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A BAYESIAN APPROACHE TO PROBABILSTIC SEISMIC DEMAND ANALYSIS OF STEEL MOMENT-RESISTING FRAMES

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ABSTRACT

In this article, using a Bayesian approach, in order to estimate the seismic demand of Steel Moment-Resisting Frames (SMRFs) at any given Intensity Measure (IM), two probabilistic models, Probabilistic Seismic Demand Model (PSDM), not included collapse probability, and Collapse Probability Model (CPM), are developed. With the aim of selecting the best PSDM, 13 different IM parameters consist of one or more spectral accelerations are defined and evaluated. The Bayesian regression results show that for all defined IM, a linear relation between the logarithm of IM and the logarithm of demand parameter, drift here, is the best form to define the PSDM, but if a single spectral acceleration is used to define the IM, it is impossible to introduce a unique parameter as IM for all type of SMRFs, because a specific spectral acceleration with the most accuracy to estimated the seismic demand of a stiff frame, may change to the weakest estimator in a deformable frame and vice versa. On the other hand, if the IM is defined by using the combination of two or more spectral acceleration, one can find a unique IM with almost same accuracy for all modeled frames. Also the results show that a normal distribution is the best probabilistic model to define the CPM.

Introduction

In recently developed performance based design engineering frameworks, estimation of seismic demand is an essential part to describe the performance of structure. The most challenging in this estimation is the large uncertainty associated with the seismic events and structural response demands. Because of this uncertainty, can be described in term of those originating from randomness (aleatory) and modeling errors (epistemic), using a probabilistic method to treatment of both randomness and uncertainty is required in estimation of seismic demand. This method is generally known as Probabilistic Seismic Demand Analysis (PSDA).

PSDA is an approach for calculating the mean annual frequency (or annual probability) of exceeding a specified seismic demand for given structure at a designated site (Cornell 1996). PSDA combines a ground motion Intensity Measure (IM) hazard curves for designated site with

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the demand results from Nonlinear Dynamic Analysis (NDA) of the given structure under a suite of earthquake ground motion records through the application of the total probability theorem (Luco 2002). If the maximum inter story drift (denoted by *DR*) is selected as the demand parameter, the following mathematical expression can be used to calculate the probability that the drift exceeds the value x, $P[DR > x]$:

$$P[DR > x] = \int_0^\infty P[DR > x \mid IM = y].\,|\,dH_{IM}(y)\,| \qquad (1)$$

In this equation the term $H_{IM}(y)$ means annual frequency that *IM* at a given site will equal or exceed the value *y* and notation $|d\ldots|$ means its differential with respect to *IM*, evaluated at *y*. This term is usually computed through a probabilistic seismic hazard analysis and it is not the object of this study. The main object of this study is the term $P[DR > x \mid IM = y]$, which means the probability of *DR* exceeding the value *x* given (i.e., conditioned on knowing) that *IM* equals *y*. In order to calculate this probability in a reliable manner, along with the probability of exceeding, the probability of total collapse of structure at any given IM level must be considered. Hence the following two part equation is proposed to calculate this probability (Tothong and Cornell 2006):

$$P[DR > x \mid IM = y] = (1 - P_{C|IM}).P_{NC|IM}(DR > x \mid IM = y] + P_{C|IM} \qquad (2)$$

As seen in this expression, the target probability at each IM level is divided into two mutually exclusive and collectively exhaustive events, the probability of Collapse ($P_{C|IM}$) and the probability of exceeding under the condition of Non-Collapse ($P_{NC|IM}$).The aim of this study is to develop a fully Bayesian framework to calculate these two probabilities for generic Steel Moment-Resisting Frames (SMRFs).

In the framework of data analysis based on probability models, three principal approaches are possible: the method of moments, the method of maximum likelihood and the Bayesian updating method. Because of its ability to simultaneous modeling of randomness and uncertainty in estimating the seismic demand, the Bayesian approach is selected to provide a framework for incorporating engineering judgment and subjective information in this study. Also an Incremental Dynamic Analysis (IDA) is applied to generate required data for statistical analysis. In the following parts, after introducing the fundamental of Bayesian statistics, modeled generic SMRFs, selected ground motion records and defining the selected IM parameters, an IDA is carried out to generate two databases. The first database, including the non-collapse results of IDA, along with 13 different IM is applied to define the best demand model and remaining results, which lead to collapse of frame and gathered in second database, are used to establish a collapse probability model, using the Bayesian approach.

Bayesian Statistical Approach

In this article, the Bayesian statistical approach is used to estimate all unknown parameters and required relations. This method can properly account for prevailing uncertainties such as statistical and model uncertainties (Gardoni et al. 2002). Here only a brief description of this method is presented, additional details of can be found in (Der Kiureghian, 1999). Let

$$y(x,\theta,\sigma) = d(x,\theta) + \sigma.\varepsilon \tag{3}$$

be a mathematical model for predicting variable y in terms of a set of observable variables x, in which $d(x,\theta)$ is the deterministic model, θ is the vector of unknown model parameters, ε is a normal random variable with zero mean and unit standard deviation, representing the uncertainty in the model and σ is the unknown model standard deviation. So the set of unknown parameters must be estimated by using Bayesian statistics and available information is $\Phi(\theta,\sigma)$. In the Bayesian approach, this is done by using the well-known updating rule:

$$f(\Phi) = c.L(\Phi).p(\Phi) \tag{4}$$

Where $p(\Phi)$ can be viewed as the prior distribution reflecting the state of knowledge about Φ prior to obtained the information, $L(\Phi)$ likelihood function, which is a function proportional to the conditional probability of making the observation on x and y for a given value of the parameters and reflect the objective information gained from the information, $f(\Phi)$ is posterior distribution reflecting the updated information about Φ and c is a normalizing factor necessary to ensure that the posterior distribution integrates to one. In this article, the regression tool relies on Markov chain Monte Carlo simulation techniques and yields fully Bayesian posterior mean or posterior mode estimation. Details can be found in (Brezger and Lang 2006 and 2008).

Definition of Used Generic Steel Moment-Resisting Frames

In this article, NDA is carried out using a family of two-dimensional single-bay generic SMRFs with number of stories equal to 3, 6, 9, 12 and 15, and first mode periods equal to 0.3, 0.6, 0.9, 1.2 and 1.5 second respectively. Some main characteristics of this family of frames are as follows, more details can be found in (Medina and Krawinkler 2005):

- The same mass is used at all floor level
- Relative stiffness are turned so that the first mode is straight line
- Plasticization just occurs at the end of the beams and the bottom of the first story columns
- Frames are designed so that simultaneous yielding at all plastic hinge locations is attained under a parabolic (NEHRP, k=2) load pattern.
- Moment-rotation hysteretic behavior is modeled by using rotational springs with peak-oriented hysteretic rules and cyclic deterioration parameter equal to 30 and 3% strain hardening.

Selection of Ground Motion Records

An appropriate estimation of seismic demand through NDA requires a suitable selection of ground motion records which must represent the seismic hazard condition of target territory at different return periods. In this article, using a bin strategy, 80 records are selected from the PEER Center Ground Motion Database (http://peer.berkeley.edu/smcat/) and are classified into four magnitude-distance bins for the purpose of time history analysis of SMRFs (Medina and Krawinkler 2003). The record bins are designated as follows:

- Large Magnitude-Short Distance Bin, LMSR, ($6.5 < Mw < 7.0$, $13 \text{ km} < R < 30 \text{ km}$),

- Large Magnitude-Long Distance Bin, LMLR, (6.5 < Mw <7.0, 30 km < R < 60 km),
- Small Magnitude-Short Distance Bin, SMSR, (5.8 < Mw < 6.5, 13 km < R < 30 km), and
- Small Magnitude-Long Distance Bin, SMLR, (5.8 < Mw < 6.5, 30 km < R < 60 km).

Definition of Selected IM Parameters

In this article, 13 different parameters are defined and evaluated as IM parameter. All of these IM parameters consist of one or more spectral parameters. The used spectral acceleration to define these IM parameters are the peak ground acceleration (*PGA*) and first, second and third mode spectral acceleration (S_{a1}, S_{a2} and S_{a3}). The following parameters are used as IM parameter:

Table 1. Definition of 13 IM parameters consist of one, two or more spectral accelerations

IM No:1	PGA	IM No:8	$\sqrt{S_{a1}^2 + S_{a2}^2}$
IM No:2	S_{a1}	IM No:9	$\sqrt{S_{a1}^2 + S_{a2}^2 + S_{a3}^2}$
IM No:3	S_{a2}	IM No:10	$\sqrt{S_{a1}^2 + S_{a2}^2 + S_{a3}^2 + PGA^2}$
IM No:4	S_{a3}	IM No:11	$\sqrt{S_{a1}.S_{a2}}$
IM No:5	$\frac{S_{a1} + S_{a2}}{2}$	IM No:12	$\sqrt[3]{S_{a1}.S_{a2}.S_{a3}}$
IM No:6	$\frac{S_{a1} + S_{a2} + S_{a3}}{3}$	IM No:13	$\sqrt[4]{S_{a1}.S_{a2}.S_{a3}.PGA}$
IM No:7	$\frac{S_{a1} + S_{a2} + S_{a3} + PGA}{4}$		

Using IDA to Generate the Databases

In order to generate two required databases, an IDA is applied to the each selected ground motion record. Each record is scaled from a lower limit equal to S_{a1}=0.05g to an upper limit, defined as the value of first mode spectral acceleration that leads to collapse of structure, with 0.05g steps. At each step, a NDA is carried out and the resulted maximum inter story drift, along with the amount of every 13 IM parameters are sent to the one of the defined databases (collapse or non-collapse), depends on the structure collapses at the monitored IM level or not. Collapse is considered here as the ultimate limit state in which dynamic sideway instability in one or several stories of structural system is attained. For instance, the distribution of data points between databases is shown in Fig. 1 for 3 and 15-story models. As seen, at the low levels of IM, which is the first mode spectral acceleration in this figure, all of the points belong to the non-collapse database, but at the upper limit IM, all of the 80 records lead to collapse of structures.

Calculation the Probability of Exceeding Under the Condition of Non-Collapse

In this section, using the database of non-collapse data points, the probability of *DR* exceeding the value *x* when the *IM* equals *y* and the collapse is not occurred is calculated. By assuming a normal distribution for dispersion, Θ, this probability can be expressed as:

$$P_{NC|IM}[DR > x \mid IM = y] = 1 - \Theta\left(\frac{\ln(x) - \mu_{\ln(IM)}(y)}{\sigma}\right) \tag{5}$$

The required parameter of this distribution, the mean $\mu_{\ln(IM)}(y)$ and standard deviation σ, are defined through an Probabilistic Seismic Demand Model (PSDM). The PSDM is a mathematical expression relates structure specific demand to the specific IM. Analogous to Eq. 3, a demand model is defined as follow:

$$D(x,\theta,\sigma) = d(x,\theta) + \sigma.\varepsilon \tag{6}$$

In this article, using Bayesian statistical, the best form and the best IM parameter are selected for PSDM. Plotting the resulted drift in IDA against any of 13 defined IM may lead to different relation and standard deviation. For example such a plotting is drawn for 3 and 15-story frame and two different IM (S_{a1} and S_{a2}) in Fig. 2.

Selecting the Form of PSDM

By using the Bayesian statistical, it is possible to select the best form of a relation. For this aim the following general expression is defined for deterministic part of demand model:

$$\ln(DR) = f(\ln(IM)) \tag{7}$$

The results of Bayesian regression show that for all types of SMRFs and IM, a linear relation between the logarithm of IM and the logarithm of demand parameter is the best form to define the demand model. Actually for 3, 6, 9, 12 and 15-story frames when the IM parameter No: 2, 2, 11, 5 and 5 are used, the defined *f*( ) function has the most similarity to the linear form and when the IM parameters No: 3, 4, 4, 4, and 2 are used the most difference is appeared. Fig. 3 shows these results. As seen in this figure, even in the case of using the weakest IM, defining a linear relation for demand model is completely rational, so the following form is selected for deterministic part of the model:

$$\ln(DR) = a.\ln(IM) + b \tag{8}$$

Selecting the Best IM Parameter

After selecting the best form for demand model, the best IM parameter must be selected. The standard deviation of models, which can represent the dispersion of data along the median value and the efficiency of model, is the best criterion to select the best IM. For this purpose, using the Bayesian estimation, standard deviation of demand model is evaluated and regarding the used IM parameter in the model represented in Fig 4.

As seen in this figure, the defined IMs have totally different accuracy in estimation of drift demand in various types of frames. In stiff low-rise 3-story frame, S_{a1} is the best estimator and S_{a2} is the weakest one. In this frame, generally it seems the multi-parameters IMs contain S_{a1}, have the same accuracy in estimation of the demand. It must be noted that in 15-story frame, the best estimator of 3-story frame, S_{a1}, is the weakest one and the weakest estimator of 3-story frame, S_{a2}, is a very accurate estimator, but similar to 3-story frame, multi-parameters IMs have the same accuracy in estimation of the demand. Regarding the same conclusion is valid for other frames, an important result can be stated; although it is impossible to find a single spectral acceleration with same accuracy as an IM parameter to estimate the seismic drift demand for all types of frames, it is feasible to set up a unique parameter, consist of two or more spectral acceleration, as a general IM parameter to estimate the drift of all SMRFs.

Calculation the Probability of Collapse

Using the second database, which consists of collapse data points, the probability of collapse at each IM level can be calculated. This probability is defined as the number of scaled record, which leads to collapse, divided to the number of all records, 80, at any given IM level. Fig. 5 shows the calculated collapse probability at each S_{a1} level for all modeled SMRFs. As seen in this figure, the distribution of collapse probability is completely normal. Hence the following expression can be defined as collapse probability model:

$$P_{C|IM}(y) = \Theta\left(\frac{\ln(y) - \mu_C}{\sigma_C}\right) \tag{9}$$

Using the Bayesian updating rules, the required parameter of this distribution is estimated and shown in Table 2. By plotting the results of defined collapse model along with observed data in Fig. 5 it seems that although all the defined models have enough accuracy to predict the collapse, by increasing the number of stories, the model error slightly increases.

Conclusions

A Bayesian probabilistic seismic demand estimation incorporating Incremental Dynamic Analysis (IDA) has been implemented for Steel Moment Resisting Frames (SMRFs) in two conditions, inclusion or exclusion of collapse diagnosed through facing a numerical instability. For cases not including collapse, 13 Intensity Measure (IM) parameters ranging from single spectral acceleration to combination of two or more spectral acceleration have been evaluated for investigating their efficiency, sufficiency and accuracy. It has been concluded that while a linear relation between the logarithm of drift demand and logarithm of IM is numerically the best choice for a demand model, their efficiency reflected in the standard deviation is not independent of the number of stories, i.e. when in 3-story frame the best estimation would be achieved from a single parameter IM (first mode spectral acceleration) in the 15-story frame it would be resulted the largest standard deviation among all defined IMs. On the other hand while a single parameter IM including the second mode spectral acceleration is the most accurate one for the 15-story frame, it would be the weakest estimator for 3-story frame. It has furthermore been shown that in the case of using multi-parameters IMs, it is possible to introduce a unique parameter, which can be used as general IM for all types of SMRFs with the same accuracy and sufficiency. Also in the case of IDA including collapse cases, using a Bayesian updating rule, it has been concluded that the best probabilistic model for collapse estimation is a normal one while with increasing the number of story its accuracy would be slightly decreasing.

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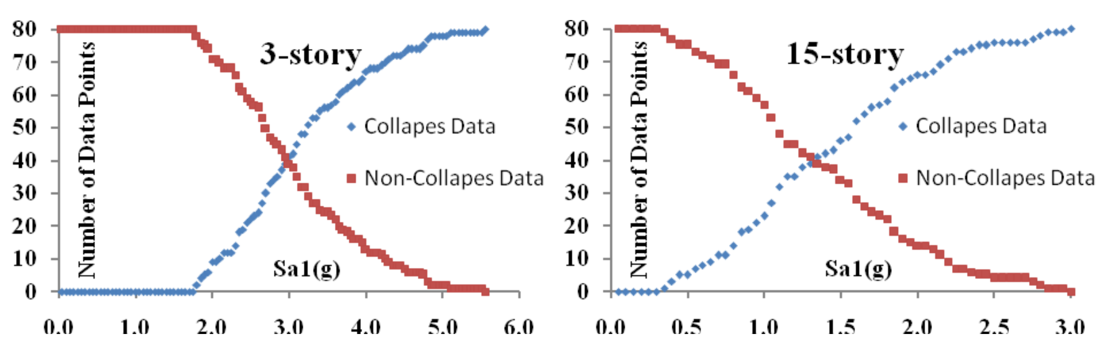

Figure 1. The distribution of data points between defined databases, collapse and non-collapse

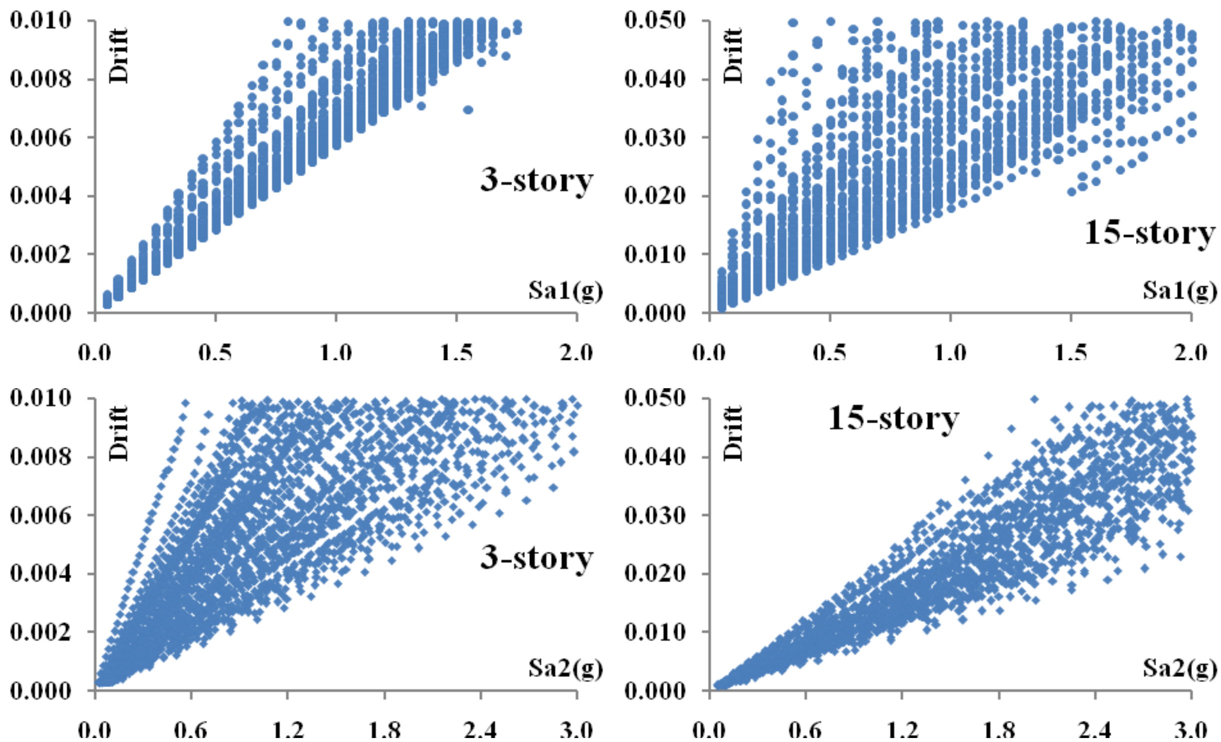

Figure 2. Examples of various relations between defined IM and drift in different SMRFs

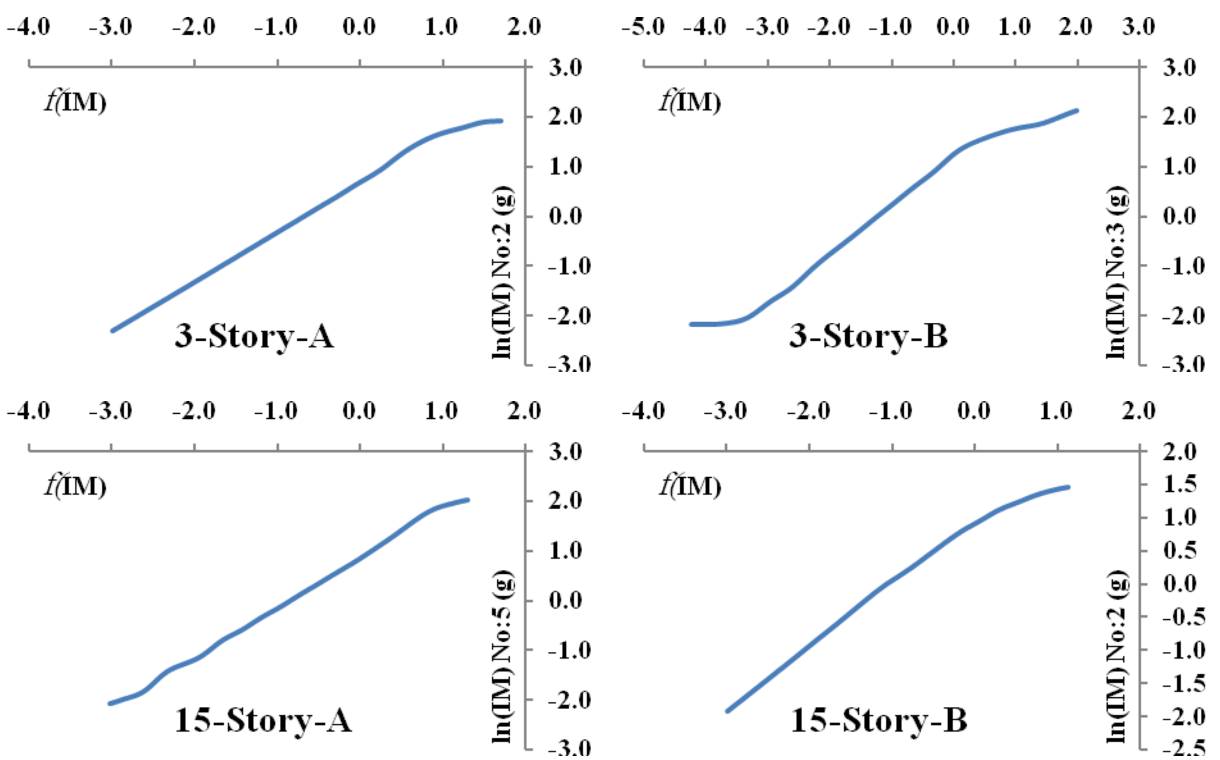

Figure 3. Evaluated forms for demand model, A: the most similarity to the linear model, B: the most difference to the linear model

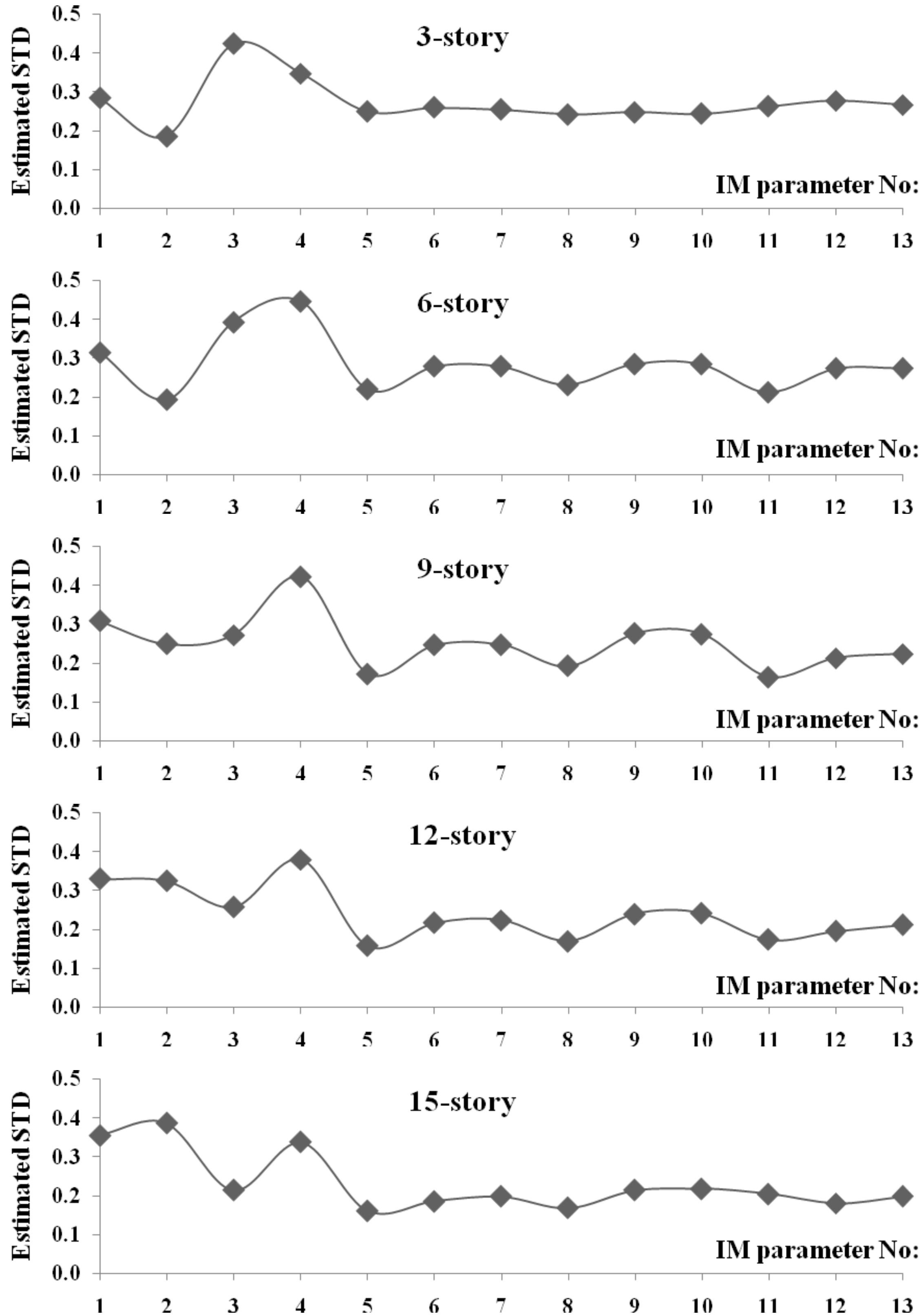

Figure 4. Estimated standard deviation for demand model with various defined IM parameters

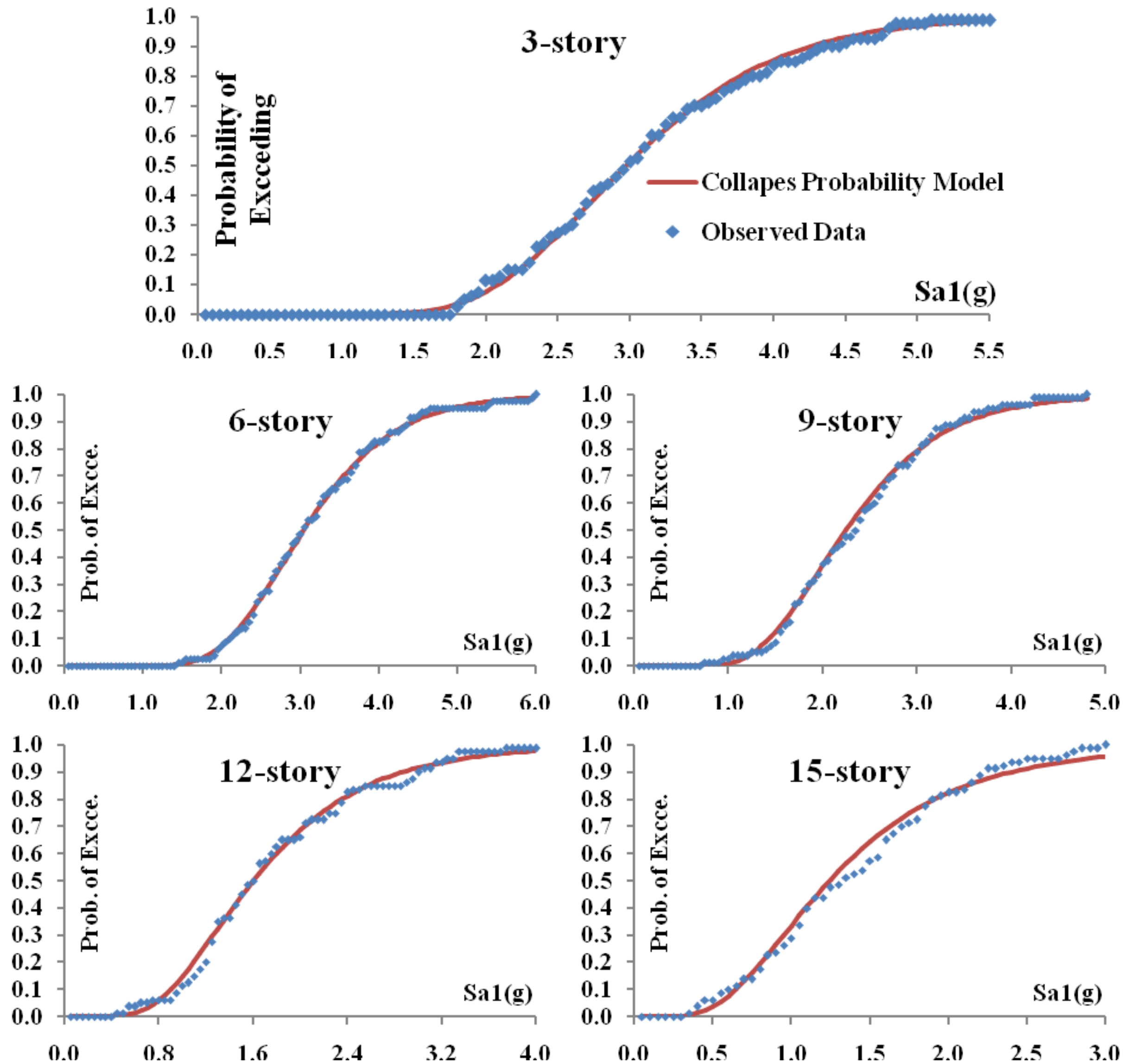

Figure 5. Observed data and defined collapse probability model for different number of stories

Table 2. Estimated parameters for defined collapse model in Eq. 9, using Bayesian estimation

Estimated Parameter	Number of Stories				
	3-story	6- story	9-story	12-story	15-story
Mean (μ_C)	1.091	1.112	0.808	0.471	0.216
Standard deviation (σ_C)	0.279	0.293	0.351	0.454	0.512