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COUPLED TUNED MASS DAMPERS FOR THE MODAL CONTROL OF ONE-WAY ASYMMETRIC BUILDINGS

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ABSTRACT

An innovative tuned mass damper, referred to as a coupled tuned mass damper (CTMD), is proposed for the control of a coupled vibration mode of one-way asymmetric-plan buildings. The CTMD simultaneously translates and rotates almost resonantly with the vibration of the controlled mode, which actually vibrates in translation as well as rotation. Thus, the CTMD can be viewed as a direct approach for controlling the modal vibration of asymmetric-plan buildings. The CTMD is developed from the two-degree-of-freedom modal system. It is illustrated that the optimum parameter values of the CTMD can be conveniently determined from those of the corresponding tuned mass damper (TMD). The effectiveness of the CTMD in reducing the vibrations of asymmetric-plan structures is verified by investigating the frequency response functions and the response histories of three 8-storey asymmetric-plan buildings with and without dampers. This study confirms that the CTMD is an effective alternative for the seismic control of asymmetric-plan buildings.

Introduction

A tuned mass damper (TMD) is a mechanical vibration absorber vibrating almost resonantly with the main structure. The input energy to the structure-TMD system is mostly dissipated through the damping of the TMD. Thus, the main structure is adequately protected. The early studies on TMDs were on undamped main structures with main mass excitation (Brock 1946; Den Hartog 1956). They were followed by studies on damped main structures with base excitation (Warburton and Ayorind 1980; Tsai and Lin 1993). The structure-TMD system studied by Tsai and Lin (1993) and several other researchers is a single-degree-of-freedom (SDOF) TMD attached to a SDOF main structure (Fig. 1a). Rana and Soong (1998) suggested that the TMD design based on harmonic excitations is the best possible optimization for real seismic ground motions. They indicated that the design of a TMD for controlling a single vibration mode of a MDOF structure is similar to the design of a TMD for controlling a SDOF structure. They verified that the TMD design for controlling a certain vibration mode of a MDOF structure can be satisfactorily achieved. In their research, for controlling the *i*-th mode of a MDOF structure by placing a TMD on the *j*-th

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floor, the mass ratio was defined as the ratio of the TMD mass to the *i*-th modal mass. Moreover, the *i*-th mode shape was normalized with its *j*-th component equal to one. Due to the modal contamination problem, the multi-TMD (MTMD) was not recommended for the simultaneous control of several vibration modes of a MDOF structure (Rana and Soong 1998).

Before about one decade ago, there was no reported work about the investigation of the effectiveness of TMDs in reducing responses of asymmetric-plan structures (Jangid and Datta 1997). The translation-only single SDOF TMD, referred to as the conventional TMD in the rest of this paper, may be improper for simultaneously controlling the translational and rotational vibrations of an asymmetric-plan structure. Intuitively, it might be accomplished by directly controlling a translation-rotation coupled vibration mode of an asymmetric structure by using a single mechanical vibration absorber, which could also be translation-rotation vibrated almost resonant with the target vibration mode of the main structure. Nevertheless, instead of researching and developing a translation-rotation coupled vibration absorber, most researchers devoted to use MTMDs to control the coupled vibration of asymmetric-plan structures. A simple two-degree-of-freedom (2DOF) asymmetric structure (Fig. 1b) controlled by using the MTMD, consisting of 21 TMDs, was investigated by Jangid and Datta (1997). Li and Qu (2006) used only five TMDs in the investigation of the optimization of the MTMD for the same simple 2DOF asymmetric structure. In addition, Ahlawat and Ramaswamy (2003) placed one TMD on each side of the rectangular floor plan of asymmetric buildings to study the optimal absorber system. From these above mentioned studies, it is evident that there seems no consensus on the logical number of TMDs used for controlling the coupled vibration of asymmetric-plan buildings. In addition to the number of TMDs, the frequency bandwidth and the placement width of the TMDs are also key parameters that need to be determined while using MTMDs to control asymmetric structures. It appears much more complicated to apply MTMDs instead of a single TMD for controlling the vibration of asymmetric structures. Thus, it may be useful to develop a new kind of tuned mass dampers which can conveniently and effectively reduce the coupled vibrations of asymmetric structures.

Lin and Tsai (2007) developed the 2DOF modal system (Fig. 1c) for the seismic analysis of one-way asymmetric-plan buildings. In the present research, rather than using several eccentrically placed TMDs, a single coupled-TMD (CTMD) developed from the 2DOF modal system is proposed for the control of a coupled vibration mode of asymmetric-plan buildings. The optimum parameters of the CTMD are introduced and the effectiveness of the CTMD in reducing coupled vibrations of asymmetric structures is then examined.

Theoretical Background

Two-Degree-of-Freedom Modal Systems

In order to be consistent with the coordinate system used in the previous study (Lin and Tsai 2007), the two plan axes in the present study are the X- and Z-axis. The Y-axis is upward (opposite to the direction of gravity). The proportionally damped one-way asymmetric-plan buildings are only symmetric about the X-axis, and the seismic ground motions are applied along the Z-axis. The equation of motion for an *N*-storey one-way asymmetric-plan building with each floor simulated as a rigid diaphragm is

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\boldsymbol{\iota}_z \ddot{u}_g(t) = -\sum_{n=1}^{2N} \mathbf{s}_n \ddot{u}_g(t) \tag{1}$$

where

$$\mathbf{u}=\begin{bmatrix}\mathbf{u}_z\\ \mathbf{u}_\theta\end{bmatrix}_{2N\times1}=\sum_{n=1}^{2N}\mathbf{u}_n=\sum_{n=1}^{2N}\Gamma_{zn}\boldsymbol{\varphi}_n D_n,\quad \boldsymbol{\varphi}_n=\begin{bmatrix}\boldsymbol{\varphi}_{zn}\\ \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{2N\times1},\quad \Gamma_{zn}=\frac{\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\iota}_z}{\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\varphi}_n},$$
$$\mathbf{s}_n=\Gamma_{zn}\mathbf{M}\boldsymbol{\varphi}_n,\quad \boldsymbol{\iota}_z=\begin{bmatrix}\mathbf{1}\\ \mathbf{0}\end{bmatrix}_{2N\times1},\quad \mathbf{M}=\begin{bmatrix}\mathbf{m}&\mathbf{0}\\ \mathbf{0}&\mathbf{I_0}\end{bmatrix}_{2N\times2N} \tag{2}$$
$$\mathbf{C}=\begin{bmatrix}\mathbf{c}_{zz}&\mathbf{c}_{z\theta}\\ \mathbf{c}_{\theta z}&\mathbf{c}_{\theta\theta}\end{bmatrix}_{2N\times2N},\quad \mathbf{K}=\begin{bmatrix}\mathbf{k}_{zz}&\mathbf{k}_{z\theta}\\ \mathbf{k}_{\theta z}&\mathbf{k}_{\theta\theta}\end{bmatrix}_{2N\times2N}$$

M, C and **K** are the mass matrix, damping matrix and stiffness matrix respectively; **u** and $\mathbf{s}_n$ are the displacement vector and the *n*-th modal inertia force distribution respectively; Γ_{zn} and $\boldsymbol{\varphi}_n$ are the *n*-th modal participation factor and the mode shape respectively; D_n and $\boldsymbol{\iota}_z$ are the *n*-th generalized modal coordinate and the influence vector respectively. By re-defining the *n*-th modal displacement response $\mathbf{u}_n$ equal to $\Gamma_{zn}\begin{bmatrix}\boldsymbol{\varphi}_{zn}&\mathbf{0}\\ \mathbf{0}&\boldsymbol{\varphi}_{\theta n}\end{bmatrix}\begin{bmatrix}D_{zn}\\ D_{\theta n}\end{bmatrix}$, the *n*-th 2DOF modal equation of motion for a one-way asymmetric-plan multi-storey building is obtained as:

$$\mathbf{M}_n\ddot{\mathbf{D}}_n+\mathbf{C}_n\dot{\mathbf{D}}_n+\mathbf{K}_n\mathbf{D}_n=-\mathbf{M}_n\mathbf{1}\ddot{u}_g(t),\quad n=1\sim 2N \tag{3}$$

where

$$\mathbf{M}_n=\begin{bmatrix}\boldsymbol{\varphi}_{zn}^T\mathbf{m}\boldsymbol{\varphi}_{zn}&0\\ 0&\boldsymbol{\varphi}_{\theta n}^T\mathbf{I_0}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{2\times2},\quad \mathbf{K}_n=\begin{bmatrix}\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn}&\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{z\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta z}\boldsymbol{\varphi}_{zn}&\boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{2\times2}$$
$$\mathbf{C}_n=\begin{bmatrix}\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zz}\boldsymbol{\varphi}_{zn}&\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{z\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta z}\boldsymbol{\varphi}_{zn}&\boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{2\times2},\quad \mathbf{D}_n=\begin{bmatrix}D_{zn}\\ D_{\theta n}\end{bmatrix}_{2\times1},\quad \mathbf{1}=\begin{bmatrix}1\\ 1\end{bmatrix}_{2\times1} \tag{4}$$

D_{zn} and $D_{\theta n}$ are referred to as the *n*-th modal translation and modal rotation, respectively. The elastic properties of the corresponding 2DOF modal system (Fig. 1c) with two degrees of freedom, D_{zn} and $D_{\theta n}$, defined at the lumped mass are

$$\mathbf{M}_n=\begin{bmatrix}m_n&0\\ 0&I_n\end{bmatrix}_{2\times2},\quad \mathbf{C}_n=\begin{bmatrix}c_{zn}&e_{dn}c_{zn}\\ e_{dn}c_{zn}&c_{\theta n}+e_{dn}^2c_{zn}\end{bmatrix}_{2\times2}$$
$$\mathbf{K}_n=\begin{bmatrix}k_{zn}&e_nk_{zn}\\ e_nk_{zn}&k_{\theta n}+e_n^2k_{zn}\end{bmatrix}_{2\times2},\quad \mathbf{D}_n=\begin{bmatrix}D_{zn}\\ D_{\theta n}\end{bmatrix}_{2\times1} \tag{5}$$

where

$$m_n=\boldsymbol{\varphi}_{zn}^T\mathbf{m}\boldsymbol{\varphi}_{zn},\quad I_n=\boldsymbol{\varphi}_{\theta n}^T\mathbf{I_0}\boldsymbol{\varphi}_{\theta n},\quad e_n=\frac{\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{z\theta}\boldsymbol{\varphi}_{\theta n}}{\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn}},\quad e_{dn}=\frac{\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{z\theta}\boldsymbol{\varphi}_{\theta n}}{\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zz}\boldsymbol{\varphi}_{zn}},\quad k_{zn}=\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn}$$
$$k_{\theta n}=\boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}-\frac{\left(\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{z\theta}\boldsymbol{\varphi}_{\theta n}\right)^2}{\boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn}},\quad c_{zn}=\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zz}\boldsymbol{\varphi}_{zn},\quad c_{\theta n}=\boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}-\frac{\left(\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{z\theta}\boldsymbol{\varphi}_{\theta n}\right)^2}{\boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zz}\boldsymbol{\varphi}_{zn}} \tag{6}$$

It has been demonstrated that only one of the two vibration modes of the 2DOF modal system is active with mode shape equal to $[1\ 1]^T$ and the vibration period equal to that of the n-th vibration mode of the original MDOF building. The other vibration mode of the 2DOF modal system is spurious with a modal participation factor equal to zero (Lin and Tsai 2007).

Optimum Parameters of a CTMD Controlling a Coupled Vibration Mode

The n-th 2DOF vibration mode of a one-way asymmetric-plan building controlled by using a CTMD is illustrated in Fig. 1d. The equation of motion for the system shown in Fig. 1d is

$$\begin{bmatrix} \mathbf{M}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{an} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{D}}_n \\ \ddot{\mathbf{D}}_{an} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_n + \mathbf{C}_{an} & -\mathbf{C}_{an} \\ -\mathbf{C}_{an} & \mathbf{C}_{an} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{D}}_n \\ \dot{\mathbf{D}}_{an} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_n + \mathbf{K}_{an} & -\mathbf{K}_{an} \\ -\mathbf{K}_{an} & \mathbf{K}_{an} \end{bmatrix} \begin{bmatrix} \mathbf{D}_n \\ \mathbf{D}_{an} \end{bmatrix} = -\begin{bmatrix} \mathbf{M}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{an} \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix} \ddot{u}_g(t) \tag{7}$$

where $\mathbf{M}_n$, $\mathbf{C}_n$, $\mathbf{K}_n$, and $\mathbf{D}_n$ are shown in Eqs. 5 and 6 and

$$\mathbf{M}_{an} = \begin{bmatrix} m_{an} & 0 \\ 0 & I_{an} \end{bmatrix}_{2\times 2}, \quad \mathbf{C}_{an} = \begin{bmatrix} c_{azzn} & c_{az\theta n} \\ c_{a\theta zn} & c_{a\theta\theta n} \end{bmatrix}_{2\times 2}, \quad \mathbf{K}_{an} = \begin{bmatrix} k_{azzn} & k_{az\theta n} \\ k_{a\theta zn} & k_{a\theta\theta n} \end{bmatrix}_{2\times 2}$$
$$\mathbf{D}_{an} = \begin{bmatrix} D_{azn} \\ D_{a\theta n} \end{bmatrix}_{2\times 1}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}, \quad \mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2} \tag{8}$$

The subscript a represents the associated physical quantity belongs to the absorber. We let

$$\mathbf{M}_{an} = \mu \mathbf{M}_n, \quad \mathbf{C}_{an} = \beta \mathbf{C}_n, \quad \mathbf{K}_{an} = f \mathbf{K}_n \tag{9}$$

where μ, β and f respectively represent the ratios of mass, damping and frequency of the CTMD to the corresponding properties of the n-th 2DOF vibration mode. Thus, if the mass ratio μ is specified, the optimum parameters of a CTMD to be determined would be only β and f. From some derivation (Lin and Tsai 2010), it is obtained that

$$f = \mu f_{0n}^2, \quad \beta = \mu f_{0n} \frac{\xi_{an}}{\xi_n} \tag{10}$$

where f_{0n} and ξ_{an} are the parameters of a conventional TMD controlling a SDOF main structure with damping ratio ξ_n; μ is the associated mass ratio. The optimum values of f_{0n} and ξ_{an} with specified values of μ and ξ_n are available in other research results (Tsai and Lin 1993). Therefore, the optimum values of f and β of a CTMD controlling the n-th coupled vibration mode (Fig. 1d) can be conveniently determined by using Eq. (10) and the other available research results (Tsai and Lin 1993).

Application of CTMDs to Physical Building Structures

The properties of the CTMD controlling the n-th coupled vibration mode of an asymmetric building are

$$\mathbf{M}_{an}=\begin{bmatrix} m_{an} & 0 \\ 0 & I_{an} \end{bmatrix}=\mu\begin{bmatrix} m_n & 0 \\ 0 & I_n \end{bmatrix},\quad \mathbf{C}_{an}=\begin{bmatrix} c_{azzn} & c_{az\theta n} \\ c_{a\theta zn} & c_{a\theta\theta n} \end{bmatrix}=\beta\begin{bmatrix} c_{zn} & e_{dn}c_{zn} \\ e_{dn}c_{zn} & c_{\theta n}+e_{dn}^2 c_{zn} \end{bmatrix} \tag{11}$$
$$\mathbf{K}_{an}=\begin{bmatrix} k_{azzn} & k_{az\theta n} \\ k_{a\theta zn} & k_{a\theta\theta n} \end{bmatrix}=f\begin{bmatrix} k_{zn} & e_n k_{zn} \\ e_n k_{zn} & k_{\theta n}+e_n^2 k_{zn} \end{bmatrix}$$

The mass, damping and stiffness matrices of the CTMD, respectively symbolized as $\mathbf{M}_{an}^s$, $\mathbf{C}_{an}^s$ and $\mathbf{K}_{an}^s$, placed on the j-th floor of an N-storey asymmetric-plan building for controlling the n-th vibration mode are

$$\mathbf{M}_{an}^s=\begin{bmatrix} m_{an} & 0 \\ 0 & \left(\dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}\right)^2 I_{an} \end{bmatrix},\quad \mathbf{C}_{an}^s=\begin{bmatrix} c_{azzn} & \dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}c_{az\theta n} \\ \dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}c_{a\theta zn} & \left(\dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}\right)^2 c_{a\theta\theta n} \end{bmatrix},\quad \mathbf{K}_{an}^s=\begin{bmatrix} k_{azzn} & \dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}k_{az\theta n} \\ \dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}k_{a\theta zn} & \left(\dfrac{\phi_{zn,j}}{\phi_{\theta n,j}}\right)^2 k_{a\theta\theta n} \end{bmatrix} \tag{12}$$

where m_{an}, I_{an}, c_{azzn}, $c_{az\theta n}$, $c_{a\theta zn}$, $c_{a\theta\theta n}$, k_{azzn}, $k_{az\theta n}$, $k_{a\theta zn}$ and $k_{a\theta\theta n}$ are defined in Eq. 11; $\phi_{zn,j}$ and $\phi_{\theta n,j}$ are the j-th components of the n-th mode shape in the translational and rotational directions, respectively. The mode shape is normalized to $\phi_{zn,j}=1$. The superscript s shown in Eq. 12 denotes that the associated quantities are related to the physical main structure rather than to the conceptual vibration mode. The proof of Eq. 12 can be found in Lin and Tsai (2010).

The difference between the proposed CTMD and the conventional TMD can be clearly observed from their physical behaviors. Recall that the conventional TMD, developed from the SDOF modal system, only vibrates in translation. Nevertheless, the vibration mode of an asymmetric-plan building actually vibrates in translation as well as rotation. That is why, to the best of the authors' knowledge, there is no research successfully applying a single conventional TMD to simultaneously control the translational and rotational vibrations of an asymmetric-plan building. The proposed CTMD, developed from the 2DOF modal system, not only translates but also rotates almost resonantly with the actual translation and rotation resulting from a coupled vibration mode of an asymmetric-plan building. Therefore, the proposed CTMD is completely different from the conventional TMD.

Validation of the Effectiveness of the CTMD

Example Buildings and Selected Ground Motions

There are three 24m (L) × 15m (W) × 8-storey example buildings investigated in this study. The center of rigidity (CR) and the center of mass (CM) of each storey are aligned in two vertical lines. The CR is located at the geometric center. The CM is 6m away from the CR along the long side of the floor plan, and the eccentricity ratio is 25%. Rayleigh damping with damping ratios of the first two vibration modes equal to 2% is assumed. The translational and rotational stiffness of the corresponding symmetric building are 4.503×10^5 kN/m and 3.84×10^7 kN-m/rad respectively. The stiffness matrices of these example buildings are shown in Table 1. The storey mass of the 3rd to the 8th storey of the example buildings is 3.456×10^5 kg. The storey mass of the 1st and the 2nd stories of the example buildings is two times 3.456×10^5 kg, i.e. 6.912×10^5 kg. The only difference between these example buildings is the mass moment of inertia of each storey, which is equal to one-third, one and five times 2.37×10^7 kg-m^2. The frequency ratios of the example buildings, which are defined as the ratio of the rotational frequency to the translational frequency of the corresponding symmetric building, are equal to 1.40, 1.07 and 0.71, respectively. Thus, these three example buildings represent torsionally-stiff, torsionally-similarly-stiff and torsionally-flexible buildings. They are respectively denoted as Building TS, Building TSS and Building TF. In addition to the example buildings without dampers, a TMD or a CTMD is placed on the roof of each example building. The mass ratio μ is chosen as 10%. From Tsai and Lin (1993), the optimum values of f_0 and ξ_a for the TMD are equal to 0.9306 and 0.188, respectively. Furthermore, from Eq. 10, the optimum values of f and β for the CTMD are found to be equal to 0.087 and 0.87, respectively. The responses of the example buildings equipped with CTMDs or TMDs are obtained by applying the step-by-step integration procedures on the equation of motion for the structure-CTMD or the structure-TMD systems. Due to the limitation of the paper length, only the results of Building TF are presented in this paper. The results of Building TS and TSS are available in other paper (Lin and Tsai 2010).

Analytical Results

The roof translational and rotational frequency response functions, denoted as H_z and H_θ, of Building TF are illustrated in Fig. 2. The notations CTMD_1 and CTMD_2 shown in Fig. 2 respectively represent the CTMD are designed for controlling the 1st and the 2nd vibration modes of the example buildings. Likewise, the notations TMD_1 and TMD_2 shown in Fig. 2 respectively represent the TMD are designed for controlling the 1st and the 2nd vibration modes of the example buildings. All these notations, CTMD_1, CTMD_2, TMD_1 and TMD_2, are also used in the rest of this paper. The properties of the CTMDs and TMDs for controlling Buildings TF are listed in Tables 2a and 2b respectively. Table 2b shows that the mass of TMD_1 for Building TF is 1003.5 ton, which is about 29% of the total mass of Building TF. It indicates that the use of a TMD for controlling the torsionally dominant vibration mode of a torsionally-flexible building seems sometimes impractical.

Figs. 2a and 2b indicate that the CTMD_1 and the TMD_1 have similar effectiveness in reducing the amplitudes of the roof translational and rotational response functions resulting from the 2nd vibration mode of Building TF. Nevertheless, the CTMD_1 is obviously more effective in reducing the amplitudes of roof translational and rotational response functions resulting from the 1st vibration mode of Building TF than the TMD_1. Nevertheless, as mentioned above, the mass of the TMD_1 is too large to be practical. Figs. 2c and 2d indicate that the effectiveness of the CTMD_2 and TMD_2 have the same effectiveness for reducing the amplitudes of the roof translational and rotational response functions resulting from the second vibration mode of Building TF. Nevertheless, the amplitudes of the roof translational and rotational response functions resulting from the first vibration mode of Building TF are harmfully increased by using the TMD_2. In other words, the use of the TMD_2 has an adverse effect on the roof response functions contributed from the first vibration mode of Building TF. Nevertheless, the CTMD_2 still effectively reduces the amplitudes of the roof response functions resulting from the first vibration mode of Building TF. Therefore, Fig. 2 indicates that using CTMDs for controlling Building TF is obviously better than using TMDs. Since frequency response functions are general transfer functions, the damper's effectiveness assessed by using the reductions of the amplitudes of frequency response functions is generally accepted and independent of the characteristics of input ground motions. Thus, from Fig. 2, it confirms the effectiveness of the CTMD for controlling the seismic responses of asymmetric-plan buildings.

Summary and Conclusions

Just like the use of a translation-only TMD to control a purely translational vibration mode of a symmetric-plan building, the use of a translation-rotation CTMD to control a translation-rotation coupled vibration mode of an asymmetric-plan building is proposed in this paper. In addition to this simple and clear thought, the advantages of using a CTMD instead of a set of several TMDs are that the associated parameters are simplified and the optimum parameter values can be conveniently obtained from the optimum parameter values of the corresponding conventional TMD. The properties of the CTMD placed on the j-th floor of an asymmetric-plan building for controlling the n-th vibration mode have been established. The effectiveness of CTMDs in reducing the seismic responses of asymmetric-plan buildings has been validated from the reduction of the amplitudes of the roof frequency response functions of three different types of asymmetric-plan example buildings.

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Table 1. The stiffness matrices of the 8-storey example buildings (units: kN, m, rad).

$\mathbf{k}_{zz}=10^5\times$

4.50							
-4.50	9.01				*symm.*		
0.00	-4.50	9.01					
0.00	0.00	-4.50	9.01				
0.00	0.00	0.00	-4.50	9.01			
0.00	0.00	0.00	0.00	-4.50	9.01		
0.00	0.00	0.00	0.00	0.00	-4.50	9.01	
0.00	0.00	0.00	0.00	0.00	0.00	-4.50	9.01

$\mathbf{k}_{z\theta}= \mathbf{k}_{\theta z}=10^5\times$

27.02							
-27.02	54.04				*symm.*		
0.00	-27.02	54.04					
0.00	0.00	-27.02	54.04				
0.00	0.00	0.00	-27.02	54.04			
0.00	0.00	0.00	0.00	-27.02	54.04		
0.00	0.00	0.00	0.00	0.00	-27.02	54.04	
0.00	0.00	0.00	0.00	0.00	0.00	-27.02	54.04

$\mathbf{k}_{\theta\theta}=10^5\times$

546.11							
-546.11	1092.22				*symm.*		
0.00	-546.11	1092.22					
0.00	0.00	-546.11	1092.22				
0.00	0.00	0.00	-546.11	1092.22			
0.00	0.00	0.00	0.00	-546.11	1092.22		
0.00	0.00	0.00	0.00	0.00	-546.11	1092.22	
0.00	0.00	0.00	0.00	0.00	0.00	-546.11	1092.22

Table 2. (a) The properties of the CTMDs used for Building TF (units: ton, kN, m, rad, sec).

Building	Damper	$\mathbf{M}_{an}^{s}$		$\mathbf{C}_{an}^{s}$		$\mathbf{K}_{an}^{s}$	
TF	CTMD_1	154.87	0	346	1374.6	5723.7	34285
		0	50907	1374.6	66041	34285	691900
	CTMD_2	163.82	0	366.65	1242.1	6070.8	30981
		0	43497	1242.1	58930	30981	653610

(b) The properties of the TMDs used for Building TF (units: ton, kN, m, sec).

Building	Damper	$\mathbf{M}_{an}^{s}$	$\mathbf{C}_{an}^{s}$	$\mathbf{K}_{an}^{s}$
TF	TMD_1	1003.5	1091.9	8404.2
	TMD_2	194.06	473.11	8158.7

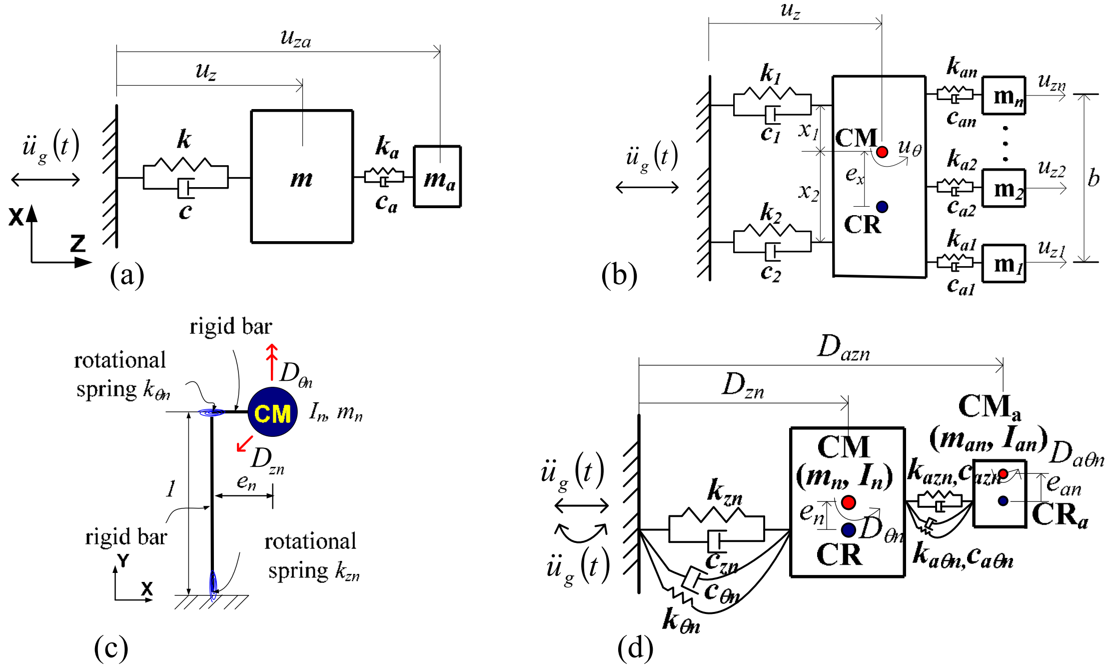

Figure 1. (a) A SDOF system controlled by using a TMD. (b) A simple 2DOF asymmetric structure controlled by using MTMDs. (c) The *n*-th 2DOF modal system. (d) A 2DOF modal system controlled by using a CTMD.

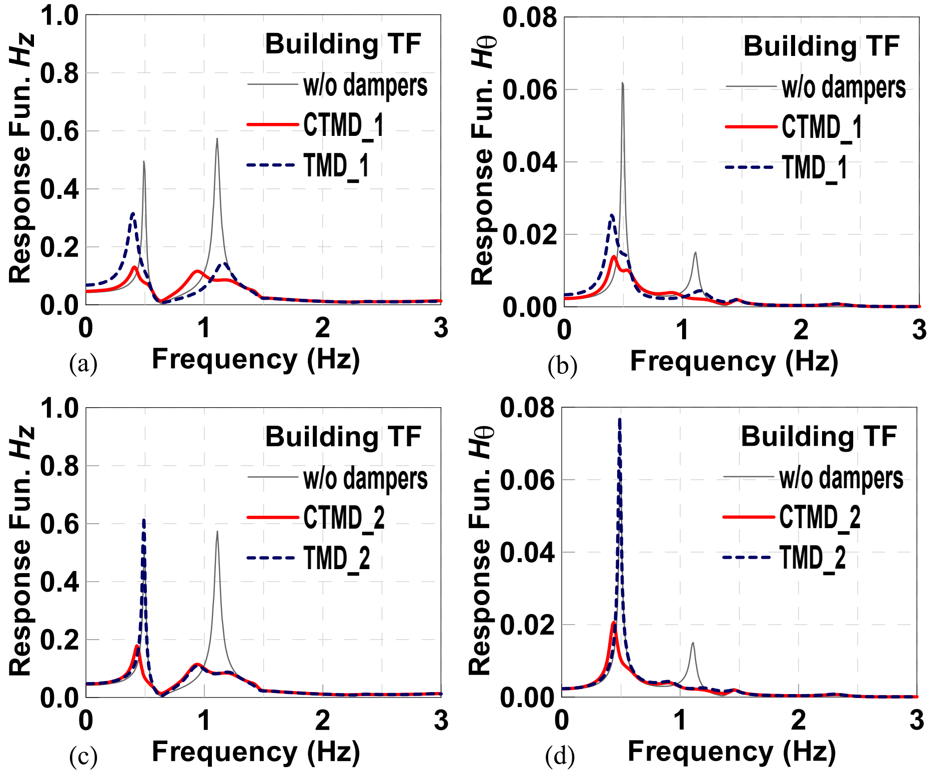

Figure 2. The roof translational and rotational frequency response functions, H_z and H_θ respectively, of (a)-(b) Building TF with the 1st-mode controlled dampers and (c)-(d) Building TF with the 2nd-mode controlled dampers.