency function. In order to account for the non-stationarity of the earthquake motions, the original record is divided into nearly stationary segments with different durations (Vanmarcke and Fenton 1991, Liao and Zerva 2006). The CPDF method is applied to each segment separately, after tapering its ends with cosine functions. The simulated time-histories for each segment extend beyond the corresponding time-window and the accelerations in the overlapping regions are combined using cosine weighting functions.

The simulated accelerograms are further processed (Boore et al 2002, Liao and Zerva 2006) by subtracting the mean value of the entire time-history, applying a short cosine taper function to set their initial values to zero, subtracting the derivative of a velocity quadratic fitting function and applying a 4<sup>th</sup> order high-pass Butterworth filter to assure zero residual velocity and displacement. The final acceleration time-histories are integrated to obtain the velocity and displacement records.

### The MSRS method

Based on the principles of random vibration theory, the MSRS combination rule evaluates the mean peak structural response under the assumption that the support motions are jointly-stationary, zero-mean, broad-band processes. For a linear structural model with  $N$  unconstrained degrees of freedom and  $m$  support degrees of freedom, the mean peak value of a generic response quantity,  $z(t)$  is approximated by (Der Kiureghian and Neuenhofer 1992)

$$\begin{aligned}
 & E[\max|z(t)|] \\
 & \approx \left[ \sum_{k=1}^m \sum_{l=1}^m a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} \right. \\
 & + 2 \sum_{k=1}^m \sum_{l=1}^m \sum_{j=1}^N a_k b_{lj} \rho_{u_k s_{lj}} u_{k,\max} D_l(\omega_j, \zeta_j) \\
 & \left. + \sum_{k=1}^m \sum_{l=1}^m \sum_{i=1}^N \sum_{j=1}^N b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} D_k(\omega_i, \zeta_i) D_l(\omega_j, \zeta_j) \right]^{1/2}
 \end{aligned} \tag{1}$$

In the preceding equation,  $a_k$  is the response quantity of interest when the  $i$ th support degree of freedom is statically displaced by a unit amount,  $b_{ki}$  is the  $i$ th modal participation factor associated with the  $k$ th support motion,  $u_{k,\max}$  is the mean peak ground displacement at the  $k$ th support degree of freedom,  $D_k(\omega_i, \zeta_i)$  is the mean displacement response spectrum for the  $k$ th support motion and the  $i$ th modal frequency and damping, and  $\rho_{u_k u_l}$ ,  $\rho_{u_k s_{lj}}$  and  $\rho_{s_{ki} s_{lj}}$  are three sets of cross correlation coefficients. The cross-correlation coefficient  $\rho_{u_k u_l}$  describes the correlation between the  $k$ th and  $l$ th support displacements, the cross-correlation coefficient  $\rho_{u_k s_{lj}}$  describes the correlation between the  $k$ th support displacement and the response of mode  $j$  to the  $l$ th support motion, while the cross-correlation coefficient  $\rho_{s_{ki} s_{lj}}$  describes the correlation between the responses of modes  $i$  and  $j$  to the  $k$ th and  $l$ th support motions, respectively. The coefficients  $\rho_{u_k u_l}$  are functions of the auto- and cross-PSDs of the support motions, whereas the coefficients  $\rho_{u_k s_{lj}}$  and  $\rho_{s_{ki} s_{lj}}$  additionally depend on the modal frequencies and damping ratios. In the MSRS method the specified response spectra at the support motions are used to evaluate the required auto-PSDs and, together with a coherency function and the set of peak ground displacements, provide full specification of the ground motion random field.

### Example application

#### Structural model

The structural model considered in this application represents the Penstock Bridge, designed by the California Department of Transportation (Caltrans). The Penstock Bridge is a prestressed-concrete, four-span Bridge with one pier per bent. The elevation and girder cross-section of the model are shown in Fig. 1. The deck has a vertical grade, varying from 0.3% to 2.1%, and a constant horizontal curvature of radius  $R = 458$  m. The piers have circular cross sections of diameter  $D = 2.13$  m.

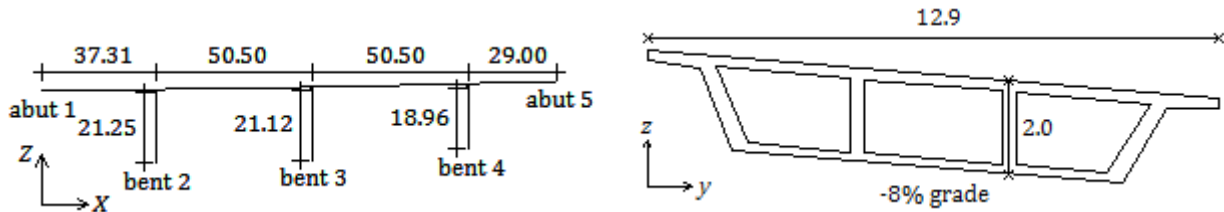


Figure 1. Penstock bridge: Elevation and girder cross-section (dimensions are in meters)

The finite element model of the bridge consists of 3 elements per pier and 6, 8, 8 and 4 elements in spans 1, 2, 3 and 4, respectively. In the non-linear analyses, non-linear elements with distributed plasticity (5 integration points per element) are used for the piers. The pier section is modeled as a fiber section with 12 subdivisions in the circumferential direction and 8 and 4 subdivisions for the core and the cover, respectively, in the radial direction. The reinforcing steel bars are specified as additional layers. The properties of the unconfined concrete and the reinforcing steel are the expected material properties defined in Caltrans Seismic Design Criteria (SDC), whereas the properties of the confined concrete are evaluated from the Mander model. The fiber model of the pier section accounts for the interaction between axial force and bending moment. The shear and torsional behaviors are modeled with aggregated uniaxial models. The yield force for the shear model is determined from Caltrans specifications, whereas the yield force for the torsional model is evaluated according to the strength of materials theory. In the

linear analyses, elastic elements with effective stiffness properties are used for the piers. Moment-curvature analysis indicated that the flexural stiffness should be reduced to 28% of its initial value to account for cracking, whereas the effective torsional moment of inertia is taken to be 20% of its uncracked value (Caltrans SDC). In both cases, the deck is modeled with elastic elements for which no stiffness reduction is required due to prestressing. Vertical rigid frame elements are used to connect the tops of the piers with the deck.

The piers are considered rigidly connected to the deck at the top and fixed in all directions at the bottom. The abutment response is modeled through two translational springs, one longitudinal and one transverse. The stiffness of the longitudinal spring is proportional to the backwall area and the stiffness of the nominal transverse spring is taken equal to 50% of the transverse stiffness of the adjacent bent (Caltrans SDC). Vertical translations at the end supports are fully constrained.

Condensing out the rotational degrees of freedom and accounting for the constraints imposed by the rigid elements, the structure has 103 translational unconstrained degrees of freedom and 15 translational support degrees of freedom. The fundamental period of the linear bridge model is  $T = 2.38$  s. The damping ratio of the  $i$ th mode of the bridge model is evaluated by the relation  $\zeta_i = (a_0/2)(1/\omega_i) + (a_1/2)\omega_i$ , where the parameters  $a_0$  and  $a_1$  are such that  $\zeta_2 = 0.05$  and  $\zeta_{10} = 0.10$  (Chopra 2000).

### Support motions

In this application, uniform soil conditions are assumed and only the transverse component of the excitation is considered. The accelerogram used for the conditional simulation of support motions is the fault normal component of the Pacoima Dam record of the 1971 San Fernando earthquake. The assumed coherency model accounts for the effects of incoherence and wave passage:

$$\gamma_{kl}(\omega) = |\gamma_{kl}(\omega)|^{incoherence} \exp[i\theta_{kl}(\omega)^{wave\ passage}] \quad (2)$$

In the above equation, the incoherence component is described by (Luco and Wong 1986)

$$|\gamma_{kl}(\omega)|^{incoherence} = \exp\left[-\left(\frac{\alpha d_{kl}\omega}{v_s}\right)^2\right] \quad (3)$$

in which  $d_{kl}$  is the distance between the support points,  $v_s$  is the shear wave velocity of the ground medium and  $\alpha$  is an incoherence coefficient estimated from data. The phase shift due to the wave-passage effect is given by (Der Kiureghian 1996)

$$\theta_{kl}(\omega)^{wave\ passage} = -\frac{\omega d_{kl}^L}{v_{app}} \quad (4)$$

where  $d_{kl}^L$  is the projected algebraic horizontal distance in the longitudinal direction of propagation of waves and  $v_{app}$  is the surface apparent wave velocity. The waves are assumed to propagate in the direction of the  $X$  axis. For the coherency function parameters, we assume the typical values  $\alpha/v_s = 1/600$  and  $v_{app} = 400$  m/s.

Linear and non-linear RHAs are performed for 10 sets of conditionally simulated support motions with the simulation at zero distance applied at abutment 1. Fig. 2 compares the acceleration, velocity and displacement time-histories of the recorded motion and the conditionally simulated motion at zero distance. The differences are due to the segmentation of the original record, the tapering of the segments and the post-processing of the resulting accelerogram. Figs. 3a, 3b and 3c show the sets of acceleration, velocity and displacement time-

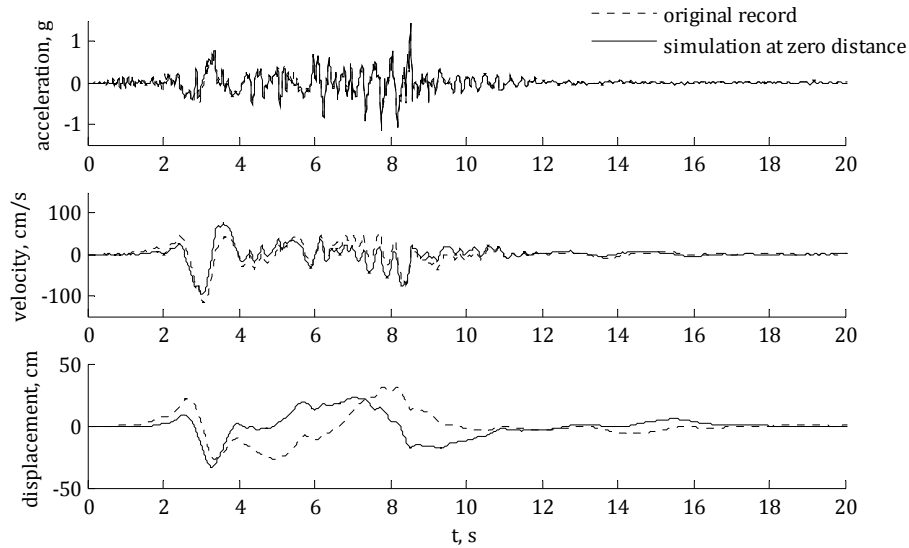


Figure 2. Comparison of the original record with the simulation at zero distance

histories, respectively, for an example simulation. Fig. 4 shows the mean response spectra (for 5% damping) of the 10 simulations for each support. The spectrum corresponding to abutment 1 is that of the conditional simulation at zero distance (same in all 10 simulations) and is more jagged compared to the average spectra for the other supports. It is characterized by two peaks, with the first characterizing the low frequency content of the motion and the second corresponding to the pulse-type motion occurring, approximately, from 2.5s to 4s. For simulations at increasing distances, the first peak becomes higher, whereas the second peak is gradually smoothed out, indicating the loss of coherency of the pulse. Fig. 5 compares the response spectrum of the original record with the response spectrum of the simulated motion at zero distance and the mean response spectrum of all simulated support motions.

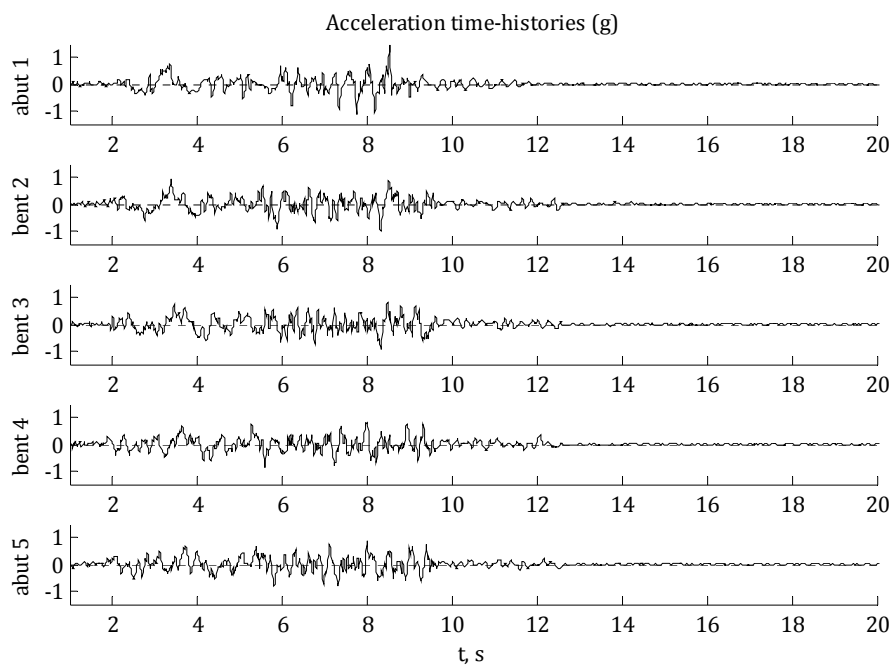


Figure 3a. Example set of simulations: Acceleration time-histories

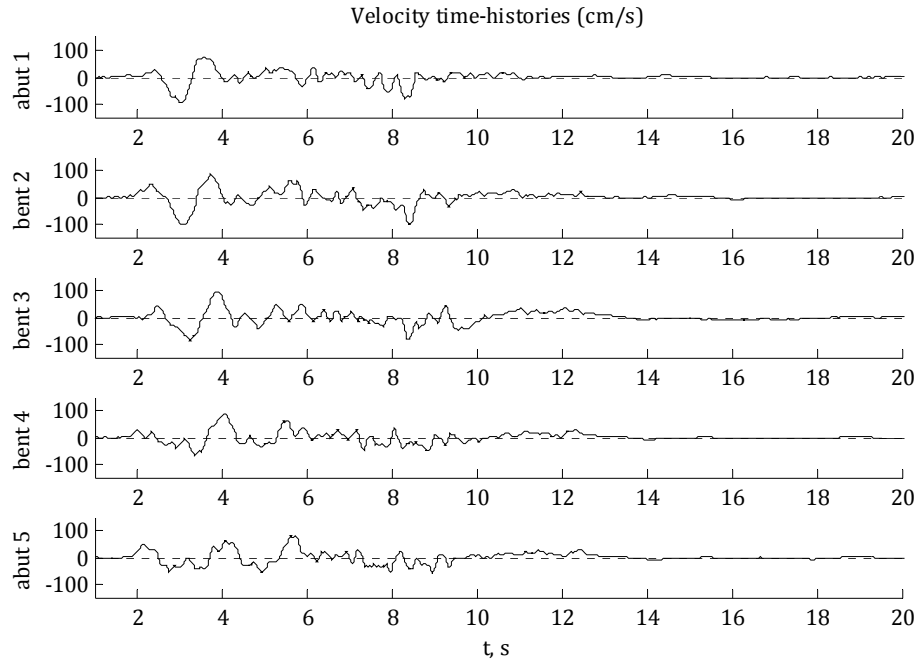


Figure 3b. Example set of simulations: Velocity time-histories

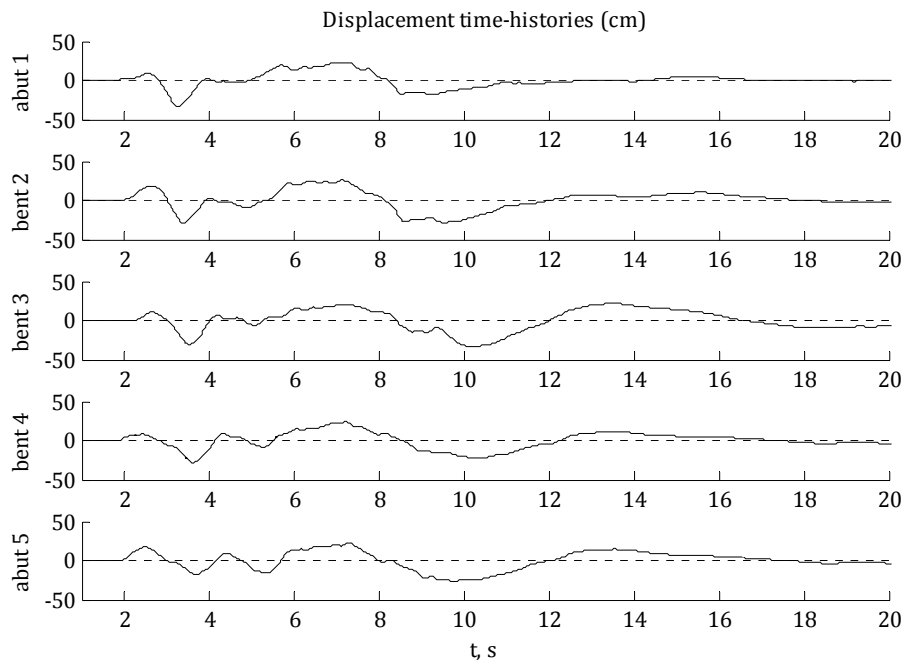


Figure 3c. Example set of simulations: Displacement time-histories

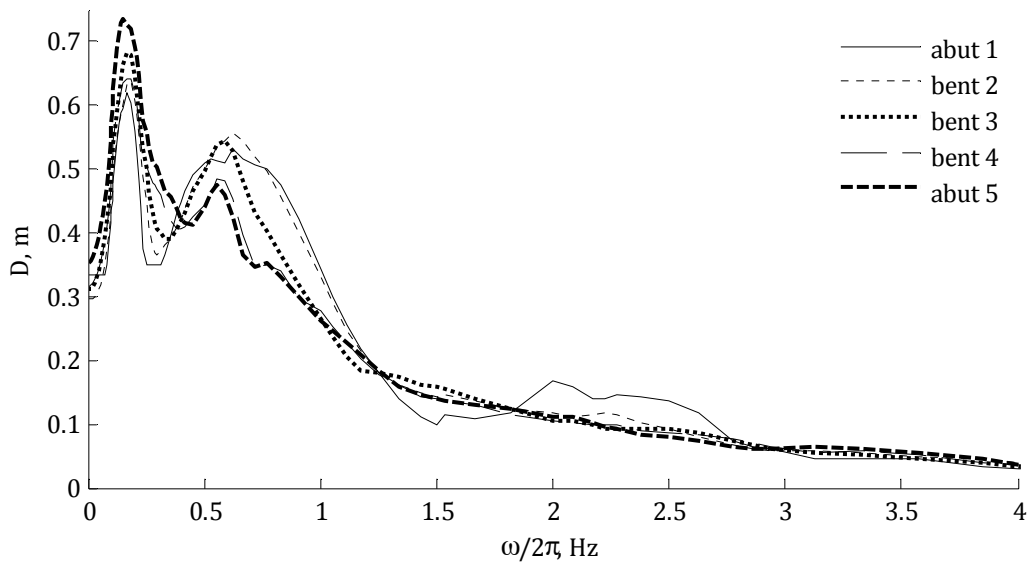


Figure 4. Mean response spectra for each support point

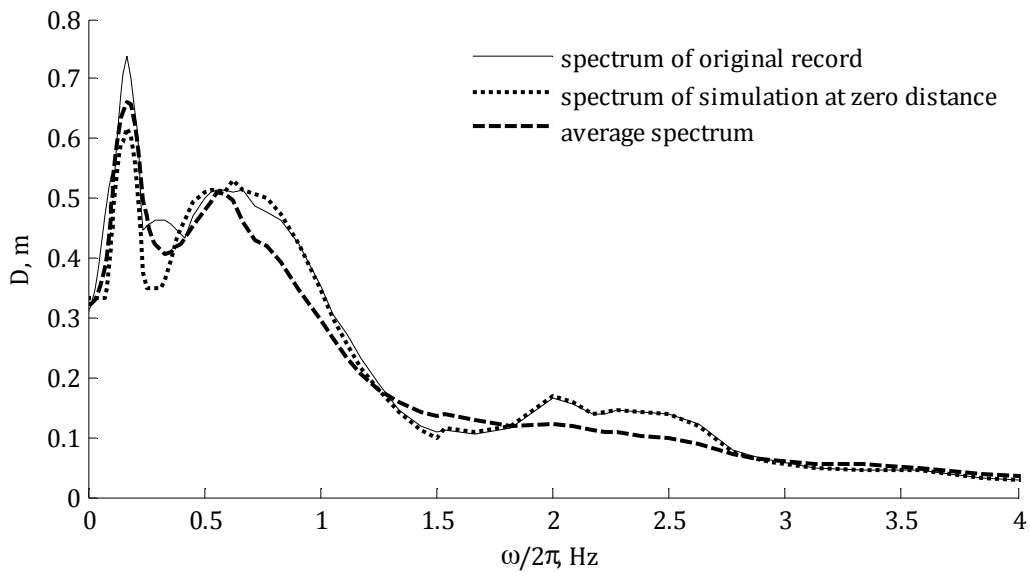


Figure 5. Comparison of response spectra

## Results

In this section, we compare the peak response estimates from MSRS with the results from linear and non-linear RHAs for the pier drifts in the transverse direction. The response spectrum considered in the MSRS method is the average spectrum of all support motions. The MSRS results are evaluated with a MATLAB code developed by the first author, whereas the RHA results are obtained with the structural analysis software Opensees.

Figs. 6 and 7 show the time-histories of the responses of the linear and non-linear models, respectively, together with the peak MSRS estimates. It is noted that all sets of simulated support

motions cause the non-linear model to deform in the inelastic region. For each pier, Table 1 compares the mean peak responses evaluated from linear and non-linear RHAs with the MSRS estimates. The standard deviations of the RHA results are also listed. Since the sets of support motions are generated by conditioning on the same motion at abutment 1, the simulations at increasing distances from this point exhibit larger variations. This justifies the larger values of standard deviation for piers 3 and 4. In assessing the performance of the MSRS method, we should consider that the latter is intended for use in conjunction with smooth design response spectra that represent an ensemble of ground motions. However, the 10 sets of support motions used in this example are simulations conditioned on the same record and the excitation at abutment 1 is the same in all 10 cases. Another source of error in the MSRS estimate is that the mean response spectrum for each support exhibits variations from the average response spectrum considered in the MSRS method, as shown in Figure 4. Considering the limitations of this application, the MSRS estimates are satisfactory: The mean errors are  $-5.5\%$  (st.d. =  $9.8\%$ ) and  $1.0\%$  (st.d. =  $11.4\%$ ) comparing with the results from linear and non-linear RHA, respectively.

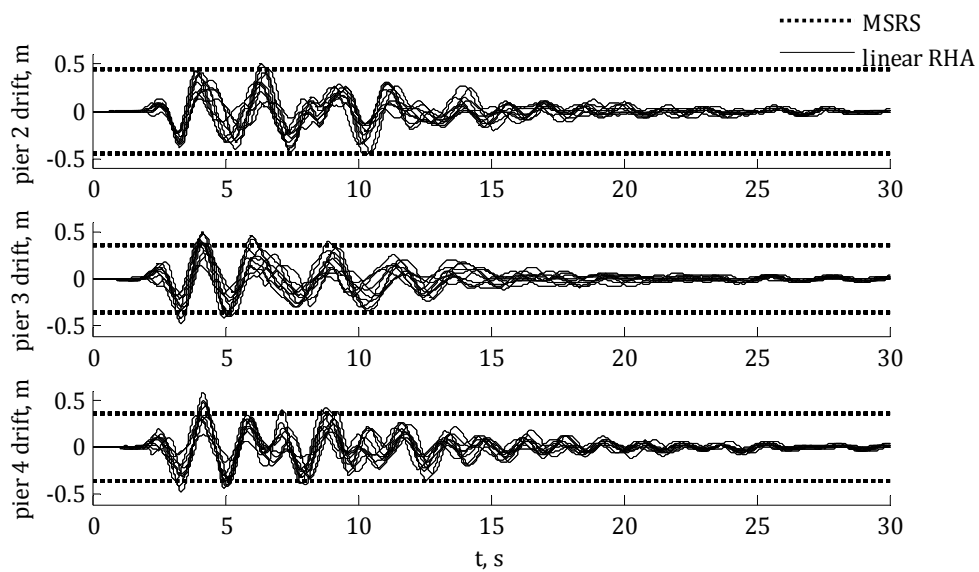


Figure 6. Comparison of the linear RHA results with the MSRS estimates

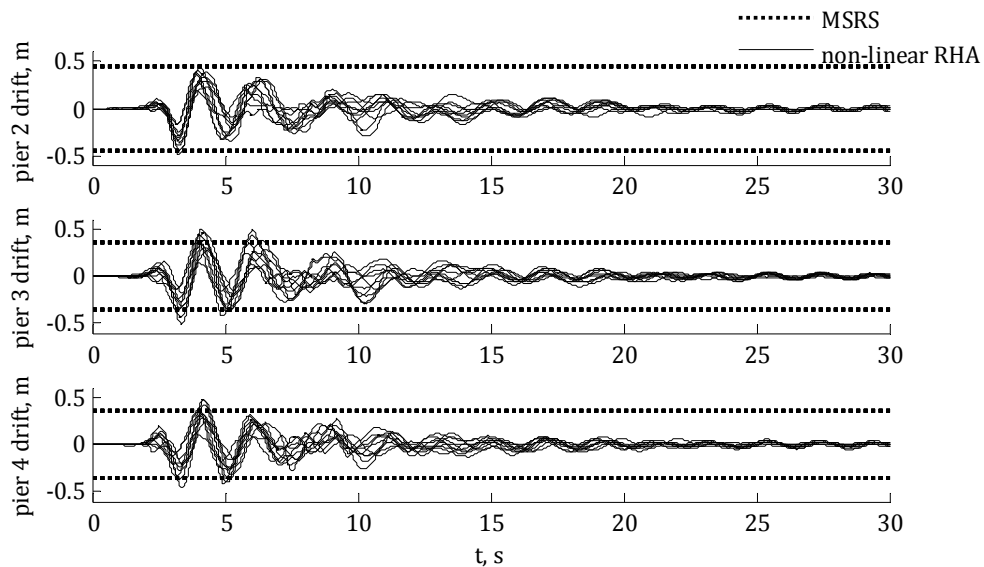


Figure 7. Comparison of the non-linear RHA results with the MSRS estimates



Table 1. Transverse pier drifts

location	MSRS (m)	linear RHA (m)		non-linear RHA (m)	
		mean	st.d.	mean	st.d.
pier 2	0.444	0.421	0.057	0.393	0.061
pier 3	0.359	0.393	0.081	0.398	0.086
pier 4	0.361	0.417	0.080	0.362	0.094

In order to compare the linear and non-linear responses, we introduce the ratio of non-linear over linear demand, denoted  $C_\mu$ . The ductility demand,  $\mu$ , defined as the ratio of the peak non-linear drift over the yield drift, is used to quantify the level of inelastic response for the non-linear model. Figure 8 shows the pairs  $(\mu, C_\mu)$  for each pier and set of support motions, evaluated by (i) using the linear demands from RHA and (ii) using the linear demands from MSRS. Some level of linear correlation is apparent in case (ii). Table 2 lists the mean and standard deviation of the ductility demand, as well as the mean and standard deviation of  $C_\mu$  for the two cases. The proximity of  $C_\mu$  to unity in both cases verifies the equivalent displacement rule and justifies the use of the MSRS method to estimate non-linear structural demands for differential support motions. Of course, these observations only refer to the particular case-study and no generalization is suggested at this point. The problem of relating the MSRS estimates with results from non-linear RHA for various cases of excitation and different bridge models is currently under investigation.

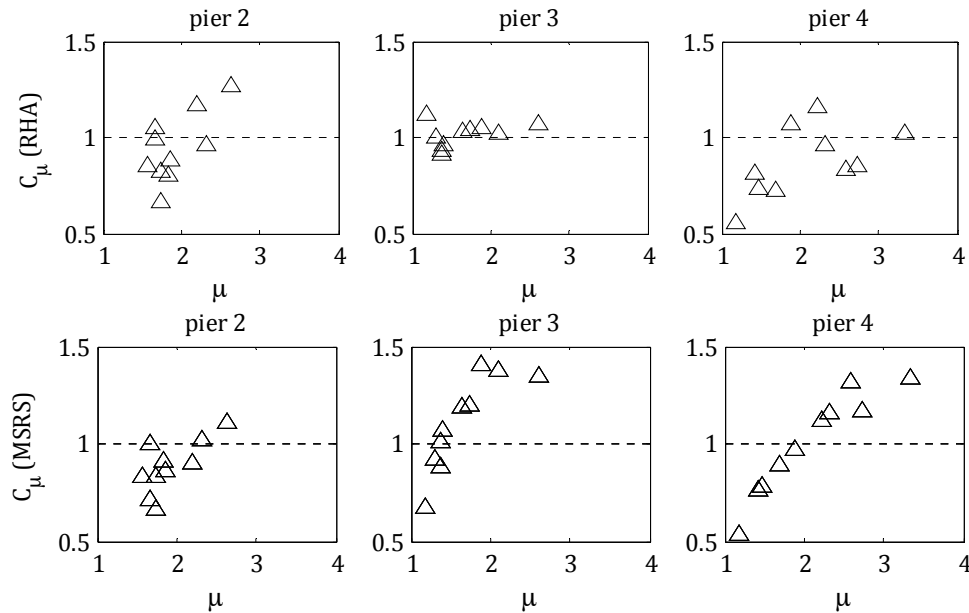


Figure 8. Ratios of non-linear over linear demands versus ductility

Table 2. Measures of non-linear response

location	$\mu$		$C_u$ (RHA)		$C_u$ (MSRS)	
	mean	st.d.	mean	st.d.	mean	st.d.
pier 2	1.92	0.35	0.95	0.18	0.88	0.14
pier 3	1.66	0.44	1.01	0.06	1.11	0.24
pier 4	2.09	0.68	0.87	0.18	1.00	0.26

## Conclusions

The peak pier drifts of an existing bridge model subjected to differential support motions were evaluated with linear and non-linear response history analyses and the MSRS method. The RHAs were performed for 10 different sets of support motions, generated with the CPDF method and conditioned on the same earthquake record. The mean values of the RHA results were compared with the results from MSRS analysis, in which the support motions were described by the average spectrum of all simulations. The MSRS method provides fairly accurate estimates of the mean peak linear and non-linear responses. The possibility of using the MSRS method in conjunction with appropriate modification factors (if necessary) to estimate the non-linear bridge response is currently under investigation.

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