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SEISMIC ANALYSIS OF CANTILEVER RCC RETAINING WALL

DR. M. A. Chakrabarti [1] and P. T. Mestri[2]

Abstract

Present state of the art for the analysis and design of retaining walls under earthquake loading is based on the method proposed by Mononobe and Matsuo (1929) and Okabe (1926) (M-O analysis). Normally, dynamic pressure induced in the soil both due to active and passive conditions are computed using a pseudo-static force acting within the failed soil wedge and equivalent dynamic coefficients of active and passive earth pressures are obtained. Also the Indian Code of Practice, IS-1893 suggests the use of α_h and α_v based on seismic coefficient without having re-course to any time-period calculation, which could again make the analysis far too conservative. As IS-1893 does not provide any rational method for calculation of natural period of the retaining wall, seismic analysis for such system is still based on what one can term as a pseudo static analysis.

In the present paper, a method is used by which it is possible to obtain the natural period of a cantilever retaining wall with leveled backfill quite accurately by use of the improved Rayleigh-Ritz method and to carry out the dynamic analysis of such walls based on modal response technique. The fundamental natural period and shear force and bending moment of the retaining wall with soil mass has been computed for two cases 1.Under active earth pressure condition, 2.Under passive force. The results of the dynamic analysis are then compared with the IS Code dynamic analysis. The comparative study of the design forces based on this dynamic analysis and the IS Code dynamic analysis is made.

Introduction

For the safe and economic design of retaining structures, correct estimation of earth pressure on retaining is very important to civil engineers. Due to its complexity in analysis, this problem has drawn the attention of researchers through the decades. Even under static conditions this is one of the most critical and complex problems of soil mechanics and geotechnical engineering. So, under dynamic condition and under seismic loading, the problem is no doubt challenging. The recent devastating earthquakes in India, like the Kashmir Earthquake in 2005, and the Bhuj Earthquake in 2001 have added important dimensions to this problem, as in the hilly regions, retaining structures are of utmost importance. Among the theories available till date for the estimation of seismic earth pressure, the Mononobe–Okabe (Mononobe N. and H. Matsuo 1929 & Okabe S. 1926) method, which is the pioneering work in this field, is commonly used.

[1] *Professor, Department of Structural Engineering, Veermata Jijabai Technological Institute, Mumbai.*
[2] *Post Graduate, Department of Structural Engineering, Veermata Jijabai Technological Institute*

Dynamic pressure induced in the soil both due to active and passive conditions are computed using a pseudo-static force acting within the failed soil wedge and equivalent dynamic coefficients of active and passive earth pressures are obtained. The pressure distribution is obtained considering the wall, as a gravity-type having infinite stiffness while the ground acceleration is considered maximum. However, a typical RCC retaining wall is much more flexible than the gravity wall and such analysis could either be too conservative. Again, the acceleration coefficient depends on the natural period of the wall-soil system, which in turn is a function of the stiffness and mass distributions.
The total forces on the wall is given in Eq. (1) under active as well as passive earth pressure conditions are as per Mononobe and Matsuo (Mononobe N. and H. Matsuo 1929) and Okabe (Okabe S. 1926) method is given IS 1893 (Part - III) , by

$$P_a = \frac{1}{2} C_a \gamma H^2 \ ; \ P_p = \frac{1}{2} C_p \gamma H^2 \tag{1a,b}$$

in which Pa and Pp are static plus dynamic force on the wall under active and passive earth pressure conditions respectively; Ca and Cp are the coefficients of dynamic active and passive earth pressures respectively; γ is the unit weight of soil and H is the height of wall/soil retained. For the case of a typical retaining wall, Ca and Cp are given by

$$C_a = \frac{(1 \pm \alpha_v)\cos^2(\phi - \lambda)}{\cos\lambda\cos(\delta + \lambda)} \times \left[\frac{1}{1 + \left\{ \frac{\sin(\phi + \delta)\sin(\alpha - \lambda)}{\cos(\delta + \lambda)} \right\}^{\frac{1}{2}}} \right]^2 ;$$

$$C_p = \frac{(1 \pm \alpha_v)\cos^2(\phi - \lambda)}{\cos\lambda\cos(\delta + \lambda)} \times \left[\frac{1}{1 - \left\{ \frac{\sin(\phi + \delta)\sin(\alpha - \lambda)}{\cos(\delta + \lambda)} \right\}^{\frac{1}{2}}} \right]^2 \tag{2a,b}$$

where ϕ is angle of internal friction, δ is angle of friction between the wall and earthfill and λ is $\tan^{-1}(\alpha_h/1-\alpha_v)$; in which α_h and α_v are maximum ground acceleration (in g) in horizontal and vertical direction respectively.

Earth Pressure Condition

A method is used of Dasgupta S. P. and Chowdhury I (Dasgupta S. P. and Chowdhury I 2003) by which it is possible to obtain the natural period of a retaining wall quite accurately and to carry out an alternate method for the dynamic analysis of such walls based on modal response technique. The wall will be assumed to be a flexural member. The fundamental natural period of the retaining wall with soil mass has been computed for two cases:

1. Under active earth pressure condition (i.e. the earthquake shock pushes the wall away from the retained soil).

2. Under passive force (i.e. the earthquake shock pushes the wall towards the retained soil)

For case 1 when the earthquake force tries to move the retaining wall away from the soil, the soil behind the retaining wall is already under incipient failure having failure profile inclined at (45^o + ϕ/2)

For the case 2, failure profile is inclined at (45^o - ϕ/2). It may be argued that during an earthquake shock.

Under Active Earth Pressure Condition

Based on the above assumption a cantilever wall is considered for analysis, which will be subjected to the following loads under static condition:

- The weight of the failed wedge having weight of $\gamma H^2 / [2 \tan(45^o + \phi/2)]$;
- A uniformly distributed load due to self-weight.

Considering the self-weight of the wall to be negligible compared to the soil mass. Shown in Fig. 1 is the mass distribution of the failed soil wedge ABD. For an elemental strip dz in vertical direction mass distribution is given by $m(z) = \gamma z \, dz / \tan\alpha$ However, if one wall he may change the expression for m(z) to $m(z) = \gamma z/\gamma \tan\alpha + \gamma c \, z \, tw/\gamma$, in which γ= unit weight of soil and γc = unit weight of concrete and t_w = average thickness of the RCC wall.

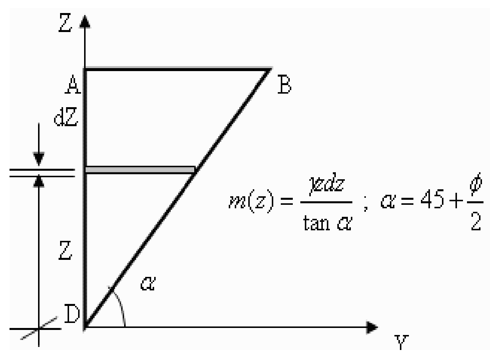


Figure 1. Mass distribution of the failed soil wedge under active soil pressure

Mass Contribution

Base on the improved Rayleigh-Ritz method, for failure wedge of the soil, the mass contribution m_{11} for the first mode is given by

$$m_{11} = \int_0^H \frac{\gamma z}{g \tan\alpha} \sin^2 \frac{\pi z}{2H} dz = \frac{\gamma H^2}{g \tan\alpha} \left[\frac{1}{4} + \frac{1}{\pi^2} \right] \quad (3)$$

in which γ is unit weight of soil and g is the gravitational acceleration.

Stiffness of Wall

If I is the average moment of inertia of the wall, the stiffness k_{11} for the first mode can be written as

$$k_{11} = EI\int_0^H \frac{\pi^4}{16H^4}\sin^2\frac{\pi z}{2H}\,dz = \frac{\pi^4 EI}{32H^3} \quad (4)$$

in which *E* is Young's modulus of concrete and *H* is height of the wall.

Fundamental Natural Period

Considering the fundamental natural period as

$$T = 2\pi\sqrt{m/k} \quad (5)$$

substituting the above values form Eq.(3) of mass (*k*) and Eq.(4) of stiffness (*m*), the fundamental natural period is

$$T = 2\pi\sqrt{\frac{8\gamma H^5(4+\pi^2)}{\pi^6 EIg\tan\alpha}} \quad (6)$$

The above eq. (6) of natural period would be sufficiently accurate. This is because we have arrived at the above natural period assuming a shape function of $\phi_1 = \sin \pi z / 2L$. How accurate the natural period will be, depends on how realistic has been the assumed shape function and there could be an error in the estimation in natural period depending on this choice.

Distribution of Nodal Mass

Knowing the design acceleration based on response spectrum as furnished in IS-1893, it is necessary to generate the nodal mass for the failed soil wedge contributing to the dynamic participation. It may be pointed out that the accuracy of the shear and its distribution on the wall will depend on how realistic has been the mass distribution and the accuracy of the mode shape. The total weight of the soil within the wedge is given by $W_s = 1/2[\gamma H2 \ \mathrm{Cot}\alpha]$ and the intensity of load is $w = H \ \mathrm{Cot}\alpha$, per linear height (Fig. 2).

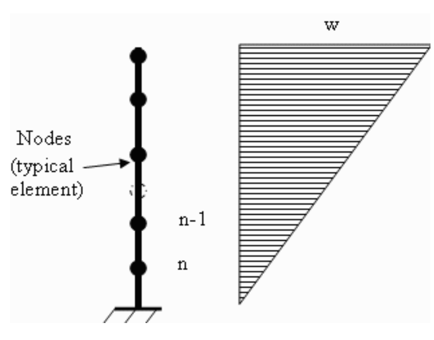
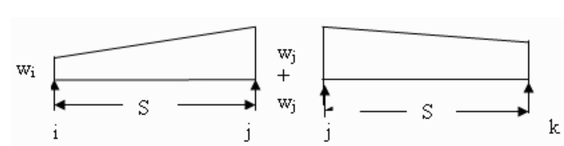


Figure 2. Distribution of the soil nodes and wall loads along the

Based on the above load distribution, the reaction transferred to subsequent stations is W_j^{ij} load transferred to station j from span i-j = $S(w_i + 2w_j)/6$; W_j^{jk}; load transferred to station j form span j-k = $S(2w_j + w_k)/6$ (Fig. 3).

The total load transferred to the node j is



$$W_j = W_{jij} + W_{jjk} = S(W_i + 4W_j + W_k)/6 \qquad (7)$$

For n number of nodes on the wall, mass contribution is given by

Figure 3. Reaction at nodes i, j, k

$$m_1 = S\,(2w_1 + w_2)/6g,\ \ m_2 = S\,(w_1 + 4w_2 + w_3)/6g,\ m_3 = S\,(w_2 + 4w_3 + w_4)/6g \qquad (8\ a,b,c)$$

$$m_{n-1} = S\,(w_{n-2} + 4w_{n-1} + w_n)/6g,\ m_n = S\,(w_{n-1} + 2w_n)/6g \qquad (9\ a,\ b)$$

Once the magnitude of the lumped masses are ascertained, the shear force at each node is obtained from the expression

$$V = \beta . I_p . F_0 . \kappa_i . S_a \sum_{i=1}^{n} m_i \phi_i \qquad (10)$$

in which V = net shear force induced due to earthquake; I_p = importance factor; β = foundation soil factor; F_0 = seismic zone factor and

$$\kappa_1 = \sum_{i=1}^{n} m_i \phi_i \,/ \sum_{i=1}^{n} m_i \phi_i^2 \qquad (11)$$

is called the modal participation factor, which for this case it can be expressed as

$$\kappa_1 = \frac{\int_0^H \frac{\gamma z}{g \tan\alpha} \sin\frac{\pi z}{2H}\, dz}{\int_0^H \frac{\gamma z}{g \tan\alpha} \sin^2\frac{\pi z}{2H}\, dz} = \frac{16}{4+\pi^2} \qquad (12)$$

Knowing the modal mass participation factor, the shear at any node i can be obtained as follows

$$V_i = \beta . I_p . F_o \frac{16}{4+\pi^2} S_a \int_0^H \frac{\gamma z}{g \tan\alpha} \sin\frac{\pi z}{2H}\, dz \qquad (13)$$

It is possible to obtain the dynamic shear force at base directly from Eq. (14) by modifying it to the form

$$V_i = \beta . I_p . F_o . \frac{64}{\pi^2 (4+\pi^2)} \frac{S_a}{g} \frac{\gamma . H^2}{\tan\alpha} \qquad (14)$$

However, the use of Eq. (13 & 14) is limited, as moments cannot be obtained directly. As such, it is better to discretise the same based on lumped mass at various nodes and obtain the shear and bending moments.

Under Passive Earth Pressure Condition

In this case, as mentioned above, steps remain the same except that the failure profile reduces from $\alpha = (45 + \phi/2)$ to $(45 - \phi/2)$ and the mass matrix and the fundamental natural period are modified accordingly.

Example

Based on the above theory, the dynamic analysis results of a 4 to 8 m high retaining wall in an earthquake zone I to V (of India) are compared with the static analysis. The acceleration is obtained based on natural period, finally for this case considering the soil-profile given in Fig. 4. Basic data considered for the retaining wall is as follows:

H = 4 to 8m; γ= 18 kN/m^3; ϕ= 28^o; Seismic zone = I to V (IS 1893 1984); Soil Profile = same as shown in Fig. 4; Average thickness of wall = 500 mm; Grade of concrete = M 25, β = 1.0; I. F = 1.5; Damping = 5 % damping.

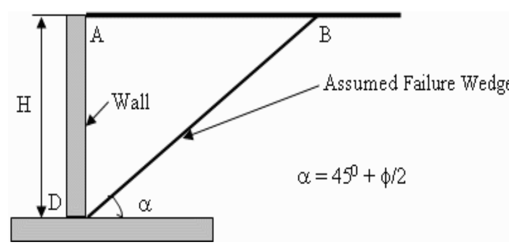


Figure 4. Retaining wall with levelled backfill

Table 1. Natural Periods for Active earth pressure case

Height of Retaining Wall (m)	Natural period in Sec.
4	0.131631
5	0.229950
6	0.362733
7	0.533279
8	0.744619

Table 2. Natural Periods for Passive earth pressure case

Height of Retaining Wall (m)	Natural period in Sec.
4	0.219071
5	0.382702
6	0.603689
7	0.887525
8	1.239254

Comparative study is carried out between this method and IS code method, as the maximum force is at the base, so the results are summarized for different height of wall and zones.

Table 3. Percentage variations of shear force at base of wall between dynamic force by this method and IS code method for Active earth pressure

Height	Percentage variation of Shear force between this method and IS code method for Zone I	Percentage variation of Shear force between this method and IS code method for Zone II	Percentage variation of Shear force between this method and IS code method for Zone III	Percentage variation of Shear force between this method and IS code method for Zone IV	Percentage variation of Shear force between this method and IS code method for Zone V
4 m	1.090	1.367	2.168	2.714	4.897
5 m	1.090	1.367	2.168	2.714	4.897
6 m	1.154	1.490	2.398	2.993	5.306
7 m	1.606	2.366	4.053	5.007	8.301
8 m	1.997	3.133	5.530	6.821	11.068

Table 4. Percentage variations of shear force at base of wall between dynamic force by this method and IS code method for Passive earth pressure

Height	Percentage variation of Shear force between this method and IS code method for Zone I	Percentage variation of Shear force between this method and IS code method for Zone II	Percentage variation of Shear force between this method and IS code method for Zone III	Percentage variation of Shear force between this method and IS code method for Zone IV	Percentage variation of Shear force between this method and IS code method for Zone V
4 m	-5.441	-6.617	-8.868	-9.933	-12.819
5 m	-5.430	-6.596	-8.826	-9.880	-12.733

6 m	-5.337	-6.408	-8.445	-9.401	-11.955
7 m	-5.275	-6.282	-8.191	-9.082	-11.437
8 m	-5.223	-6.178	-7.979	-8.816	-11.005

Table 5. Percentage variation of bending moment at base of wall between static force and dynamic force for Active earth pressure

Height	Percentage variation of Bending Moment between this method and IS code method for Zone I	Percentage variation of Bending Moment between this method and IS code method for Zone II	Percentage variation of Bending Moment between this method and IS code method for Zone III	Percentage variation of Bending Moment between this method and IS code method for Zone IV	Percentage variation of Bending Moment between this method and IS code method for Zone V
4 m	1.923	2.367	3.632	4.481	7.784
5 m	1.923	2.367	3.632	4.481	7.784
6 m	2.037	2.583	4.025	4.950	8.453
7 m	2.844	4.127	6.877	8.389	13.452
8 m	3.547	5.496	9.476	11.563	18.224

Table 6. Percentage variation of bending moment at base of wall between static force and dynamic force for Passive earth pressure

Height	Percentage variation of Bending Moment between this method and IS code method for Zone I	Percentage variation of Bending Moment between this method and IS code method for Zone II	Percentage variation of Bending Moment between this method and IS code method for Zone III	Percentage variation of Bending Moment between this method and IS code method for Zone IV	Percentage variation of Bending Moment between this method and IS code method for Zone V
4 m	-7.959	-8.536	-9.475	-9.825	-10.310
5 m	-7.976	-8.569	-9.542	-9.908	-10.445
6 m	-8.124	-8.865	-10.133	-10.648	-11.642
7 m	-8.222	-9.061	-10.523	-11.135	-12.422
8 m	-8.304	-9.224	-10.846	-11.537	-13.062

Conclusions

The present analysis gives a solution both for the general and particular cases of dynamic response of RCC retaining wall based on improved Rayleigh-Ritz method. The Indian Code of Practice, IS-1893[3] suggests the use of α_h and α_v based on seismic coefficient without having re-course to any time-period calculation, so in this analysis the natural time period of the retaining wall are found out from which the response on the retaining wall are found out from IS 1893[3] and used the response spectrum method for shear force and bending moment analysis.
Based on the above results, it can be concluded that the present IS code analysis is gives additional forces on retaining wall as this method based on a pseudo static forced-based approach and hence is only dependent on the maximum amplitude, not on the frequency of ground motion. The shear force and bending moment calculated by IS code method are the maximum.

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