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APPROXIMATE METHODS FOR ESTIMATING HYSTERETIC ENERGY DEMAND ON UNIAXIAL PLAN-ASYMMETRIC BUILDINGS

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ABSTRACT

Hysteretic energy dissipated in a structure due to plastic deformation during seismic ground motion gives a much better estimate of damage in the structure as compared to the maximum displacement or maximum inter-story drifts. However, the accurate estimation of hysteretic energy demand requires a complete nonlinear response-history analysis of the multi-degree of freedom model of the structure. This method is computationally intensive and hence not suitable for incorporating in design procedures. In this paper, two different multiple equivalent systems based approximate methods using the concepts of modal pushover analysis, in short MPA method, and 2D-MPA method are used to compute the hysteretic energy demand on a structure with uniaxial plan-asymmetry. The effectiveness of the proposed methods is measured by comparing energy estimates obtained using these methods with those obtained from the nonlinear response history analysis of the original multi-degree of freedom model for various earthquake scenarios. The applicability of the proposed methods is also checked for different heights of structure in addition to different degree of plan-asymmetry. The proposed methods are found to be conceptually very simple, light on computation and reasonably accurate alternatives that can be adopted for energy-based design and evaluation procedures.

Introduction

There can be several ways to express damage in a building when it is subjected to earthquakes. The most common philosophy adopted by researchers till now is the displacement-based approach. However, many researchers suggested that the hysteretic energy dissipated due to cyclic-plastic deformations occurring in a structure gives a better estimation of seismic damage as compared to peak displacements or inter-story drifts occurring in a structure (Fajfar 1996). This demand can take into account the dynamic nature of earthquake forces like duration, magnitude and frequency content of that particular earthquake, number of deformation cycles, cumulative damage at plastic hinges, etc. For example, if the structure undergoes several inelastic load

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reversals without having a large deformation in any of these cycles (more likely for a pulse-type ground motion), the cumulative damage can only be measured using a parameter based on hysteretic energy demand, and not by peak drift or ductility demands. The philosophy of energy-based seismic design gained importance after the publication of the Vision 2000 document (SEAOC 2000). This document recommended energy-based design as one of the advanced design options for future earthquake codes. The first step in an energy-based seismic design is the estimation of the hysteretic energy demand on a structure. This demand can be estimated using a nonlinear response history analysis (NLRHA) of the multi-degree of freedom (MDOF) model of the structure. This method is not complex but is very heavy computationally and also not suitable for design implementation. Therefore, alternative simple procedures for estimating hysteretic energy demand are needed. These simple methods will be equally useful for a hysteretic energy-based performance evaluation of an existing structure, as well.

Prasanth et al. (2008) proposed a modal pushover analysis (MPA)-based (Chopra and Goel, 2002) approximate method for estimating hysteretic energy demand on symmetric-plan structures. This method is computationally less demanding and gave satisfactory results when tested for low- to high-rise steel moment frame buildings. Their multiple modal equivalent systems based methodology is a computationally efficient alternative to the NLRHA of the MDOF system. Two approximate methods are proposed in this paper by extending the method proposed by Prasanth et al. to hysteretic energy estimation of uniaxial plan-asymmetric structures: one is based on the concept of MPA for plan-asymmetric structures (Chopra and Goel, 2004) and the other is based on the 2D-MPA method developed by Lin and Tsai (2007). The proposed methods are validated by comparing the approximate estimates to the 'exact' estimates from the NLRHA of the MDOF systems for low- to high-rise steel moment frame buildings with varying degrees of plan asymmetry.

MPA-Based Method for Plan-Asymmetric Structures

The modal pushover analysis (MPA) was developed to estimate the force/moment and displacement/rotation demands of MDOF system using multiple equivalent SDOF systems for symmetric structures (Chopra and Goel 2002) and for asymmetric structures (Chopra and Goel 2004). An MPA-based method was then developed by Prasanth et al. (2008) to estimate the hysteretic energy demand on symmetric structures. The fundamental concepts of this method are now used to develop a new method to estimate seismic energy demand of asymmetric structures.

The governing equation for an N-story lumped mass structure subjected to unidirectional ground motion is given as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{f_s}(\mathbf{u}, sign\dot{\mathbf{u}}) = -\mathbf{M}\boldsymbol{\iota}_x \ddot{u}_{gx}(t) \tag{1}$$

where $\mathbf{M}$ is a $2N \times 2N$ diagonal mass matrix of the structure and includes two diagonal sub-matrices $\mathbf{m}$ and $\mathbf{I_o}$, each of order $N \times N$. $\mathbf{m}$ is the diagonal mass matrix containing the translational floor masses and $\mathbf{I_o}$ is the diagonal mass matrix of the rotational floor masses i.e. the polar moment of inertia of floor masses about a vertical axis through the center of mass at each floor. $\boldsymbol{\iota}_\mathbf{x}$ is the influence vector associated with an x-direction ground motion.

$$\mathbf{M} = \begin{bmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_0 \end{bmatrix} \text{ and } \boldsymbol{\iota} = \begin{Bmatrix} \boldsymbol{\iota}_\mathbf{x} \\ \boldsymbol{\iota}_\boldsymbol{\theta} \end{Bmatrix} \tag{2}$$

The entire structure is divided into a number of modal equivalent systems using multiple pushover analyses, each corresponding to a specific mode of vibration. The free vibration properties of structure such as the mode shape $\boldsymbol{\varphi}_\mathbf{n}$, frequency ω_n and participation factor Γ_n in the nth mode are obtained from an eigenvalue analysis of the structure. The lateral force distribution for pushover analysis is expanded as a summation of modal inertia force distributions $\mathbf{s}_\mathbf{n}$:

$$\mathbf{M}\boldsymbol{\iota}_x = \sum_{n=1}^{2N} \mathbf{s}_\mathbf{n} = \sum_{n=1}^{2N} \begin{Bmatrix} \mathbf{s}_\mathbf{xn} \\ \mathbf{s}_\mathbf{\theta n} \end{Bmatrix} = \Gamma_n \sum_{n=1}^{2N} \begin{Bmatrix} \mathbf{m}\phi_{\mathbf{x}\mathbf{n}} \\ \mathbf{I}_0 \phi_{\theta \mathbf{n}} \end{Bmatrix}_{2N\times 1} \tag{3}$$

The displacement vector is assumed as

$$\mathbf{u} = \begin{Bmatrix} \mathbf{u}_\mathbf{x}(t) \\ \mathbf{u}_\boldsymbol{\theta}(t) \end{Bmatrix}_{2N\times 1} = \sum_{n=1}^{2N} \begin{Bmatrix} \mathbf{u}_\mathbf{nx}(t) \\ \mathbf{u}_\mathbf{n\boldsymbol{\theta}}(t) \end{Bmatrix}_{2N\times 1} = \sum_{n=1}^{2N} \begin{Bmatrix} \phi_{n\mathbf{x}} \\ \phi_{n\boldsymbol{\theta}} \end{Bmatrix}_{2N\times 1} q_n(t) \tag{4}$$

Substituting the expressions from Eq. 4 in Eq. 1 and after pre-multiplying with $\boldsymbol{\varphi}_\mathbf{n}$, the equation of motion of the nth modal ESDOF system can be written in terms of the modal coordinate $q_n(t)$:

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \frac{F_{sn}}{M_n} = -\Gamma_n \ddot{u}_{gx}(t) \tag{5}$$

where

$$\begin{aligned} &M_n = \boldsymbol{\varphi}_\mathbf{n}^\mathbf{T} \mathbf{M} \boldsymbol{\varphi}_\mathbf{n}, \Gamma_n = \frac{L_n}{M_n}, L_n = \boldsymbol{\varphi}_\mathbf{xn}^\mathbf{T} \mathbf{m} \boldsymbol{\iota}_\mathbf{n} \\ &F_{sn} = F_{sn}(q_n, sign\dot{q}_n) = \boldsymbol{\varphi}_\mathbf{n}^\mathbf{T} \mathbf{f}_\mathbf{s}(q_n, sign\dot{q}_n) \end{aligned} \tag{6}$$

For each mode the lateral force distribution for pushover analysis $\mathbf{f}_\mathbf{n}$ consists of story shears as well as story torques:

$$\mathbf{f}_\mathbf{n} = \begin{Bmatrix} \mathbf{f}_\mathbf{xn} \\ \mathbf{f}_\mathbf{\theta n} \end{Bmatrix} = \begin{Bmatrix} \mathbf{m}\phi_{xn} \\ \mathbf{I}_0 \phi_{\theta n} \end{Bmatrix}_{2N\times 1} \tag{7}$$

$\mathbf{f}_\mathbf{n}$ is applied at the centre of mass of each story. The structure is pushed up to some predefined displacement parameter. Based on previous recommendations (Prasanth et al. 2008), the pushover analyses are carried out up to a maximum inter-story drift of 2.5%. Thus we obtain the base shear versus roof displacement pushover plot for each mode. These plots are then bilinearized such that the areas under the original and approximating curves remain equal, and significant parameters (e.g., yield base shear V_{ny}, yield roof displacement U_{rny} and strain-hardening stiffness ratio α_{kn}) are obtained from these bilinearized plots. Stiffness and yield parameters of the nth mode equivalent SDOF (ESDOF) system are derived from these parameters as

$$K_n = \frac{\phi_{rnx} K}{\Gamma_n}, \frac{F_{sny}}{M_n} = \frac{V_{ny}}{L_n} \text{ and } q_{ny} = \frac{u_{rny}}{\phi_{rn}} \tag{8}$$

The modal hysteretic energy (E_{hn}) demand on the nth mode ESDOF system is calculated using NLRHA subjected to the specific ground acceleration. Hysteretic energy demand on the structure (E_{MPA}) is obtained by summing up the hysteretic energy demands of the different modal ESDOF systems at the end of the earthquake:

$$E_{MPA} = \sum_{n=1}^{N} E_{hn} \tag{9}$$

2D-MPA Based Estimation of Hysteretic Energy Demand on Asymmetric Structures

The second approximate method for energy demand estimation is based on the 2D modal pushover analysis (2D-MPA) for uniaxial plan-asymmetric structures proposed by Lin and Tsai (2007). The first few steps, upto the pushover analyses, in this method remain same as in the MPA-based method discussed earlier. However, here two pushover curves are plotted for each mode: roof displacement (D_{xn}) vs. base shear (V_{bn}) and roof rotation (θ_{rn}) vs. base torque (T_{bn}). These curves are converted to the *Acceleration-Displacement Response Spectrum* (ADRS) format using the following equations:

$$A_{xn} = \frac{V_{bn}}{\Gamma_{xn}^2 M_n}; A_{\theta n} = \frac{T_{bn}}{\Gamma_{\theta n}\Gamma_{xn} M_n}; D_{xn} = \frac{u_{rn}}{\Gamma_{xn}\phi_{rxn}}; D_{\theta n} = \frac{\theta_{rn}}{\Gamma_{xn}\phi_{r\theta n}} \tag{10}$$

where, A_{xn} and $A_{\theta n}$ are the nth modal "accelerations" in the translational and rotational directions, respectively, and D_{xn} and $D_{\theta n}$ are the corresponding "displacement" functions in the ADRS format pushover curve. These curves are then bilinearized, similar to the MPA-based method, to obtain initial slopes (K_{ixn} and $K_{i\theta n}$), yield accelerations (A_{yxn} and $A_{y\theta n}$), and strain-hardening stiffness ratios (α_{xn} and $\alpha_{\theta n}$) in translational and rotational directions, respectively.

The structure stiffness matrix **K**, usually obtained by inverting the flexibility matrix, is written as

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{xx} & \mathbf{k}_{x\theta} \\ \mathbf{k}_{\theta x} & \mathbf{k}_{\theta\theta} \end{bmatrix} \tag{11}$$

This **K**, along with the **M** and $\boldsymbol{\varphi}_{\mathbf{n}}$ matrices are used to formulate the elastic properties of the equivalent two degree of freedom systems as per Lin and Tsai (2007):

$$m = \boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{m} \boldsymbol{\phi}_{xn}; I = \boldsymbol{\phi}_{\theta n}^{\mathrm{T}} \mathbf{I}_0 \boldsymbol{\phi}_{\theta n}; e = \frac{\boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{k}_{x\theta} \boldsymbol{\phi}_{n}}{\boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{k}_{xx} \boldsymbol{\phi}_{xn}}$$

$$k_x = \boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{m} \boldsymbol{\phi}_{xn}; k_\theta = \boldsymbol{\phi}_{\theta n}^{\mathrm{T}} \mathbf{k}_{\theta\theta} \boldsymbol{\phi}_{\theta n} - \frac{(\boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{k}_{x\theta} \boldsymbol{\phi}_{\theta n})^2}{\boldsymbol{\phi}_{xn}^{\mathrm{T}} \mathbf{k}_{xx} \boldsymbol{\phi}_{xn}} \tag{12}$$

where m = translational mass, I = rotational mass, e = plan eccentricity, and k_x and k_θ are the initial stiffness of springs representing lateral and rotational movement of the structure. The inelastic stiffnesses and the yield “forces” of the equivalent 2DOF systems are obtained from the bilinearized ADRS pushover plots (x represents the translational DOF and θ the rotational):

$$k'_\theta = \alpha_\theta k_\theta; k'_{\mathrm{x}} = \frac{m}{\dfrac{\dfrac{m}{k_x} - \dfrac{(I - me)e}{k_\theta}}{\alpha_x} + \dfrac{(I - me)e}{k'_\theta}}; \; F_{y\theta} = A_{\theta ny} I; F_{yx} = A_{\mathrm{x}ny} m \tag{13}$$

Detailed derivation of above equations and description of equivalent systems may be obtained from the work of Lin and Tsai (2007).

The contribution of hysteretic energy demand from each mode (E_{hn}) is obtained from an NLRHA of the nth modal 2DOF equivalent system. The structural hysteretic energy demand ($E_{2D\text{-}MPA}$) is obtained, again, by adding the modal contributions:

$$E_{2D-MPA} = \sum E_{hn} \tag{14}$$

Validation Case Studies for the Proposed Methods

The proposed approximate methods are tested on 3-, 9- and 20-story steel moment frame buildings. These buildings are based on the SAC Steel Project “Pre-Northridge” buildings from Los Angeles, USA (Gupta and Krawinkler 1999). These originally symmetric structures are made uniaxial plan-asymmetric structures by shifting the center of mass at each floor along one of the axes of symmetry. The plan eccentricity (e) of these buildings is varied from 0% to 40% of the lateral dimension to check the effectiveness of the proposed method for different degrees of eccentricity. These buildings are checked for a set of 18 strong motion records with varying magnitude, PGA and frequency content. For each building with a specific eccentricity and subjected to a specific earthquake the hysteretic energy demand is obtained using the MPA-based method (E_{MPA}), 2D-MPA-based method ($E_{2D\text{-}MPA}$), and NLRHA of the MDOF model (E_{NLRHA}). Accuracy of an approximate method is measured with a bias factor defined as

$$N_{MPA} = \frac{E_{NLRHA}}{E_{MPA}} \text{ or } N_{2D-MPA} = \frac{E_{NLRHA}}{E_{2D-MPA}} \tag{15}$$

Statistics of this bias factor is studied for all the 18 records for a selected test building. Summary of all the studies for the MPA-based method is provided in Table 1. Table 2 presents the summary of all results for the 2D-MPA-based method. A mean bias close to its ideal value 1 signifies the estimates to be good overall, and a low standard deviation or coefficient of variation implies that the approximate estimates are consistently good (or consistently bad). A bias larger that 1 signifies an underestimation of the hysteretic energy demand on the structure by the respective approximate method proposed here, and vice versa. Figure 1 presents sample scatterplots for the 9-story buildings with varying eccentricity for a quick and easy graphical comparison of the two methods with respect to the "exact" method based on an NLRHA of the MDOF system. Each point on these plots provides comparisons for a specific earthquake.

Table 1. Summary of bias (N_{MPA}) statistics for the MPA-based method.

Structure	Parameter	Eccentricity					
		0%	5%	10%	20%	30%	40%
3-story	Mean	1.20	1.22	1.26	1.25	1.20	1.25
	Std. Dev.	0.201	0.213	0.263	0.442	0.291	0.455
	CoV	0.167	0.174	0.208	0.354	0.242	0.365
	Max % Error	55.3	58.0	83.9	187	115	202
9-story	Mean	1.35	1.35	1.34	1.33	1.36	1.30
	Std. Dev.	0.500	0.472	0.428	0.396	0.379	0.306
	CoV	0.370	0.351	0.320	0.298	0.279	0.235
	Max % Error	180	172	153	156	140	107
20-story	Mean	1.43	1.46	1.51	1.71	1.86	1.84
	Std. Dev.	0.615	0.633	0.650	0.582	0.669	0.606
	CoV	0.430	0.435	0.431	0.341	0.360	0.330
	Max % Error	255	265	278	234	250	219

Table 2. Summary of bias ($N_{2D\text{-}MPA}$) statistics for the 2D-MPA-based method.

Structure	Parameter	Eccentricity				
		5%	10%	20%	30%	40%
3-story	Mean	1.11	1.01	0.936	0.968	1.11
	Std. Dev.	0.127	0.203	0.303	0.300	0.418
	CoV	0.115	0.202	0.323	0.310	0.376
	Max % Error	39.2	45.2	72.2	72.9	77.5
9-story	Mean	1.14	1.04	0.992	0.979	0.992
	Std. Dev.	0.148	0.239	0.267	0.307	0.375
	CoV	0.130	0.230	0.269	0.313	0.378
	Max % Error	43.6	47.9	54.3	71.3	66.8
20-story	Mean	1.25	1.11	0.938	0.862	0.778
	Std. Dev.	0.274	0.127	0.152	0.214	0.294
	CoV	0.218	0.114	0.162	0.249	0.377
	Max % Error	99.9	40.2	43.6	46.2	69.1

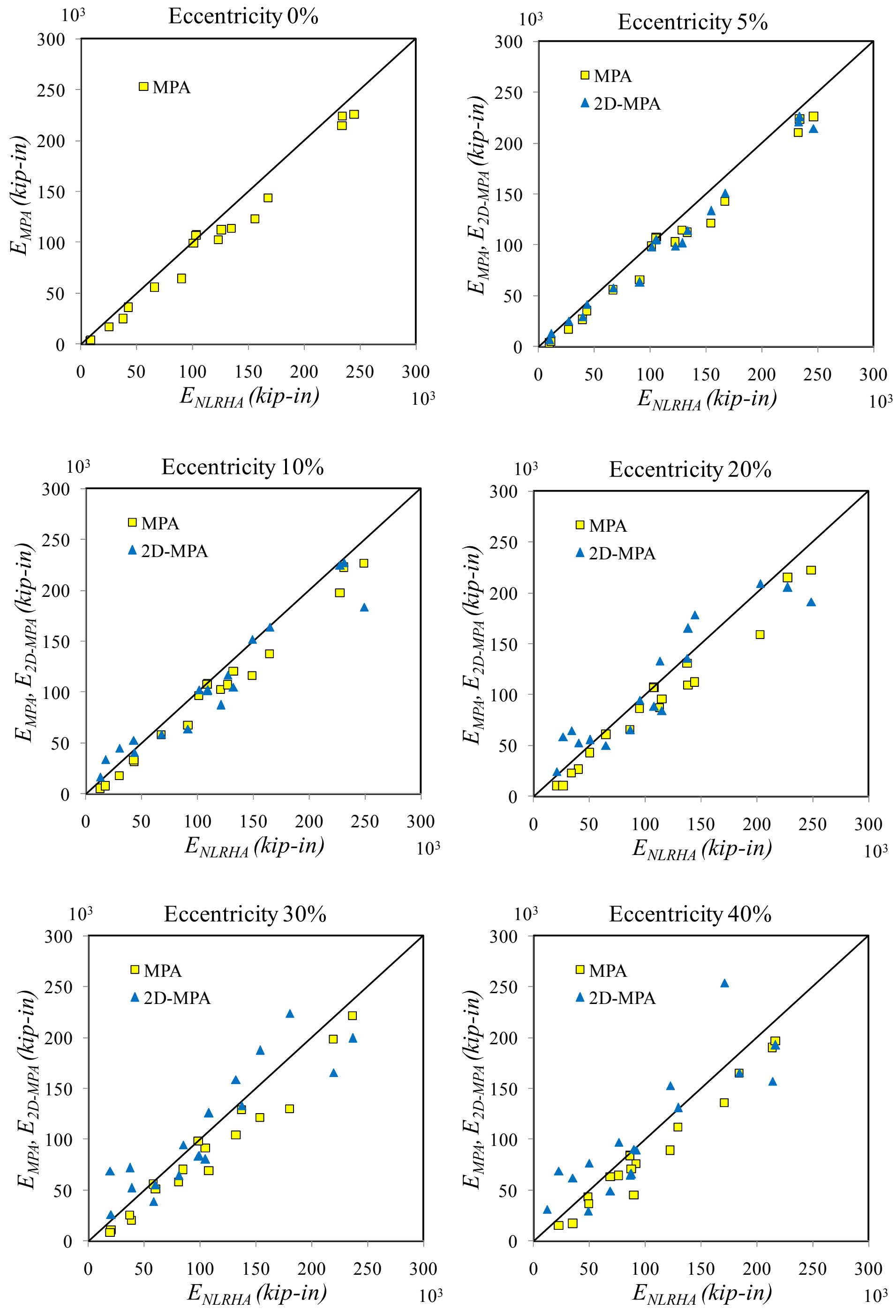

Figure 1. Scatterplots for the 9-story buildings with various eccentricities.
(1 kip-in = 0.113 kNm)

Tables 1 and 2 clearly show that both the methods are effective in estimating the hysteretic energy demand in plan-asymmetric structures, although with varying degrees of accuracy depending on building height and degree of eccentricity. The diagonal line across scatterplots indicates an ideal estimate, and any point above this line signifies an overestimation and vice-versa. Although not presented here for lack of space, mode-wise contribution to the total energy demand is also studied for each of the cases. Table 3 presents a sample mode-wise distribution of energy (E_{hn}) expressed as percentages of the total demand (E_{MPA}).

Table 3. Mode-wise distribution of hysteretic energy demand for the 20-story building with 20% plan-eccentricity.

Ground motion record	E_{hn}/E_{MPA} (where $E_{MPA} = \sum_{n=1}^{5} E_{hn}$)				
	Mode 1 (%)	Mode 2 (%)[R]	Mode 3 (%)	Mode 4 (%)	Mode 5 (%)[R]
s549	98.9	0	1.08	0	0
s621	100	0	0	0	0
s640	100	0	0	0	0
syl90	100	0	0	0	0
syl360	100	0	0	0	0
tcu0659	99.8	0	0.169	0	0
tcu06536	100	0	0	0	0
chy0809	0	0	100	0	0
chy08036	0	0	100	0	0
newh360	76.8	0	23.2	0	0
nh	100	0	0	0	0
nr	54.2	0	45.7	0	0
ns	34.2	0	65.8	0	0
s050	99.8	0	0.199	0	0
s065	100	0	0	0	0
s212	100	0	0	0	0
s305	73.8	0	26.2	0	0
s503	100	0	0	0	0

[R] Primarily torsional mode.

Conclusions

The following conclusions can be drawn from the research work presented in this paper on approximate methods for the estimation hysteretic energy demand on plan-asymmetric structures:

- The proposed MPA-based and 2D-MPA-based methods give good estimations of hysteretic energy demand for uniaxial plan-asymmetric structures.
- The proposed methods are computationally very light, conceptually simple and reasonably accurate for adopting for the purpose of energy-based design or performance evaluation.
- For almost all cases, considering the first three translational modes for energy calculations produce reasonably accurate results for both the methods. However, the first torsional mode may also be included for tall structures with very high eccentricity cases.
- 2D-MPA-based method gives almost equally accurate results for low- to high-rise structures. The MPA-based method gives good results for low- to mid-rise structures and the level of

accuracy decreases for taller structures.

- The MPA-based method underestimates hysteretic energy demand almost for all the cases. This is true for 2D-MPA based method for low eccentricity cases, and there is a weak trend of overestimation for large eccentricity cases.
- Overall, the 2D-MPA-based method gives more accurate results, whereas the MPA-based method is conceptually simpler.
- The MPA-based methods can also use energy response spectra to determine E_{hn} values and make computations even simpler for the user.

It should however be noted that these three buildings, even with a 40% plan eccentricity, are torsionally stiff structures. Therefore, there is a need to validate the proposed methods for torsionally flexible systems. Also these methods need to be checked for other building configurations (such as braced frames, shear walls, etc.), and for the inclusion of geometric nonlinearity (Roy Chowdhury and Ghosh 2007). Future extension of the proposed work can be in the line of the 3D-MPA method (Lin and Tsai 2008) for energy demand estimation in biaxial plan-asymmetric buildings.

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