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CRITICAL ASPECTS OF FINITE ELEMENT MODELING OF RC STRUCTURES FOR SEISMIC PERFORMANCE ASSESSMENT

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ABSTRACT

Finite-element modeling of RC structures involves making a series of assumptions related to the distribution of forces and deformations within the structure. In an extensive variety of possible modeling strategies, the engineer is challenged to build an effective numerical model which can be used to quantify the performance of the structure in an efficient manner. In this study a fundamental component of the FE modeling which is the discretization of the model is discussed. The influence of the adopted discretization scheme on the simulated response is presented for a set of representative RC components. Both displacement-based and force-based beam-column finite-elements with two nodes are considered in the analysis. The results obtained using alternative meshing schemes are compared to estimates obtained using the empirical approach of the equivalent plastic hinge length method, as well as against experimental evidence. Additionally, the impact of the adopted meshing scheme on the predicted dynamic response of structures is investigated. The response of a series of RC columns and walls tested on shaking table are considered for this purpose. The simulated maximum and residual displacements are compared to the measured values. Based on the obtained results, preliminary recommendations are developed for the identification of appropriate meshing schemes.

Introduction

During the FE analysis of the inelastic response of RC structures by means of beam-columns elements, discretization scheme and element formulation have a direct influence on the simulated distribution of curvatures along the structural elements, i.e. beam, columns and walls. Many researchers have investigated localization phenomena related to the distribution of post-elastic strains in nonlinear finite-element analysis under monotonic loads (e.g. Bathe 1996, Coleman and Spacone, 2001). In this study, a series of discretization strategies are developed for modeling slender RC structures and their performance in predicting the dynamic inelastic response is assessed using shaking table test data. Particularly, important aspects relevant to the

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selection of discretization schemes for capacity designed reinforced concrete (RC) structures deforming predominantly under flexure are investigated.

The analyses presented here where carried out in the framework of a broader study aiming at developing a methodology to improve the seismic performance assessment of RC structures making use of post-earthquake residual displacements (Yazgan 2009). Key questions at the beginning of the study where: (1) How reliably can residual displacement be predicted and (2) which numerical models are better suited to predict residual displacement. Both questions are addressed in this paper.

Beam-Column Finite Element Models

Slender RC structures are typically modeled using beam-column finite elements with 2 nodes per element (CEB 1996). In beam-column element modeling, the response of individual structural components is defined according to functions called "element formulation". These functions relate the assumed local force-deformation or stress-strain behavior to the force-deformation behavior of the whole element. Post-elastic flexural deformations typically do not localize to a critical section but spread along the RC components. Several finite element formulations have been proposed to capture this spreading (CEB 1996). These formulations can be grouped into two classes based on the approach that is adopted in their derivation: (1) the displacement- and (2) the force-based formulation.

Displacement-based Formulation

In the displacement-based element formulation (i.e. stiffness-based approach), the curvature and the axial deformation at any section along the element are expressed as a function of the member end-deformations (Bathe 1996). This relationship is defined by means of displacement shape interpolation functions. Often, the curvature is assumed to vary linearly in a single element. Therefore to be able to capture the nonlinear spread of plasticity along the RC component, the latter must be discretized into a number of displacement-based elements.

Force-based Formulation

The force-based element formulation (i.e. flexibility-based approach) is based on the interpolation of member end-forces to identify the internal section forces (Spacone *et al.* 1996). As a result, for a linear variation of the moment along the component, the behavior of the latter can be captured by utilizing a single element even for deformations reaching beyond the elastic limit. However, due to localization issues related to deformations, the adopted meshing scheme has still an influence on the simulated response (Coleman and Spacone 2001). Therefore, regions where post-elastic deformations are likely to be computed must be discretized such that these deformations do not localize abruptly or spread excessively.

Important remark

It is important to note that beam-column elements implemented according to both the displacement based and the force based formulations typically assume that plane sections along the element remain plane during the analysis. This assumption implies that only the component of the spread of plasticity due to hardening can be captured correctly while the component due to tension shift, which is typical for RC structural elements, can not.

Shaking Table Test Data

The results of shaking table tests are utilized to assess the reliability and the sensitivity of local and global deformations simulated numerically. To this end, 12 shaking table tests are considered. Important properties of the relevant RC test units are presented in Table 1. The first 4 test units (i.e. A1, A2, B1, B2) are the RC columns tested by Hachem *et al.* (2003). Column EBII07 is an RC unit tested at the ETH Zurich. WDH1 to 6 are the RC walls tested by Lestuzzi *et al.* (1999). CAMUS3 is a 5 story RC wall tested by Combescure and Chaudat (2000).

Table 1. Important properties of the test units (columns 3-7), yield and ultimate drift limits (columns 8-9), and measured maximum and residual drift ratios (columns 10-12).

#	*Unit*	*Scale*	n_s	ρ_l	n_{ax}	L_p	Average drift, d_a [%]				$d_{a,r}/d_{a,m}$
		[-]	[-]	[%]	[%]	[mm]	Yield, $d_{a,y}$	Ultimate, $d_{a,u}$	Maximum, $d_{a,m}$	Residual, $d_{a,r}$	[-]
1	A1	1/4.5	-	1.17	5.7	336	1.03	9.2	5.10	0.55	0.11
2	A2	1/4.5	-	1.17	5.7	336	1.03	9.2	3.56	0.12	0.03
3	B1	1/4.5	-	1.17	5.7	336	1.03	9.2	4.98	0.62	0.13
4	B2	1/4.5	-	1.17	5.7	336	1.03	9.2	2.98	0.07	0.02
5	EBII07	-	-	0.96	9.0	192	2.11	16.8	8.12	0.38	0.05
6	WDH1	1/3	3	0.48	2.0	311	0.39	1.7	0.71	0.01	0.02
7	WDH2	1/3	3	0.48	2.5	307	0.39	1.8	2.22	0.08	0.04
8	WDH3	1/3	3	0.47	1.8	352	0.39	2.1	1.84	0.13	0.07
9	WDH4	1/3	3	0.47	1.8	356	0.39	2.6	1.52	0.21	0.14
10	WDH5	1/3	3	0.60	1.8	354	0.45	1.9	1.2	0.11	0.09
11	WDH6	1/3	3	0.60	1.6	370	0.45	2.1	1.22	0.12	0.10
12	CAMUS3	1/3	5	0.72	4.0	539	0.24	2.7	0.83	0.13	0.15

n_s: Number of stories; ρ_l: Longitudinal reinforcement ratio; n_{ax}: Axial load ratio; the plastic hinge length L_p as well as $d_{a,y}$ and $d_{a,u}$ are estimated according to the recommendations by Priestley *et al.* (2007).

For each unit, only the first dynamic test that resulted in inelastic deformations is considered in this study. All test units are slender and were provided sufficient transverse reinforcement to prevent a shear failure. In the relevant test reports it is reported that the cracks that opened during the considered tests were mainly horizontal and located in the bottom segment of the test units, hence confirming that they behaved predominantly in flexure.

Modeling of the Test Units

The finite-element models considered here are built using OpenSees (McKenna *et al.* 2007). A fiber-section model is employed to represent the flexural hysteretic behavior at the section level. The uniaxial stress-strain behaviors of the reinforcement and of the concrete fibers are modeled using the *Steel02* and the *Concrete04* models, respectively. Two alternative element formulations, i.e. (1) the displacement-based and (2) the force-based formulation, are adopted to model the test units. Discretization strategies are established to achieve a consistent modeling for the entire set of tests. When establishing these discretization strategies, two issues are considered: (1) the strain localization in FE models and (2) the spread of plasticity in RC members. The strain localization issue in FE modeling refers to the sensitivity of the simulated strain distribution to the adopted displacement interpolation functions and the numerical integration scheme. The spread of plasticity refers to the spreading of deformations in plastic hinge regions of RC components that yield in flexure. The processes of establishing the discretization strategies for the force- and displacement-based element models are presented below.

Discretization Strategies

Displacement-based Element Models (DF)

The models employing the displacement-based fiber-section (DF) element formulation are modeled using the *dispBeamColumn* element of OpenSees (McKenna *et al.* 2007). Cubic Hermitian polynomials are utilized as the shape interpolation functions for the displacements transverse to the element axis. Thus, the curvature varies linearly along each element. In the models considered here, two Gauss-Legendre integration points are utilized in each element.

In order to demonstrate the strain localization issue, three alternative models of column EBII07 are considered (Fig. 1a). In all three models, two displacement-based elements are used. In all three models, the lowest node (i.e. the fixed support) is located at the strain penetration depth L_{sp} computed according to Priestley et al. (2007). The length L_1 of the bottom element is different for each model. The equivalent plastic hinge length L_p, also computed according to Priestley et al. (2007), is employed as an index value for L_1. For the first, second and third model, L_1 is equal to $0.5L_p$, L_p and $2L_p$, respectively. The horizontal force applied at the top node versus average drift behaviors obtained using the three models are plotted in Figure 1b. The average drift ratio corresponding to a significant drop in shear resistance is marked for each model. The drop corresponds to the exceedance of ultimate compressive strain for the confined concrete fibers at the lowest integration section (*S1*). The average drift exhibited when the resistance drops, increases with the increasing of L_1. The primary reason behind this is the dependence of the simulated curvature distribution to the discretization (Fig. 1c). In all models, the plastic deformations are localized within the lowest element. Hence, the drift corresponding to the failure at *S1* is the largest for the model with the longest L_1.

As already mentioned, in RC members deforming under flexure, post-elastic deformations spread along the length of the member. The exact prediction of the distribution of those inelastic deformation is a challenging task and the estimation of the corresponding global deformations is made difficult by the fact that due to tension shift sections do not remain plane, hence – strictly speaking – curvatures can no longer be defined. However, the concept of curvature as a measure of local deformation and the double integration thereof to obtain global deformations is very appealing to engineers. For this reason, a common approach for predicting the ultimate displacement capacity of such an RC member consists in assuming the idealized curvature distribution shown in Fig. 2a and whose integration allows the estimation of the ultimate drift capacity shown in Fig. 2b. In this so called *plastic hinge method*, the length over which a constant plastic curvature is assumed is called the plastic hinge length L_p.

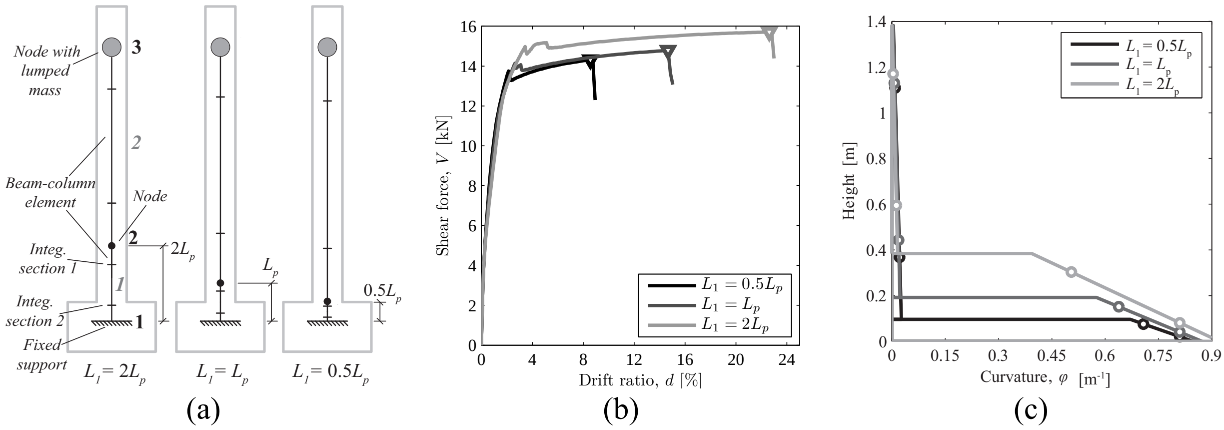

Figure 1. Discretization schemes for modeling column EBII07, where L_1 is the length of the bottom element (a), base shear versus average drift relationship obtained using displacement-based elements (b), and curvature profiles simulated when the lowest integration section *S1* reaches the ultimate curvature ϕ_u (c).

The idealized curvature distribution in Fig. 2a has a shape similar to that of the simulated curvature distributions in Fig. 1c. This similarity suggests L_I=L_p to be a suitable discretization strategy. For the DF model with L_I=L_p, Fig. 1c shows that the curvature ϕ_I at the integration section *S1* reaches ϕ_u when the average drift of the column is equal to 14.9%. This value is very close (89%) to the d_u predicted using the plastic hinge method shown in Fig. 2b. The suitability of this discretization scheme (L_I=L_p) was also investigated for the other test units listed in Table 1. Pushover analyses were carried out for each test unit. In each analysis, L_I was made equal to a series of values. Figure 2c shows the drifts obtained in the analyses when ϕ_I=ϕ_u at *S1* divided by the ultimate drifts d_u estimated using the plastic hinge method, confirming the discretization scheme L_I=L_p to be suitable.

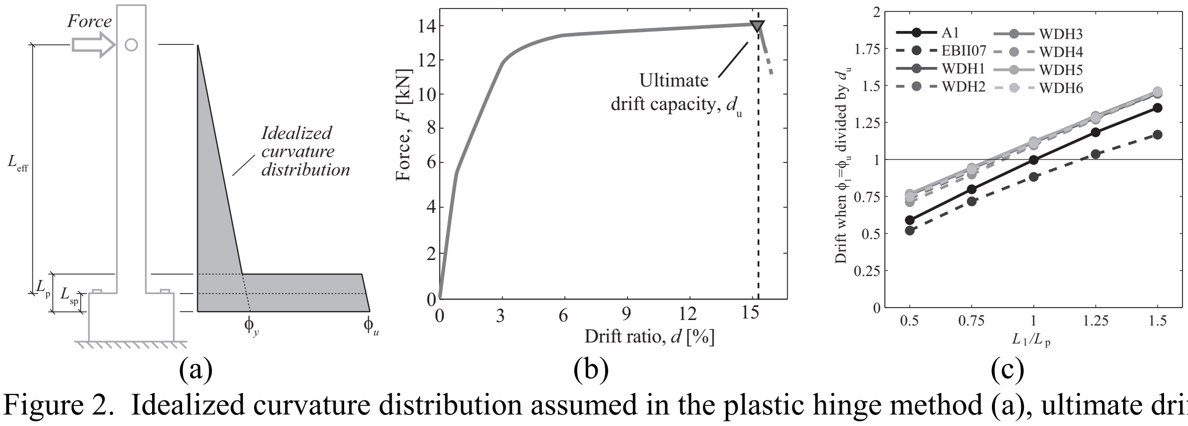

Figure 2. Idealized curvature distribution assumed in the plastic hinge method (a), ultimate drift capacity (b), and ratio of the drift – simulated when the curvature at the lowest integration section ϕ_I reaches the ultimate ϕ_u – to the ultimate drift d_u estimated using the plastic hinge length method (c) (L_1: Length of the bottom element).

Apart from the discretization scheme, the simulated curvature distribution is also sensitive to several other factors such as: the section hysteretic model (e.g. fiber-section model, Takeda hysteretic model), the numerical integration rule and the number of integration points.

An extensive investigation campaign was carried out to address these issues. Because of space limitations not all results can be presented here and for more details the reader is referred to Yazgan (2009).

Force-based Element Models (FF)

The models employing the force-based fiber-section (FF) element formulation are discretized using the *nonlinearBeamColumn* element available in OpenSees (McKenna *et al.* 2007). This element is based on the linear interpolation of forces and moments along its length. The distribution of curvatures is not constrained by any specific deformation shape function and the end displacements are related to the section deformations in an integral sense (Coleman and Spacone 2001).

The sensitivity of the predicted response to the discretization scheme is demonstrated by analyzing again the alternative models of column EBII07 shown in Fig. 1a. However, in this case FF instead of DF elements are used, and a 2-point Gauss-Lobatto integration scheme is utilized for all elements. Figure 3a displays the base shear versus average drift relationship for all three models, while the curvatures simulated at the integration sections *S1* and *S2* are plotted against the average drift in Fig. 3b. The models fail when the curvatures at their lowest integration section S1 exceed ϕ_u. For all discretization schemes, after the curvature ϕ_1 at *S1* exceeds the yield curvature ϕ_y, the curvature increments at the two sections start to diverge significantly. The largest portions of the inelastic curvatures localize at S1.

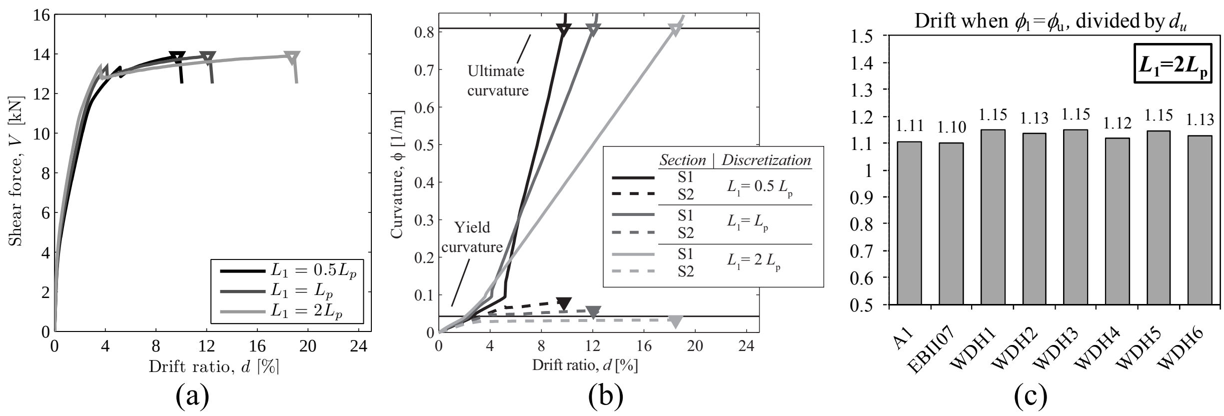

Figure 3. Base shear versus average drift relationships obtained using force-based elements (a), section curvature versus average drift (b), and ratio of the drift – simulated when the curvature ϕ_l at the lowest integration section *S1* reaches ϕ_u – to the ultimate drift d_u estimated using the plastic hinge length method (c) (L_1: Length of the bottom element).

The element end-displacements that are free of the rigid body modes can be identified by numerically integrating the section deformations. For the numerical integration, a tributary length L_{tr} which is proportional to the weight of the integration section is considered for each section. Simulated curvature distributions similar to the idealized profile shown in Fig. 2a can be obtained by setting $L_{tr,S1}$ of the section *S1* equal to the plastic hinge length L_p. Accordingly, L_1 must be equal to $2L_p$ when a 2-point Gauss-Lobatto integration scheme is utilized. Note that this modeling approach was also recommended by Coleman and Spacone (2001).

In order to assess the simulation results obtained using the L_1=2L_p discretization scheme, test units of Table 1 were modeled using force-based elements. For each unit, the drift ratio simulated when the curvature ϕ_1 at S1 reaches the ultimate curvature ϕ_u is identified. As depicted in Fig. 3c these drift ratios are found to be in good agreement with the d_u's estimated using the plastic hinge length method, showing an overestimation ranging between 10 and 15%.

Other Model Idealization Parameters

The results of the numerical simulations are sensitive to a number of idealization parameters other than the adopted element formulation and the discretization scheme. Those parameters were established consistently for every test unit. Details related to establishing the uniaxial material models, the damping models, the boundary conditions, the second order effects and the analysis algorithm are reported extensively in Yazgan (2009).

Evaluation of the Models

In this section, the simulated maximum and residual average drifts are compared against the experimental evidence. The ratio of the value measured during the test to the value predicted by means of the time-history simulations is calculated for each simulation. This ratio being larger than unity implies that the numerical model underestimates the measured value. The probabilistic character of the ratio is assessed for each modeling approach[2].

Accuracy of the Simulated Maximum and Residual Drift Ratios

The simulated maximum average drift ratio values are plotted against the measured values in Fig. 4. The estimated probabilistic character of the test-to-simulation ratios are presented by a dashed line and a shaded region. The dashed line corresponds to the median ratio and the shaded area highlights the region lying between the 10 and the 90 percentile values. At the top of each figure the median and the coefficients of variation (COVs) are presented.

In Fig. 5, the predicted residual average drift values are compared to the measured values. The numerical models tend to underestimate the residual drifts. The accuracy of the models in terms of predicting the residual deformations is significantly lower compared to the accuracy achieved for the maximum deformations; this applies for both element formulations (Figs. 5a and 5b). This is expectable because the accuracy of the simulation at any time step depends on the accuracy of the response predicted from the beginning of the simulation up to that time step. As a result, the accuracy of the predicted residual displacements depends on the errors exhibited during the full-course of the simulation process. On the contrary, the peak deformation is typically attained at a much earlier time during the response. Moreover, the lower accuracy is also due to the fact that the residual displacements are much smaller compared to the maxima. For the test units considered in this study, the residual average drift ratios are in

[2]The results from all test units except WDH2 are considered in the identification of the probabilistic character of the ratio. WDH2 failed during the first test due to rupture of a longitudinal reinforcement bar. However, its results are included in the scatter plots of Figs. 4 and 5 to display as an example the probable inaccuracy exhibited in this case.

the range of 2 to 15% of the maxima (Table 1). As a result, if the simulation fails to predict both the maximum and the residual deformations by the same amount, the resulting relative error associated with the predicted residual value is much larger than that associated with the maximum. The reasons behind the inaccuracy of the simulated residual displacements are investigated and extensively discussed in Yazgan (2009).

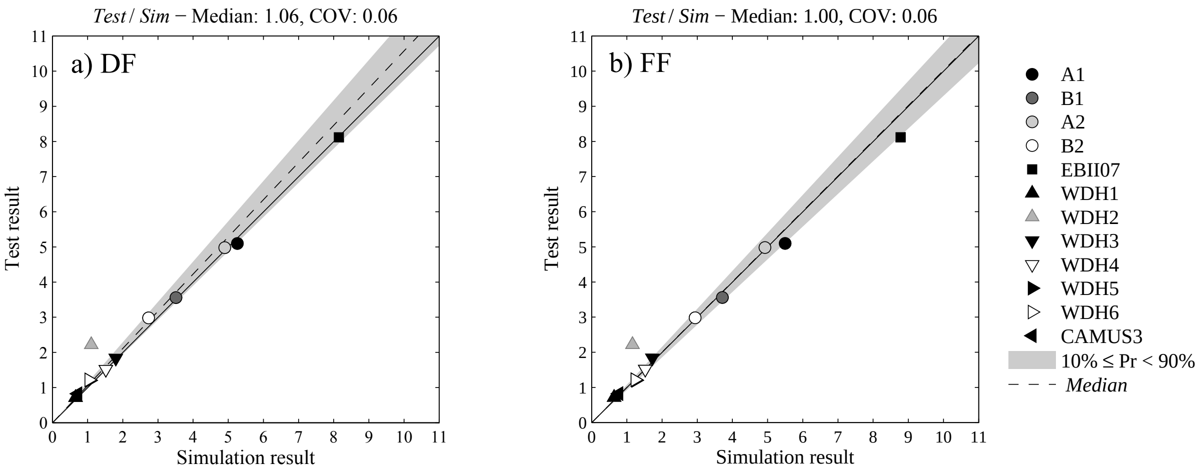

Figure 4. Comparison of the measured and the simulated maximum average drift ratios $d_{a,m}$ [%] obtained using displacement-based (a) and force-based (b) elements.

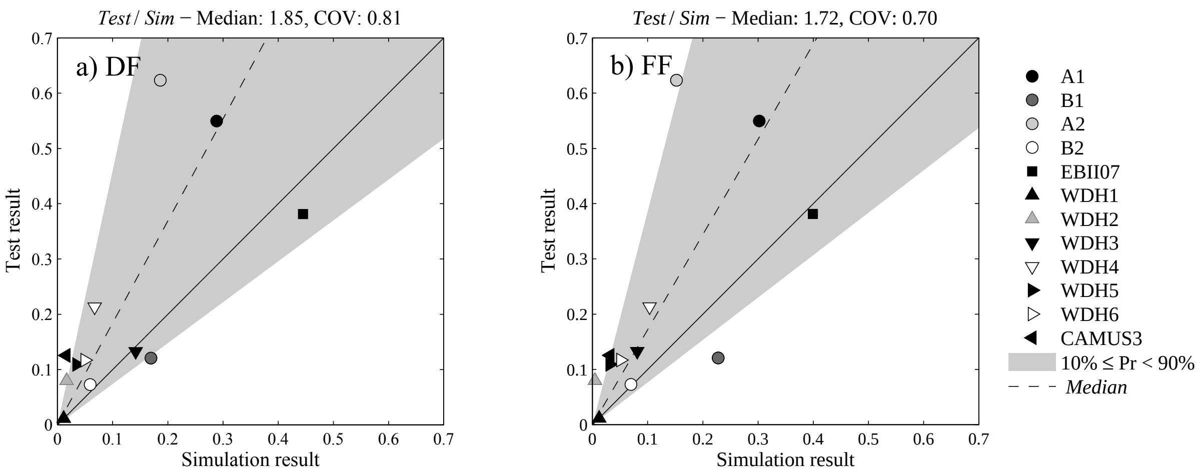

Figure 5. Comparison of the measured and the simulated residual average drift ratios $d_{a,r}$ [%] obtained using displacement-based (a) and force-based (b) elements.

Sensitivity of the Accuracy of the Simulated Drift Ratios

In order to assess the sensitivity of the accuracy of the predicted maximum and residual average drift ratios to the adopted discretization scheme, the time-history simulations are repeated for each test unit using different values of the length L_1 of the bottom element, i.e. the element where all plastic deformations are concentrated. For each model, the length L_1 calculated according to the discretization strategies explained in the previous section are referred to as the reference length $L_{1,Ref}$. For the sensitivity analyses, the length L_1 for each unit is set equal to specific ratios of $L_{1,Ref}$ and the response history simulations are repeated. For each set of simulation results, the median and the COV of the test-to-simulation ratios are calculated. The accuracy of the residual displacements is found to be significantly sensitive to the discretization scheme (Fig. 6). On the other hand, this sensitivity is less pronounced in the case of maximum displacements. The results obtained for the displacement-based (DF) and force-based (FF) element models show similar trends.

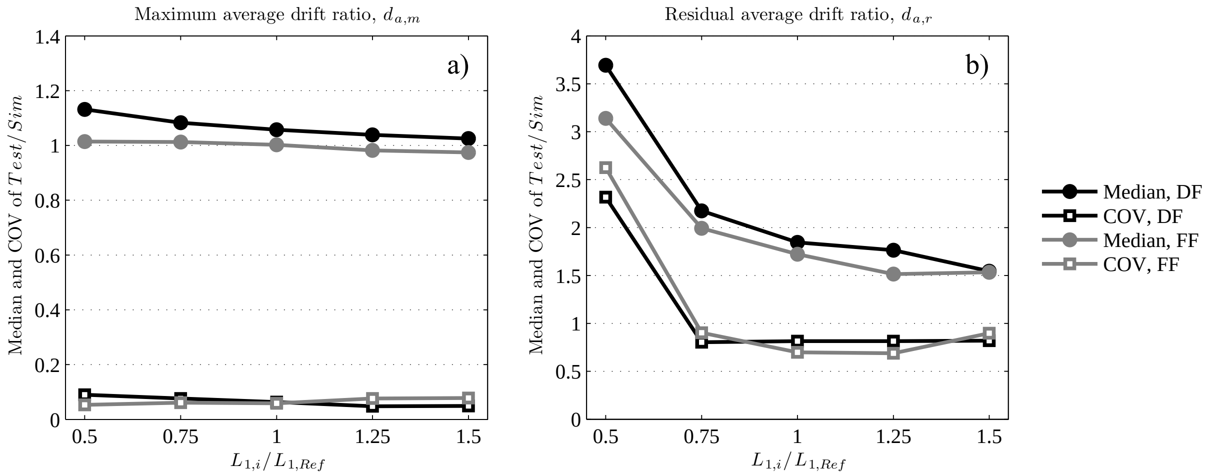

Figure 6. Sensitivity of the accuracy of the simulated maximum (a) and residual (b) average drift ratios to the discretization scheme (L_1: Length of the bottom element).

Conclusions

The discretization of the structure into finite-elements is an important step in the numerical assessment of the inelastic response of structures. Several factors such as the location of the essential degrees of freedoms of the model, the element formulation and the localization of strains in the numerical model should be taken into account appropriately when discretizing structures (Bathe 1996, Coleman and Spacone 2001). Spreading of plasticity affects the deformation capacity of RC structures significantly, hence needs also to be taken into account appropriately. Common beam-column element formulations assume plane sections remaining plane, hence not allowing a direct consideration of spreading of plasticity.

To circumvent this limitation, the discretization strategies proposed here aims at achieving a simulated curvature distribution that is in agreement with the idealized distributions assumed in the plastic hinge length method. The accuracy of the simulated response histories obtained using the proposed discretization schemes are investigated by numerically reproducing a set of shaking table tests. The comparison of the calculated response histories against those measured during shaking table tests indicates the following points:

- The adopted finite-element discretization scheme has a strong influence on the accuracy of the simulated residual displacements. Its influence on the simulated maximum displacements is smaller than the one on residuals.
- The accuracy of the considered finite-element models, in terms of estimating the residual displacements is found to be much lower than the accuracy of the peak displacements.

- The statistical comparison of the experimental and the numerical results allowed the quantification of the model error associated with the estimation of both maximum and residual displacements.
- The idealized curvature distributions assumed in the plastic hinge length method can be used as a benchmark to assess the soundness of the curvature distributions obtained using a given discretization scheme and element formulation.
- Given that the components are discretized to yield a simulated curvature distribution that agrees with the plastic hinge length method, the adopted element formulation do not significantly influence the accuracy of the simulated displacements.

Acknowledgments

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