



## **A THERMOPLASTIC DAMAGE MODEL FOR METALLIC ENERGY DISSIPATION DEVICES**

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### **ABSTRACT**

Metallic dampers represent one class of effective passive energy dissipation devices used to protect structures during earthquake excitation. In some cases, these devices can become partially or fully damaged, which in turn could lead to failure of the structural system. In the present study, a two surface damage thermoplasticity model is proposed to understand this inelastic behavior and to evaluate a potential damaged state of the metallic dampers. This model is formulated through a thermodynamic approach to damage mechanics based upon entropy production. Finally, the proposed model is implemented as a user subroutine in the finite element software ABAQUS and numerical results are compared with experimental data.

### **Introduction**

During the past several decades, passive energy dissipative systems have been developed to alleviate or to avoid damage of structures caused by earthquakes (e.g., Soong and Dargush, 1997). Metallic dampers, which function by absorbing energy through the yielding of steel plates or bars, represent one class of effective energy dissipation devices. Ultimately, the energy dissipation may improve the overall performance of the building during earthquakes. However, there is a need to understand the behavior and potential failure mechanisms of these dampers. In particular, a number of different cyclic plasticity models may be applicable. Valanis (1971, 1980) developed endochronic theories considering intrinsic time to make the response rate independent, Chaboche and Rousselier (1983) developed internal variable theories using some quantities that can not be measured by direct experiment, and Mroz (1967), Dafalias and Popov (1975), and Krieg (1975) developed a multisurface plasticity models. Banerjee et al. (1987), and Chang and Lee (1987) developed a two surface plasticity model to represent both kinematic and isotropic hardening behavior. This two surface model is characterized by an inner surface that follows a kinematic hardening rule and an outer surface, which provides for isotropic hardening. This model was applied in Dargush and Soong (1995) to help understand inelastic behavior of steel plate dampers and also was able to give a good match with experimental force-

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displacement data for the initial cycles (Tsai et al, 1993).

Under large amplitude cyclic loading, however, the structural steel of the metallic damper may undergo significant excursions into the inelastic range, which may be accompanied by increases in temperature of the steel and the accumulation of damage. Thus, in the present work, we extend the two surface model to incorporate damage mechanics concepts in order to consider these thermal effects and material degradation processes. Recently, Basaran and Nie (2004) developed a damage model using a damage parameter based on the second law of thermodynamic law to determine the fatigue life of material.

In this study, we propose a two surface thermoplasticity model and add a damage parameter to understand inelastic behavior of metallic dampers, considering thermal effects and material damage. This damage model is implemented as a user subroutine in the finite element software ABAQUS. Finally, numerical results are compared with the well-known nonlinear kinematic hardening model of Lemaitre and Chaboche (1994), an existing a two surface model, and experimental data.

### **Damage Mechanics and Thermodynamic Approach**

To consider the deterioration of structural steel members in general and metallic dampers in particular during cyclic loading, a scalar field variable  $D$  is introduced as a damage index at each point. Within the present model, under constant amplitude cyclic loading, the component tends to deteriorate gradually but at an increasing rate until failure occurs. This cumulative damage concept is suitable to predict a damage of component or structure involving a range of failure mechanisms, such as the growth of microcracks and microcavities. The basic damage mechanics idea was originated by Kachanov (1986), and then developed further by Lemaitre and Chaboche (1985) and Krajcinovic (1989), among others. At each material point, the scalar quantity  $D$  is simply interpreted as a dimensionless number between zero and one, where  $D = 0$  corresponds to an undamaged state, while  $D = 1$  represents a fully damaged state or fracture. Thus, the relation between an effective damaged stress ( $\bar{\sigma}$ ) and undamaged stress ( $\sigma$ ) can be expressed by

$$\bar{\sigma} = (1 - D)\sigma \quad (1)$$

Basaran and Yan (1998) introduced an entropy-based damage evolution function founded on the principles of thermodynamics. According to the second law, entropy is a monotonically increasing function, which is always positive for irreversible transformations of the system. Thus, entropy production can be used for evolution of accumulative damage. Boltzmann (1898) expressed disorder and entropy of system via the relations

$$S = k_0 \ln W \quad (2.a)$$

or

$$s = \frac{R}{m_s} \ln W \quad (2.b)$$

where  $k_0$  is the Boltzmann constant and  $W$  is a disorder parameter, which can be described as the probability that the system exists in a given state compared to all the possible states. In Eq. (2.b), the entropy per unit mass is related to the disorder parameter, where  $R$  is the gas constant and  $m_s$  is the specific mass. Then, according Basaran and Yan (1998), the damage parameter ( $D$ ) is defined as the ratio of the change in the disorder parameter to the initial reference state disorder as follows:

$$D = D_{cr} \left( \frac{\Delta W}{W_0} \right) = D_{cr} [1 - e^{-(m_s / R)\Delta s}] \quad (3)$$

where  $D_{cr}$  is the critical damage parameter which can be calculated from experiments. The entropy production ( $\Delta s$ ) is calculated by the summation of mechanical dissipation, thermal dissipation due to conduction of heat and thermal dissipation due to internal heat source per unit mass ( $r$ ).

$$\Delta s = \int_{t_0}^t \frac{\bar{\sigma} : \dot{\bar{\epsilon}}^p}{T\rho} dt + \int_{t_0}^t \left( \frac{k}{T^2 \rho} |\text{grad}T|^2 \right) dt + \int_{t_0}^t \frac{r}{T} dt \quad (4)$$

where  $\rho$  is density and  $T$  is absolute temperature. As shown in Eq. (4),  $D = 0$  when  $\Delta s = 0$ , and  $D = D_{cr}$  when  $\Delta s$  goes to infinity. Thus, with dissipation,  $D$  is always larger than zero, because the change in entropy has a nonnegative value.

Then, the constitutive relation can be written in the following incremental form in terms of the undamaged and damaged stress:

$$\dot{\bar{\sigma}} = (1 - D)\dot{\sigma} = (1 - D)C^e \dot{\bar{\epsilon}}^e = (1 - D)C^e (\dot{\bar{\epsilon}} - \dot{\bar{\epsilon}}^p - \dot{\bar{\epsilon}}^{th}) \quad (5)$$

where  $C^e$  is the elastic constitutive matrix,  $\dot{\bar{\epsilon}}^e$  is the elastic strain increment,  $\dot{\bar{\epsilon}}^p$  is the plastic strain increment, and  $\dot{\bar{\epsilon}}^{th}$  is the incremental thermal strain. If a two-surface thermoplasticity model (Banerjee et al., 1987; Chopra and Dargush, 1994) is adopted, the constitutive relation can be represented by the following pseudo code:

For elastic loading or unloading,

$$\dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\epsilon}_{kk} + 2\mu \dot{\epsilon}_{ij} \quad (6.a)$$

Else for inelastic loading inside the outer surface,

$$\dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\epsilon}_{kk} + 2\mu \dot{\epsilon}_{ij} - \frac{3\mu \bar{S}_{ij} \bar{S}_{kl} \dot{\epsilon}_{kl}}{(\sigma_y^L)^2 \left[ 1 + \frac{H^p}{3\mu} \right]} \quad (6.b)$$

Else for inelastic loading on the outer surface,

$$\dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\epsilon}_{kk} + 2\mu \dot{\epsilon}_{ij} - \frac{3\mu S_{ij} S_{kl} \dot{\epsilon}_{kl}}{(\sigma_y^B)^2 \left[ 1 + \frac{H^P}{3\mu} \right]} \quad (6.c)$$

where  $\sigma_y^L$  is the inner yield strength,  $\sigma_y^B$  is the outer yield strength,  $S_{ij}$  is the deviatoric stress, and  $H^P$  is a hardening modulus, dependent on the hardening parameters  $H_0^B, h_1^B, n$ . Figure 1 shows two distinct yield surfaces in deviatoric stress space. The inner surface, which separates the elastic range and inelastic range, is composed of its center and radius expressed by the back stress ( $\alpha$ ) and inner yield strength ( $\sigma_y^L$ ). Meanwhile, the outer surface, which always contains inner surface, is located on the center of stress space with a radius represented by the outer yield strength ( $\sigma_y^B$ ). The translation of inner surface corresponds to kinematic hardening, while the expansion of outer surface produces isotropic hardening.

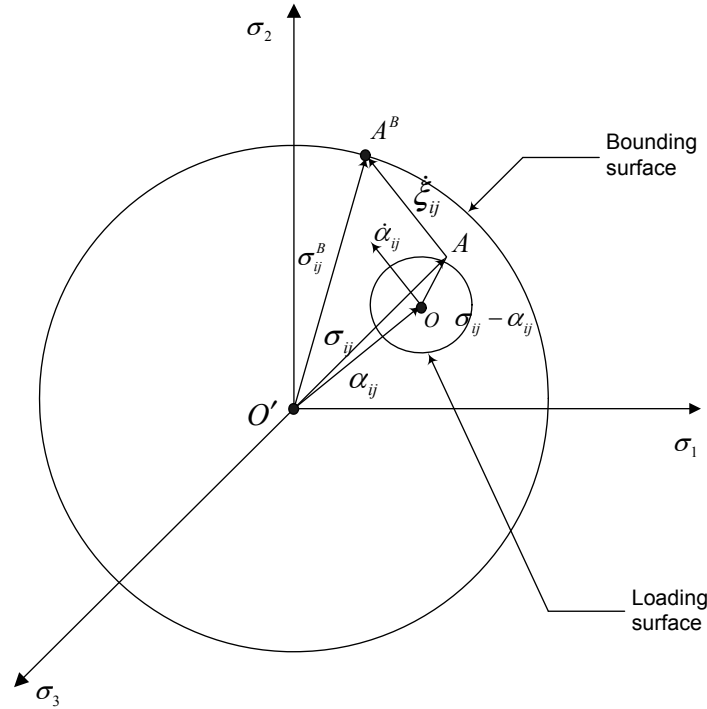


Figure 1. Two surface plasticity model definition

By substituting Eq. (6) into Eq. (5) instead of the undamaged incremental stress, the coupled damaged stress-strain relationship of a two surface plasticity model was formulated as shown in Fig. 2. Thus, the proposed thermoplastic two surface damage model is characterized by the two surface plasticity model with a yield surface, flow rule and hardening rule on both loading and bounding surfaces, and a damage evolution function based on entropy production.

The resulting model can be applied for a variety of metals with progressive damage under cyclic loading. Also, thermal strain is added to this two surface model to consider thermal effects, as indicate in Fig. 2.

$$\dot{\bar{\sigma}} = (1 - D)\dot{\sigma} = (1 - D)C^e(\dot{\bar{\epsilon}} - \dot{\bar{\epsilon}}^p - \dot{\bar{\epsilon}}^{th})$$

$$\begin{array}{l} \rightarrow \dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\bar{\epsilon}}_{kk} + 2\mu \dot{\bar{\epsilon}}_{ij} - (3\lambda + 2\mu)\delta_{ij}\alpha\dot{T} \quad \text{Elastic region} \\ \rightarrow \dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\bar{\epsilon}}_{kk} + 2\mu \dot{\bar{\epsilon}}_{ij} - \frac{3\mu \bar{S}_{ij} \bar{S}_{kl} \dot{\bar{\epsilon}}_{kl}}{(\sigma_y^L)^2 \left[1 + \frac{H^p}{3\mu}\right]} - \frac{H^p}{3\mu + H^p} (3\lambda + 2\mu)\delta_{ij}\alpha\dot{T} \quad \text{Meta-elastic region} \\ \rightarrow \dot{\sigma}_{ij} = \lambda \delta_{ij} \dot{\bar{\epsilon}}_{kk} + 2\mu \dot{\bar{\epsilon}}_{ij} - \frac{3\mu S_{ij} S_{kl} \dot{\bar{\epsilon}}_{kl}}{(\sigma_y^B)^2 \left[1 + \frac{H^p}{3\mu}\right]} - \frac{H^p}{3\mu + H^p} (3\lambda + 2\mu)\delta_{ij}\alpha\dot{T} \quad \text{Plastic region} \end{array}$$

Figure 2. Proposed two surface damage model

### Numerical Results of Simple Shear Problem with Two Surface Damage Model

The two surface damage model described above was implemented as a user subroutine (UMAT and UMATHT) in the ABAQUS (2008) finite element software. Once the small increment of strain is given, new updated state variables such as stress, back stress and plastic strain are obtained by integrating constitutive equations. For this analysis, a higher-order adaptive step size Runge-Kutta method is used to integrate the constitutive equations until a high level of accuracy is maintained. Prior to applying a two surface damage model to metallic dampers, simple shear problem was first used to consider several effects of this model, such as fatigue by cyclic loading and by temperature. The geometry and dimension of undeformed and deformed elements is shown in Fig. 3, which includes four 4-node plane strain elements and coupled displacement-temperature elements, if temperature is considered. The loading and unloading at the top is displacement controlled sine function with a maximum displacement of 0.1 units.

Figure 4 shows the cyclic response of undamaged and damaged material using the two surface model and a two surface damage model. After several cycles, entropy production is accumulated by mechanical dissipation. As a result, the damage parameter increases gradually at first and then more rapidly, until failure of material is identified, when the damage parameter approaches to one. If not only mechanical dissipation but also thermal dissipation is considered to the entropy production, the material can be damaged more severely and more rapidly than the case with only mechanical dissipation, as shown in Fig.5. Although here damage by thermal dissipation is comparatively smaller than damage by mechanical dissipation in Fig. 5, in general damage by thermal dissipation could be significant, especially if there are high temperatures or strong thermal gradients.

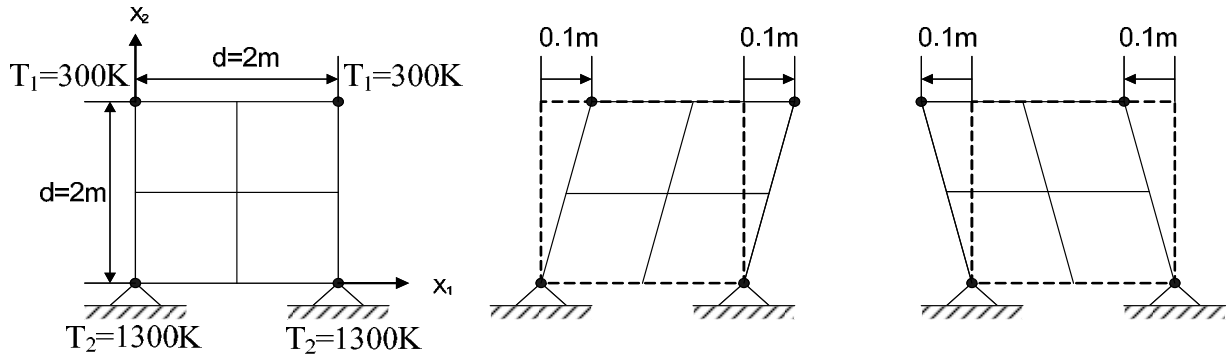
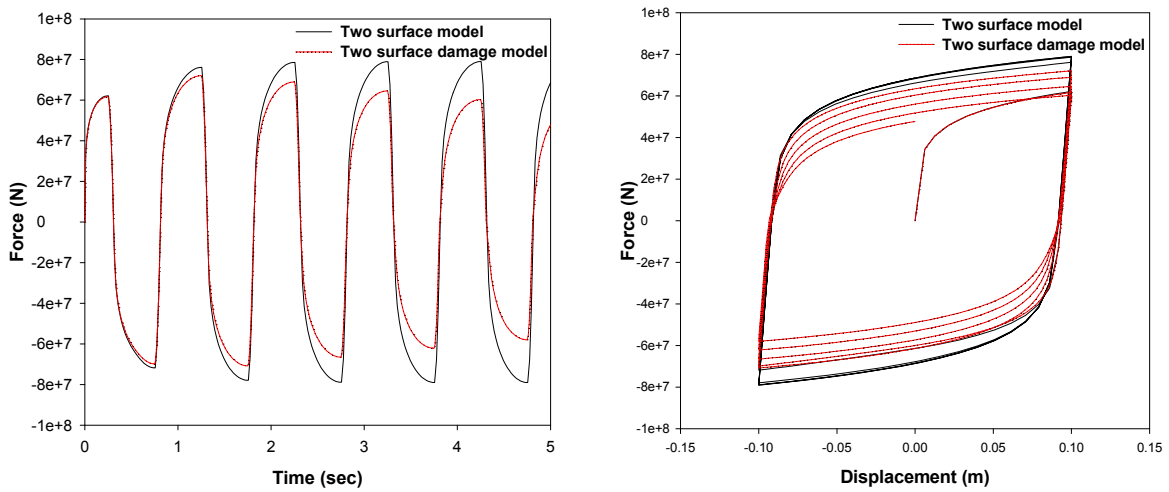


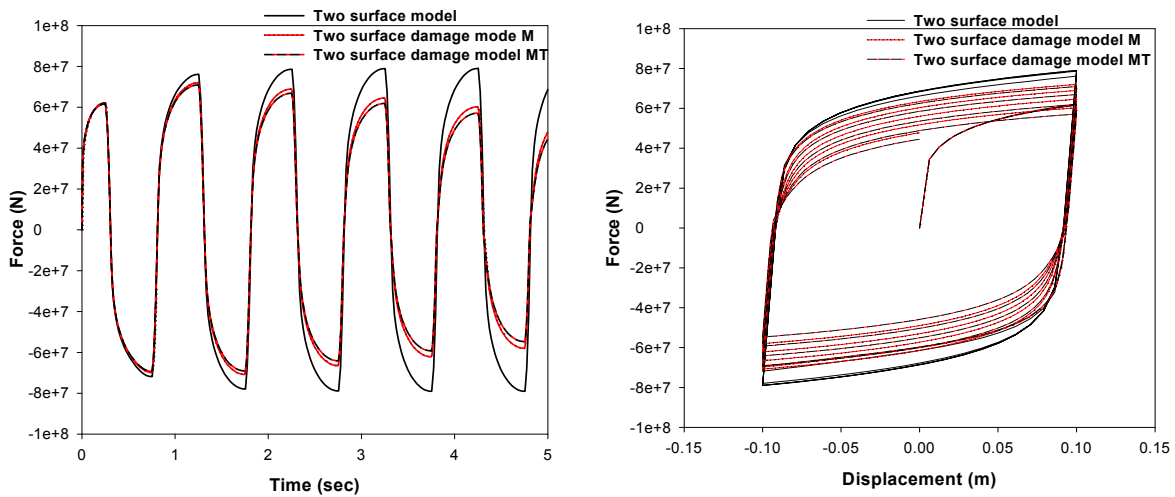
Figure 3. The geometry of simple shear problem



(a) Force-time response

(b) Force-displacement response

Figure 4. Displacement analysis with mechanical dissipation



(a) Force-time response

(b) Force-displacement response

Figure 5. Coupled temperature-displacement analysis with mechanical and thermal dissipation

## Numerical Results of Metallic Dampers with Two Surface Damage Model

Metallic plate dampers were modeled by 8-node solid brick elements (C3D8) in ABAQUS (2008) using a user subroutine (UMAT) in which the two surface damage model was implemented. Material properties and model parameters for the A36 structural steel of metallic dampers are shown in Table 1. The loading and unloading at the free edge is specified as a displacement-controlled sine function with the maximum displacement at top and bottom set at  $0.1L$ , where  $L$  represents the length of the plate. Selected results are shown in Fig. 6 for a model representing one-half of one plate of the damper. Notice that the damage parameter is large at fixed edge and is small at free edge, because the strain energy by mechanical dissipation at the fixed edge is larger than that at the free edge. Thus, a damage parameter can be used as an index to express the damaged configuration.

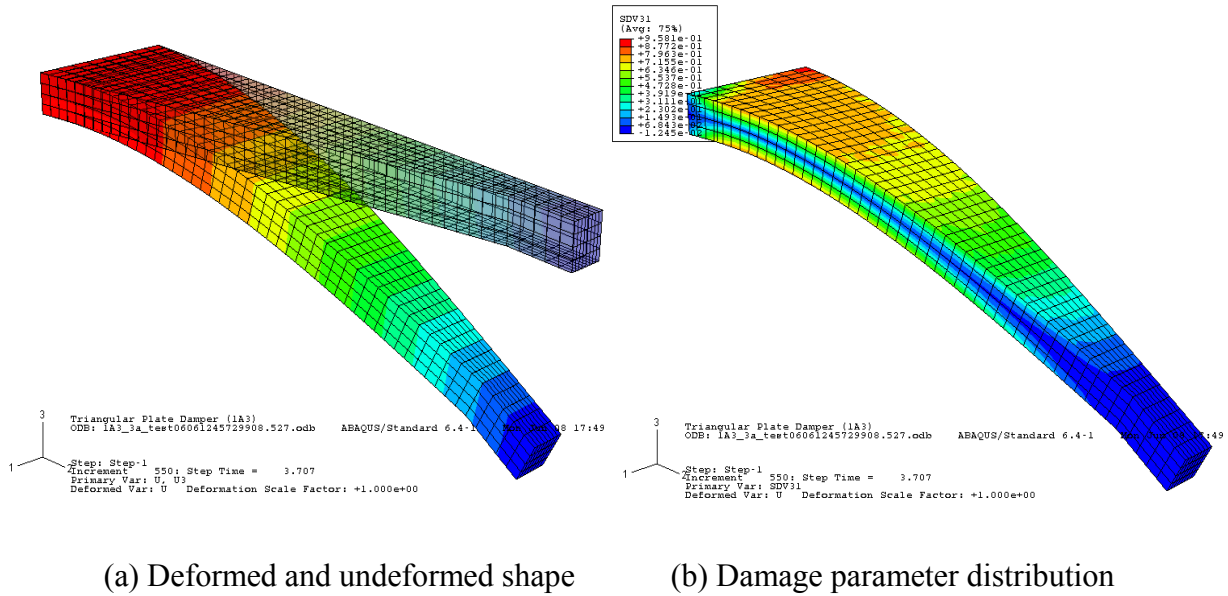


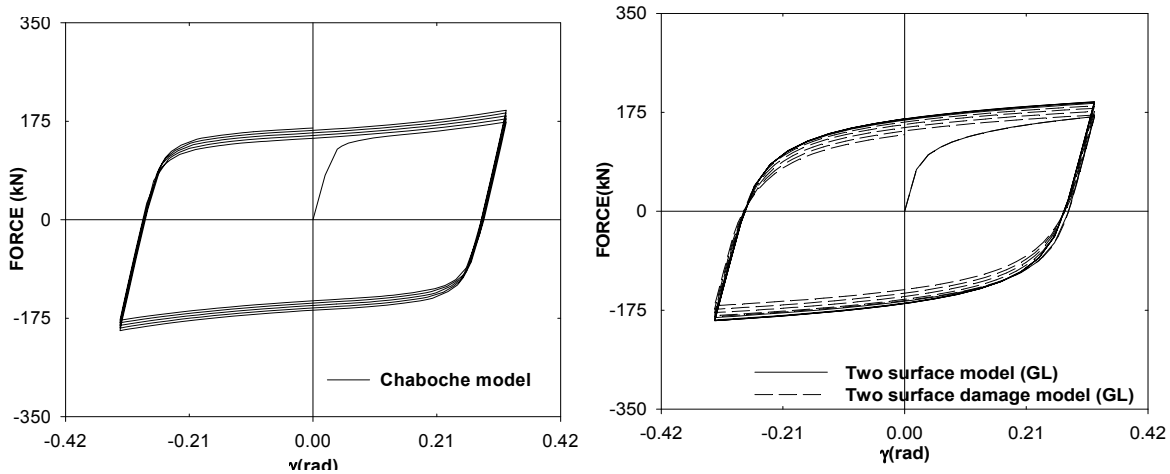
Figure. 6 Numerical results of metallic dampers

Table. 1 Material properties and model parameters for A36 structural steel

Young's Modulus ( $E$ ) = 200,000 MPa	Poisson's ratio ( $\nu$ ) = 0.3
Inner yield strength ( $\sigma_y^L$ ) = 198 MPa	Outer yield strength ( $\sigma_y^B$ ) = 427 MPa
Hardening parameter ( $H_0^B$ ) = 6450 MPa	$h_1^B = -8.47, n = -10.4$

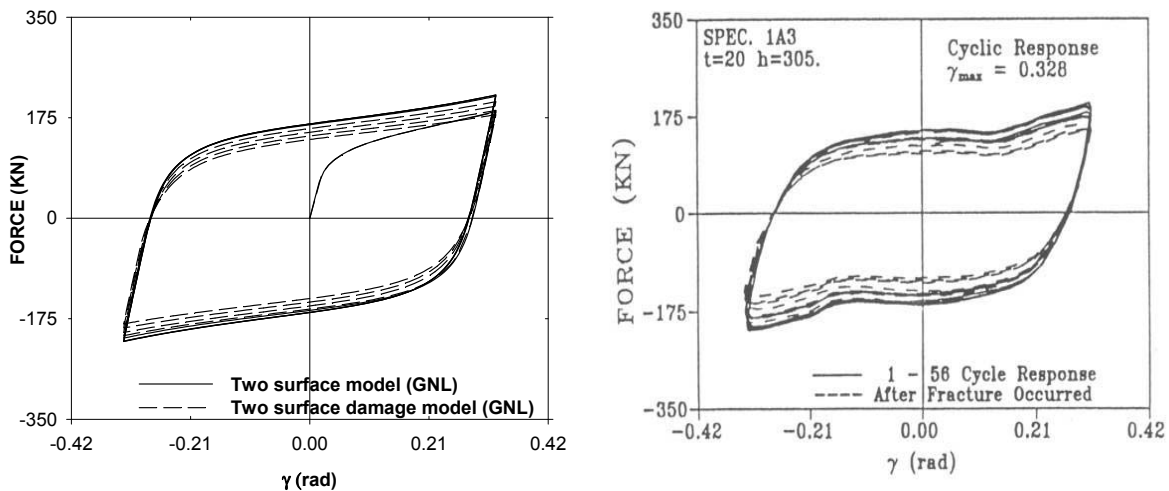
Force-displacement plots of numerical models (Chaboche model, two surface model and two surface damage model with geometrically linearity and nonlinearity) and experimental results are plotted in Fig. 7. If the metallic dampers are analyzed by a Chaboche model, it shows

that strength is increasing as the cycles progress. This model can not predict any damage, as shown in Figure 7(a). Figure 7(c) gave mostly good agreements with experiments both before and after fracture occurred, though some differences exist. If the geometrically nonlinearity is considered to a two surface model and a two surface damage model, an additional stiffening exists at the higher range of displacement, as shown in Fig. 7(b) and Fig. 7(c). This behavior is very useful for designing metallic dampers, because it proves that these dampers have a higher capacity, however the potential for locking must be considered.



(a) Chaboche model

(b) Two surface damage model with geometrically linearity



(c) Two surface damage model with geometrically nonlinearity

(d) Experimental results (Tsai et al., 1992)

Figure 7. Comparison between experimental and numerical results

## Conclusions

A two surface thermoplastic damage model is formulated and implemented as a UMAT subroutine within ABAQUS. Numerical results give reasonable correlation with experimental



results both before and after damage has occurred. Consequently, this approach is helpful for understanding the cyclic behavior of metallic dampers and for designing metallic dampers considering large deformation. The damage parameter is governed by dissipation due to plastic strain for large strain case or thermal dissipation via conduction of heat for high temperature. Thus, this model is also useful to understand the behavior of metallic dampers and other metallic components, which experience significant excursions in temperature. The proposed two surface damage model has path dependent characteristics, because entropy production is accumulated from zero until the damage parameter equals to its critical value. Finally, this model may help to estimate the fatigue life of metallic dampers under cyclic loading and also more broadly to represent a range of failure processes in metals.

## Appendix

The following symbols are used in this paper.

$C_e$	Elastic constitutive matrix
$D$	Damage parameter
$E$	Young's Modulus
$D_{cr}$	Critical damage parameter
$H^p$	Hardening parameter
$k_0$	Boltzmann constant
$m_s$	Specific mass
$R$	Gas constant
$r$	Internal heat source per unit mass
$S$	Entropy
$S_{ij}$	Deviatoric stress
$T$	Absolute temperature
$W$	Disorder parameter
$\alpha_{ij}$	Back stress
$\alpha$	Thermal expansion coefficient
$\dot{\epsilon}^e$	Elastic strain increment
$\delta_{ij}$	Kronecker delta
$\dot{\epsilon}^p$	Plastic strain increment
$\dot{\epsilon}^{th}$	Thermal strain increment
$\bar{\sigma}$	Damaged stress
$\sigma$	Undamaged stress
$\sigma_y^L$	Inner yield strength
$\sigma_y^B$	Outer yield strength
$\lambda$	Lamé constants
$\mu$	Shear modulus

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