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AN EXAMPLE OF REGIONAL SEISMIC RISK ASSESSMENT WITH RELIABILITY ANALYSIS AND PROBABILISTIC MODELS

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ABSTRACT

In this paper, the aggregate regional seismic loss curve for a portfolio of buildings is obtained. Reliability methods are employed in conjunction with probabilistic models that simulate the earthquake hazard, infrastructure performance, and damage and loss consequences. By utilizing the first-order reliability method, efficient and accurate event probabilities are computed. Furthermore, the most important sources of uncertainty are identified. The region under consideration is the campus of the University of British Columbia, Vancouver. The buildings in this region are visually inspected to obtain input to the probabilistic models. An important contribution of this paper is the utilization of reliability methods in conjunction with probabilistic models in large-scale regional risk analyses.

Introduction

The objective of this paper is to assess the regional seismic risk to buildings. This research is motivated by the fact that the safety of the society is contingent on the safety and reliability of infrastructures. Earthquakes are of the most important sources of jeopardy to this safety. Indeed, correlated damage in many buildings during an earthquake may result in catastrophic losses. To evaluate and expose the seismic risk, it is needed to simulate possible future events and evaluate the probability of their occurrence. To account for unavoidable uncertainty, "probabilistic models" are needed for hazards, infrastructures, and consequences. The methodology, software, and general format of probabilistic models in this research are introduced in a companion paper (Mahsuli and Haukaas 2010). In the present paper, the specific models that are employed in the regional seismic risk analysis are explained, and a regional reliability analysis is carried out.

The first comprehensive framework for assessment of the regional seismic loss was developed by the Applied Technology Council (ATC 1985). Two decades later, the Federal Emergency Management Agency and the National Institute of Building Sciences (FEMA–NIBS 2003) developed a more comprehensive loss estimation methodology. The methodology was

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implemented in the HAZUS software application. While most of the studies in the literature on regional analysis of civil infrastructure consider a portfolio of hypothetical structures as examples, some researchers have employed databases of buildings that exist in reality. Ventura *et al.* (2005) developed the BC-31 classification of buildings in accordance with construction in British Columbia, Canada. This classification accompanied by damage-probability function was built upon the methodology of ATC-13. The database of building inventory used to develop this classification was collected using rapid screening and visual inspection procedures. These databases cover a noticeable part of building inventory in Vancouver, New Westminster, and Victoria. Onur *et al.* (2005) applied the ATC-13 methodology and BC-31 classification on the databases of building inventories to assess the regional seismic risk in British Columbia.

The overall methodology in this research consists of employing "probabilistic models" in a reliability analysis performed with reliability methods. These probabilistic models are implemented in a comprehensive software application developed in this research for risk analysis, named Rt. This software is freely available from the Software page at www.inrisk.ubc.ca. The models are then employed by the software to assess the regional seismic loss at the campus of the University of British Columbia (UBC). Using the first-order reliability method (FORM) (Ditlevsen and Madsen 1996), the most important sources of uncertainty are identified and the buildings are ranked in accordance with their contribution to the uncertainty.

Hazard Models

The hazard investigated in this paper is earthquake ground motion. The earthquake hazard is divided in three interacting probabilistic models: magnitude model, location model, and intensity model.

The earthquake magnitude is represented by a random variable with a bounded exponential distribution (McGuire 2004). The probability density function (PDF) for this random variable is as follows:

$$f(m) = \frac{b \cdot e^{-b(m-M_{\min})}}{1-e^{-b(M_{\max}-M_{\min})}} \tag{1}$$

where m is the magnitude random variable, $M_{\min}$ and $M_{\max}$ are the lower and upper bounds of engineering interest for magnitude, respectively, and b is a parameter depending on the relative occurrence of different magnitudes. The PDF presented in Eq. (1) is illustrated in Figure 1a.

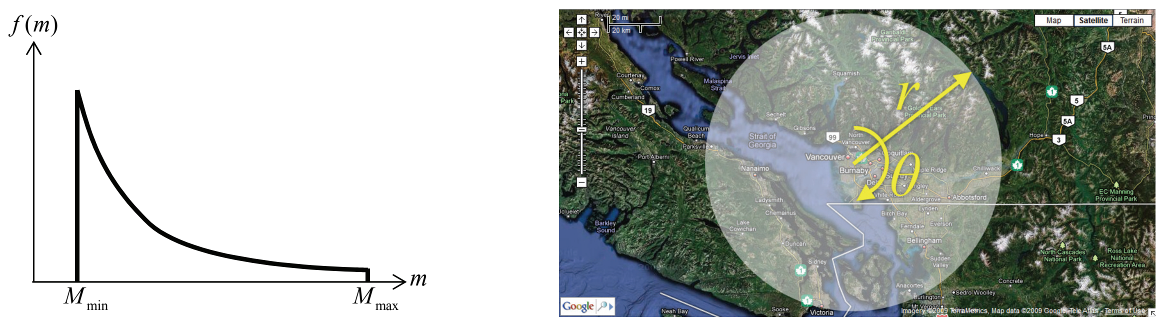
The model representing the earthquake location is a circular seismic area source. This model is attributed to sub-crustal earthquakes that can occur anywhere within an area surrounding the region under study. The model generates realizations of earthquake hypocentre location specified by latitude, longitude, and depth. Given the centre of the area source, it is assumed that the epicentre of the earthquake is equally likely to happen anywhere within a circle of radius R. The shape of this area source is depicted in Figure 1b. The uncertainty in the epicentre location of the earthquake is represented by two radial and angular random variables. The radial random variable, r, is linearly distributed between 0 and R as follows:

$$f(r) = \frac{2r}{R^2} \tag{2}$$

The angular random variable, θ, is uniformly distributed between 0 and 2π. The depth of the earthquake hypocentre, h, is evaluated by employing the following empirical relationship proposed by Atkinson (2005):

$$h = 10^{0.15M - 0.05} \tag{3}$$

where M is the magnitude evaluated by the upstream magnitude model, and h is the depth in km.



(a) (b)

Figure 1. (a) The PDF of magnitude random variable, and (b) circular seismic area source.

The last model in the chain of earthquake hazard models is the earthquake intensity model. The intensity model employed here for regional risk analysis is based on an attenuation relationship proposed by Atkinson (1997). The format of this model is as follows:

$$\log_{10}(S_a) = c_0 + c_1(M-6) + c_2(M-6)^2 + c_3 h + c_{a1} \cdot \log_{10}(R) - (c_{a3} + c_{a4}h)R + c_s S + c_\sigma \sigma \tag{4}$$

where S_a is the spectral acceleration, M is the earthquake magnitude, h is the hypocentral depth, R is the distance between the structure location and the earthquake epicentre, S is a factor for soil type, σ is the model error term, and c_0, c_1, c_2, c_3, c_{a1}, c_{a3}, c_{a4}, c_s, and c_σ are model coefficients. These coefficients are each a discrete function of magnitude M and the natural frequency of the structure $f = 1/T$, where T is the natural period of the structure. In order to use FORM analysis, these discrete functions are transformed to smoothed continuous and thus, differentiable curves through curve-fitting. The magnitude M and the hypocentral depth h in this model are provided by the upstream earthquake magnitude and location models, respectively. The distance R is evaluated using the latitude and longitude of the hypocentre provided by the location model and those of the structure. The uncertainties in this model include an epistemic random variable for the model error σ and eight other epistemic random variables for each of the model coefficients c_0 to c_σ representing the model uncertainty in the equation of the fitted curves.

Infrastructure Model

Regional risk analysis is both a computational and data intensive study owing to the large number of infrastructures involved. A vast number of buildings should be surveyed in order to compile a database for the analysis. Thus, for each building, a limited number of data of high importance can be collected. The models that utilize these data, therefore, need to be simplified models. They take general information as input, such as number of storeys, material, and structural system. Moreover, due to the high number of the buildings in the analysis, the feasibility of employing *elaborate* structural models is questionable. In summary, the need for massive amount of data and computational resources motivates the use of simplified models in regional analysis.

The structural model in this research builds upon the FEMA-NIBS (2003) methodology. In this methodology, the buildings are classified into 36 building prototypes based on their structural system and height. For instance, there are three prototypes for low, moderate, and high rise reinforced concrete frame buildings. For each building prototype, four code design levels are defined: high, moderate, low, and pre-code. Pre-code buildings encompass the constructions before the introduction of seismic codes, *e.g.*, prior to 1940's. Moreover, buildings are classified in 33 occupancy classes based on their usage, such as family housing or commercial use.

The underlying concept of determining the peak structural response and the corresponding damage to each of the building prototypes is based on three main constituents: demand spectrum, capacity spectrum, and damage fragility. These three components are briefly explained in the following sections.

Demand Spectrum

The first step in constructing a demand spectrum is to build a 5% damped elastic demand spectrum. This curve is built using spectral accelerations provided by the upstream earthquake intensity model. Subsequently, nonlinear demand spectrum is built based on the linear spectrum. For this purpose, the energy dissipated by the hysteretic behaviour of the material in the structure is represented by an equivalent viscous damping referred to as hysteretic damping. The 5% damped elastic spectrum is then reduced by reduction factors R_A and R_V in the constant acceleration and constant velocity regions, respectively. The factors R_A and R_V are functions of the effective damping, β_{eff}, which is the sum of the elastic damping, β_E, and the hysteretic damping, β_H:

$$R_A = \frac{2.12}{3.21 - 0.68\ln(\beta_{eff})} \quad , \quad R_V = \frac{1.65}{2.31 - 0.41\ln(\beta_{eff})} \tag{5}$$

$$\beta_{eff} = \beta_E + \beta_H \tag{6}$$

The value of the elastic damping β_E is determined based the material damping right before the yield point of the material. The value of the hysteretic damping is determined based on the area enclosed by the hysteresis loops of the structure as follows:

$$\beta_E = \kappa \cdot \left(\frac{\text{Area}}{2\pi \cdot D \cdot A} \right) \tag{7}$$

where D is the peak displacement response, A is the acceleration at the peak displacement response, "Area" is the area enclosed by the hysteresis loop defined by a symmetrical push-pull of the building capacity curve up to peak positive and negative displacements $\pm D$, and κ is a degradation factor representing the fraction of the equivalent viscous damping that is considered as hysteretic damping. For the degradation factor κ, the values proposed by FEMA-NIBS as a function of the building prototype and the earthquake duration represented by the earthquake magnitude are adopted. Figure 2a depicts an example of elastic and inelastic demand spectra.

Capacity Spectrum

Capacity curves of buildings are established based on nonlinear static-equivalent analyses, also known as pushover analyses. These curves are the plot of the base shear versus roof displacement of the structure subjected to incremental lateral loads. In order to make these curves comparable with a demand spectrum, the abscissa of these curves is transformed to spectral displacement, S_d, and the ordinate to spectral acceleration, S_a, using the modal properties of the structure (Kircher *et al.* 1997). Two control points determine the shape of the capacity spectrum: the yield point and the ultimate capacity point. The spectral ordinates of these points are characterized by spectral displacements of D_y and D_u and spectral accelerations of A_y and A_u, respectively. Figure 2b illustrates an example capacity spectrum. In this study, the characteristic values of the capacity spectrum proposed by FEMA-NIBS as a function of building prototype and code design level are adopted.

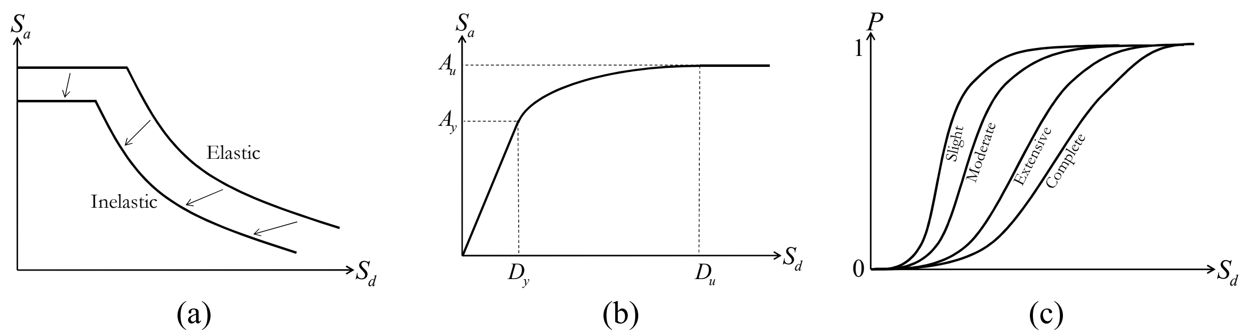


Figure 2. (a) Demand spectrum, (b) capacity spectrum, and (c) damage fragility curve.

The peak response that the building will experience is defined as the intersection of the demand and the capacity spectra. This point is referred to as "performance point."

Damage Fragility

In the FEMA-NIBS approach, four damage states are defined as follows: (1) slight, (2) moderate, (3) extensive, and (4) complete. Each damage state is characterized by the exceedance of a certain drift ratio of the building. The fragility curve associated with each damage state has a lognormal distribution. The parameters of this distribution are a median spectral displacement and a variability (standard deviation of the logarithm of spectral response), which are dependent on the building prototype and code design level. Given the peak spectral response evaluated in previous steps, these fragility curves provide the probability of being in or exceeding each damage state. Thus, it is possible to calculate the probability of falling only in damage state i,

$P(DS_i)$, where i=1, 2, 3, 4. Figure 2c depicts an example set of fragility curves.

On the other hand, each damage state i is associated with a damage factor range and thus, a central damage factor CDF_i as the centre of the damage range. The damage factor DF is defined as the cost of the repair divided by the cost of the replacement of the building. The central damage factors depend on the building occupancy classes and are adopted here based on the recommendations of FEMA-NIBS. Having the probability of falling in each damage state and the central damage factor of each damage state, the mean and standard deviation of the damage factor, namely μ_{DF} and σ_{DF}, are evaluated as follows:

$$\mu_{DF} = \sum_{i=1}^{4} P(DS_i) \cdot CDF_i \tag{8}$$

$$\sigma_{DF} = \sqrt{\sum_{i=1}^{4} P(DS_i) \cdot \left(CDF_i - \mu_{DF}\right)^2} \tag{9}$$

Damage factor DF is a positive value. Hence, it is assumed here that it is lognormally distributed with the mean μ_{DF} and the standard deviation σ_{DF}. Thus, according to the realization of the random variable representing DF, the damage factor of the structure is evaluated.

The importance of accounting for non-structural damage in seismic loss estimation is pointed out by several researchers. On average, only 25% of the total loss is due to structural damage and the rest is due to non-structural damage (FEMA–NIBS 2003). In this analysis, three sets of fragility curves and central damage factors for each damage state are considered, namely damage to structural components, non-structural drift-sensitive components, and non-structural acceleration sensitive components. The input to the structural and non-structural drift-sensitive fragilities is spectral displacement whereas the input to the non-structural acceleration-sensitive fragility is spectral acceleration. Following the same approach as above, three mean values and three standard deviation values for these three components are evaluated. Considering that the overall damage factor including all components is a function of random variables as the sum of the three component damage factors, the overall DF of the building incorporating structural and non-structural damage is evaluated.

Consequence Model

The consequence sought in this article is monetary loss. The direct monetary loss due to damage to a building is the repair cost of the building. The model for the repair cost has the following format:

$$l = DF \cdot A \cdot N \cdot C \cdot \sigma \tag{10}$$

where l is the loss, DF is the damage factor as the ratio of the repair cost to the replacement cost, A is the footprint area, N is the number of storeys, C is the replacement cost per unit area of the building, and finally, σ is the model uncertainty. The damage factor DF is provided by the upstream infrastructure model. The footprint area A and number of storeys N are obtained from the database of the building inventory. The replacement cost represents the full replacement cost of the building including the structural members and acceleration-sensitive and drift-sensitive non-structural components. The model uncertainty σ is an epistemic random variable that accounts for the uncertainty in the cost model.

Limit-State Function

The limit-state function for the reliability problem of calculating the probability of total monetary loss of the region exceeding a threshold takes the following format:

$$g = l_t - \sum_{\text{Region}} l(\mathbf{x}) \tag{11}$$

where l_t is the threshold and l is the monetary loss of individual buildings as a function of random variables, $\mathbf{x}$.

Regional Reliability Analysis of the UBC Campus

In this section, the proposed methodology and software introduced in the companion paper are employed to probabilistically assess the seismic loss of the campus of UBC. In this study, 622 buildings on UBC campus are considered. The total properties at stake comprising the structural and non-structural components of these buildings are estimated at $4.5 billion. These buildings are visually inspected using sidewalk surveys to determine their primary use, construction material, lateral load bearing system, age, height (number of storeys), and shape. The location and the footprint area of these buildings were determined using satellite images from Google Maps® in Rt. A database is compiled using this information and employed to systematically generate Rt input files to run reliability analysis. Figure 3 demonstrates this building inventory inside the map view in the Rt user interface.

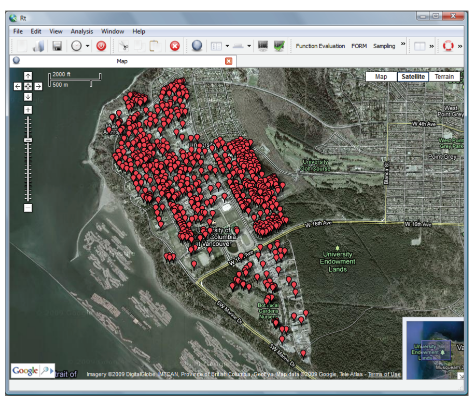
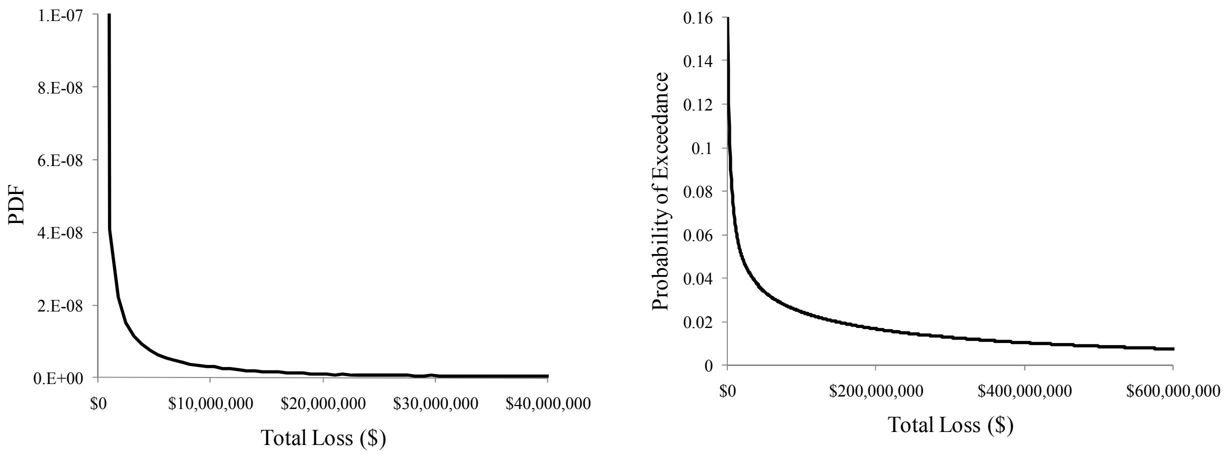


Figure 3. The building inventory used in this study including 622 buildings on the campus of UBC visualized through Google Maps® in Rt user interface.

The hazard model is established as a circular area source with the centre located at the centre of the UBC campus and a radius of 90 km. This is the only hazard source considered in this example and is deemed to represent the sub-crustal earthquakes over the region. This paper is part of an ongoing research and risk assessment considering multiple sources of hazard such as crustal, sub-crustal, and subduction earthquakes will be addressed in near future. The minimum and maximum bounds of the magnitude in the magnitude model are selected as 4 and 8, respectively. The soil type in the earthquake intensity model is considered as firm soil. The database information on construction material, lateral load bearing system, and height is used to determine the building prototypes and the age of each building is translated to its code design level. Finally, the information on the primary use of each building determines the occupancy class of that building.

In the first step, a mean-centred Monte-Carlo sampling analysis is performed on a function comprising the total loss over the region. As a result, a histogram of the probability distribution of the total loss is obtained. Figure 4 displays the PDF and the probability of exceedance of the total monetary loss over the region of the UBC campus. These curves are based on 1,000,000 samples. Albeit the probabilities in Figure 4 are obtained using a large number of samples, the coefficient of variation (C.O.V.) of the tail probabilities may not be small enough. This implies the possibility of a fairly low level of accuracy in the tail probabilities.



(a) (b)

Figure 4. (a) The PDF of total monetary loss, and (b) the probability of exceedance of the total monetary loss.

To efficiently evaluate the probability of extreme losses, FORM is employed. In this analysis, the probability of total seismic loss exceeding \$1 billion over the region is sought. Thus, the limit-state function for this analysis is the following:

$$g = \$1{,}000{,}000{,}000 - \sum_{i=1}^{622} l_i \tag{12}$$

where l_i is the loss of the building i. A total number of 635 random variable objects and 1047 probabilistic model objects are involved in this FORM analysis in Rt. The analysis converged in 45 seconds on the same machine as it took about 3.5 hours to run the sampling analysis. The search for the "most probable point" is carried out by the improved HLRF algorithm (Zhang and Der Kiureghian 1995). The analysis yields a probability of exceeding \$1 billion loss equal to 0.00126. The importance measure that is computed as a by-product of the FORM analysis indicates that the magnitude, M, introduced in Eq. (1), is the most important random variable. See Table 1 for a summary of the most important random variables according to the "α-vector," which is commonly used in structural reliability analysis. Using the importance measure, it is possible to also rank the buildings in accordance with their contribution to the uncertainty and determine the most important buildings. The importance vector indicates that the UBC hospital is the most important building on the campus followed by the TRIUMF National Laboratory for Particle and Nuclear Physics. This information can be employed to adaptively refine the building models with higher importance to achieve a balance between the computational cost on one hand and accuracy and epistemic uncertainty on the other (Haukaas and Mahsuli 2009). The angular random variable of the circular hazard source is of insignificant importance owing to the fact that the building locations are concentrated in a relatively small area at the centre of the circular source and therefore, the angle θ does not affect the results noticeably.

Table 1. Ranking of random variables and the associated α-vector elements.

Random Variable	m in Eq. (1)	r in Eq. (2)	σ in Eq. (4)	c_1 in Eq. (4)	σ in Eq. (10)
Rank	1	2	3	4	5
α-Vector Element	0.82	-0.40	0.35	0.17	0.07

Table 2. Comparison of the failure probability, reliability index, and CPU time for the mean-centred sampling, importance sampling, and FORM analyses.

	Mean-Centred Sampling	**Importance Sampling**	**FORM**
Failure Probability	0.00174	0.00174	0.00126
Reliability Index	2.921	2.921	3.020
CPU Time (sec.)	6477	203	34

To validate the result of FORM, an importance sampling analysis is conducted. In this analysis, the realizations of the random variables are generated by shifting the origin in the space of standard-normal random variables to the most probable point obtained in the FORM analysis. The target C.O.V. of the sampling analysis is set to 2%. This importance sampling analysis achieved its target C.O.V. quickly in 203 seconds and after 18,373 samples with a failure probability of 0.00174. Compared with the FORM result, this indicates a weakly nonlinear limit-state function. As recommended in the companion paper, the strategy of first obtaining a coarse histogram, followed by FORM analysis, and concluded by importance sampling is an efficient and prudent approach to obtain accurate probability results for extreme events. Table 2 compares the failure probability, reliability index, and CPU time of the mean-centred sampling, importance sampling, and FORM analyses for the limit-state function of the Eq. (12).

Conclusions

In this study, probabilistic models are employed to simulate hazard, infrastructures, and consequences in a regional seismic reliability analysis. The earthquake hazard model is divided in three probabilistic models for magnitude, location, and intensity. The building infrastructure model is built upon the FEMA-NIBS methodology. The consequence considered is the seismic loss due to damage to the structural and non-structural components of the building. These models are utilized by the developed software, Rt, to perform regional reliability analysis on the buildings located at the campus of UBC. These buildings are inspected through sidewalk surveys to generate a database of building inventory at UBC. As a result of the analysis, a regional aggregate seismic loss curve is obtained and the most important buildings in terms of uncertainty are identified.

Acknowledgments

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