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MULTI-MODE PUSHOVER ANALYSIS WITH GENERALIZED FORCE VECTORS

H. Sucuoğlu[1] and M. S. Günay[2]

ABSTRACT

A generalized pushover analysis procedure is developed for estimating the inelastic seismic response of structures under earthquake ground excitations. The procedure comprises of applying a generalized force vector to the structure in an incremental form with increasing amplitude until a prescribed seismic demand is attained. A generalized force vector is expressed as a combination of modal forces, and simulates the instantaneous force distribution acting on the system when a given interstory drift reaches its maximum value during dynamic response to a seismic excitation. The proposed generalized pushover analysis does not suffer from the statistical combination of inelastic modal responses obtained separately. The results obtained from building frames have demonstrated that generalized pushover analysis is successful in estimating maximum member deformations and member forces with reference to nonlinear response history analysis.

Introduction

Considering the simplicity and conceptual appeal of conventional pushover analysis with a single mode, several researchers have attempted to develop multi-mode pushover analysis procedures in order to replace nonlinear response history analysis with an "inelastic" response spectrum analysis (Sasaki et al. 1998, Chopra and Goel 2002). Adaptive lateral force distribution schemes have further been proposed for overcoming the limitations of conventional pushover analysis arising from an invariant lateral static load distribution (Gupta and Kunnath 2000, Aydınoğlu 2003, Antoniou and Pinho 2004) which however require rigorous computations in the implementation. All multi mode pushover analysis procedures published in literature so far have two common features. First, they are adaptive except MPA (Chopra and Goel, 2002) hence require an eigenvalue analysis at each loading increment. Moreover, an adaptive algorithm cannot be implemented with a conventional nonlinear structural analysis programming code. Second, all procedures combine modal responses statistically by SRSS, which is an approximate rule developed for combining linear elastic modal responses. Internal forces should be checked at each load increment and be corrected if they exceed the associated capacities.

A practical nonlinear static procedure is developed herein which accounts for the contribution of all significant modes to inelastic seismic response. The procedure consists of

[1]Professor, Dept. of Civil Engineering, Middle East Technical University, Ankara 06531, Turkey
[2]Research Engineer, Dept. of Civil Engineering, University of California, Berkeley, California

conducting a set of pushover analyses by employing different generalized force vectors. Each generalized force vector is derived as a different combination of modal lateral forces in order to simulate the effective lateral force distribution when the interstory drift at a selected story attains its maximum value during seismic response. Hence the proposed procedure is called generalized pushover analysis (GPA). Target seismic demands for interstory drifts at the selected stories are calculated from the associated generalized drift expressions where nonlinear response is considered in the first mode only. Finally, the maximum value of a response parameter is obtained from the envelope values produced by the set of generalized pushover analysis conducted separately for each interstory drift. GPA can be implemented with any structural analysis software capable of performing displacement controlled nonlinear incremental static analysis. Seismic response of a twelve story reinforced concrete frame structure under twelve ground motion records are estimated by GPA in this study, and compared with the results obtained from NRHA as well as from the conventional pushover analysis.

Generalized Force Vectors

Different response parameters attain their maximum values at different times during seismic response. An effective force vector acts on the system instantaneously at the time when a specific response parameter reaches its maximum value. This effective force vector is in fact a generalized force since it has contributions from all modal forces at the time of maximum response for the specified response parameter. Accordingly, if this force vector can be defined, then it can be applied either directly or incrementally to the investigated structural system in order to produce the maximum value of this response parameter.

The derivation of generalized effective force vectors is based on the dynamic response of linear elastic MDOF systems to earthquake ground excitation $\ddot{u}_g(t)$, by employing the modal superposition procedure. The effective force vector f(t_{max}) at time t_{max}, when an arbitrarily selected response parameter reaches its maximum value, can be expressed as the superposition of modal forces $\mathbf{f}_{\mathrm{n}}$ (t_{max}):

$$\mathbf{f}(t_{max}) = \sum_n \mathbf{f}_n(t_{max}) \quad (1)$$

The n'th mode effective force in Eq. (1) at time t_{max} is given by

$$\mathbf{f}_n(t_{max}) = \Gamma_n \, \mathbf{m}\boldsymbol{\varphi}_n \, A_n(t_{max}) \quad (2)$$

Here, $\Gamma_{\mathrm{n}} = L_{\mathrm{n}} / M_{\mathrm{n}}$; $L_n = \boldsymbol{\varphi}_n^{\mathbf{T}} \mathbf{ml}$; $M_n = \boldsymbol{\varphi}_n^{\mathbf{T}} \mathbf{m}\boldsymbol{\varphi}_n$; $\boldsymbol{\varphi}_n$ is the n'th mode shape, **m** is the mass matrix, **l** is the influence vector, and A_n (t_{max}) is represented with Eq. (3).

$$A_n(t_{max}) = \omega_n^2 D_n(t_{max}) \quad (3)$$

Here ω_n^2 is the n'th mode vibration frequency and D_n (t_{max}) is the modal displacement amplitude at t_{max} which satisfies

$$\ddot{D}_n(t_{max}) + 2\varsigma_n \omega_n \dot{D}_n(t_{max}) + \omega_n^2 D_n(t_{max}) = -\ddot{u}_g(t_{max}) \tag{4}$$

$D_n\ (t_{max})$ cannot be determined from Eq. (4) unless t_{max} is known. t_{max} is the time when the selected response parameter becomes maximum, which depends on all modal responses. This response parameter is selected as the interstory drift Δ_j at the j'th story. Then,

$$\Delta_{j,\,max} = \Delta_j\,(t_{max}) \tag{5}$$

Its associated modal expansion is

$$\Delta_j(t_{max}) \;=\; \sum_n \Gamma_n\ D_n(t_{max})\,(\varphi_{n,\,j} - \varphi_{n,\;j\text{-}1}) \tag{6}$$

where $\varphi_{n,\,j}$ is the j'th element of the mode shape vector φ_n. Eq. (6) can be normalized by dividing both sides with $\Delta_j\ (t_{max})$, which yields

$$1 \;=\; \sum_n \;\Gamma_n \frac{D_n(t_{max})}{\Delta_j(t_{max})}\,(\varphi_{n,\,j} - \varphi_{n,\;j\text{-}1}) \tag{7}$$

Each term on the right hand side of Eq. (7) under summation expresses the contribution of n'th mode to the maximum interstory drift $\Delta_j(t_{max})$ at the j'th story in a normalized form.

The maximum value of interstory drift at the j'th story in Eq. (5) can also be estimated by RSA through SRSS of the related spectral modal responses.

$$(\Delta_{\,j,\,\mathbf{max}})^2 \;\approx\; \sum_n \left[\,\Gamma_n\ D_n\ (\varphi_{n,\,j} - \varphi_{n,\;j\text{-}1})\right]^2 \tag{8}$$

D_n in the above equation is the spectral displacement of the n'th mode, which is directly available from the displacement response spectrum of the earthquake ground excitation $\ddot{u}_g(t)$. Eq. (8) can also be normalized similarly, by dividing both sides with $(\Delta_{\,j,\,max})^2$.

$$1 = \sum_n \left[\Gamma_n \frac{D_n}{\Delta_{j,max}}(\varphi_{n,j} - \varphi_{n,j\text{-}1})\right]^2 \tag{9}$$

Accordingly, the respective terms on the right hand sides of Equations (7) and (9) are the normalized contributions of the n'th mode to the maximum interstory drift at the j'th story. Equating these terms, and assuming the equality $\Delta_{j,\,max} = \Delta_j\,(t_{max})$ from Eq. (5) leads to

$$D_n(t_{\mathbf{max}}) \;=\; \frac{D_n}{\Delta_{j,\mathbf{max}}}\left[\Gamma_n D_n(\varphi_{n,j} - \varphi_{n,j\;\;1})\right] \tag{10}$$

Since the term in the parentheses in Eq. (10) is equal to $\Delta_{j,n}$ from Eq.(8),

$$D_n(t_{max}) = \frac{\Delta_{j,n}}{\Delta_{j,max}} D_n \tag{11}$$

It should be noted that $\Delta_{j,n}$ in Eq. (11) is the n'th mode contribution to the maximum interstory drift at the j'th story determined from RSA, and $\Delta_{j,max}$ in Eq. (11) is the quadratic combination of the $\Delta_{j,n}$ terms as given by Eq. (8). The generalized force vector is obtained by first calculating A_n (t_{max}) by substituting $D_n(t_{max})$ from Eq. (11) into Eq. (3), then substituting A_n (t_{max}) from Eq. (3) into Eq. (2), and finally substituting $\mathbf{f}_n$ (t_{max}) from Eq. (2) into Eq. (1).

$$\mathbf{f}_j(t_{max}) = \sum_n (\Gamma_n \; \mathbf{m} \; \boldsymbol{\varphi}_n \; A_n \frac{\Delta_{j,n}}{\Delta_{j,max}}) \tag{12}$$

A_n is the pseudo spectral acceleration of the n'th mode in Eq. (12). Since the formulation that is developed in Eqs. (1- 11) is employed for obtaining the generalized force vector which acts on the structural system when the interstory drift at the j'th story becomes maximum, the associated generalized force vector in Eq. (12) is identified with the subscript j.

Target Seismic Deformation Demand

In linear elastic response spectrum analysis, the target drift demand at the j'th story Δ_{jt} is calculated from the SRSS combination of modal drifts $\Delta_{n,j}$ expressed by

$$\Delta_{jt}^{\;2} = \sum_n \left[\Gamma_n \; (\varphi_{n,j} - \varphi_{n,\,j-1}) D_n\right]^2 \tag{13}$$

The displacement shape of a nonlinear system during seismic response can be expanded in terms of the linear elastic mode shapes if the modal amplitudes (coordinates) can be calculated appropriately. It has been observed by Chopra and Goel that the coupling between modal coordinates due to yielding of the system is negligible. Therefore Eq. (13) may be employed for estimating the inelastic drift demands, provided that linear elastic modal spectral displacement demands D_n in Eq. (13) are replaced by the inelastic modal spectral displacement demands D_n*.

Replacing only D_1 in Eq. (13) with D_1* while retaining the linear elastic modal spectral displacements for the second and higher modes improves the target displacement demand significantly. D_1* can then be estimated from either the NRHA of the nonlinear SDOF system representing the first mode contribution, or from the associated R-μ-T relations. Then the target drift demand Δ_{jt} in GPA becomes;

$$\Delta_{jt} = \left(\left[\Gamma_1 \; (\varphi_{1,j} - \varphi_{1,\,j-1}) D_1^*\right]^2 + \sum_{n=2}^{N} \left[\Gamma_n \; (\varphi_{n,j} - \varphi_{n,\,j-1}) D_n\right]^2 \right)^{1/2} \tag{14}$$

The contribution of higher modes to a maximum interstory drift parameter is more significant than their contribution to a maximum displacement parameter. GPA uses the interstory drift parameters as target demands. Accordingly, interstory drift is not obtained from GPA, but from an independent response spectrum analysis. When the associated generalized force vector pushes the system to this target drift, the system adopts itself in the inelastic deformation range while the further higher order deformation parameters (rotations, curvatures) take their inelastic values as in modal pushover, but by receiving appropriate contributions from the higher modes.

Generalized Pushover Algorithm

The GPA algorithm is composed of the five basic steps summarized below.

1. *Eigenvalue analysis*: Natural frequencies ω_n (natural periods T_n), modal vectors φ_n and the modal participation factors Γ_n are determined from eigenvalue analysis.

2. *Response spectrum analysis*: Modal spectral amplitudes A_n, D_n are obtained from the corresponding linear elastic spectra and modal interstory drift ratios at the *j*'th story, $\Delta_{j,n}$ are determined from RSA. The maximum interstory drift ratio at the *j*'th story, $\Delta_{j,max}$ is obtained by SRSS.

3. *Generalized force vectors*: Generalized force vectors $\mathbf{f}_j$ which produce the maximum response Δ_j are calculated from Eq. (12).

4. *Target interstory drift demands*: Maximum inelastic modal displacement demand D_1* for the first mode under an earthquake excitation is obtained from either NRHA or inelastic response spectrum of the inelastic SDOF system idealized with a bi-linear force–displacement relation. For the higher modes n=2-N, D_n values are obtained from the linear elastic response spectrum. Finally, D_1* and D_n (n=2, N) are substituted into Eq. (14) for calculating the target interstory drift demands Δ_{jt}.

5. *Generalized pushover analysis*: A total number of N generalized pushover analyses are conducted. In the *j*'th GPA (j=1-N), the structure is pushed in the lateral direction incrementally with a force distribution proportional to $\mathbf{f}_j$. At the end of each loading increment i, the interstory drift Δ_{ji} obtained at the *j*'th story is compared with the target interstory drift Δ_{jt} calculated from Eq. (14). Displacement controlled incremental loading (i=1, 2…) at the *j*'th GPA continues until Δ_{ji} reaches Δ_{jt}.

All member deformations and internal forces are directly obtained from the *j*'th GPA at the target interstory drift Δ_{jt}. Once all GPA is completed for j=1-N, the enveloping values of member deformations and internal forces are registered as the maximum seismic response values.

Test Case: 12 Story RC Building Frame

The proposed GPA procedure is tested on a 12 story reinforced concrete building with symmetrical plan, where the floor plan is shown in Figure 1. The building is designed according to the regulations of the 2007 Turkish Earthquake Code in accordance with the capacity design principles. An enhanced ductility level is assumed for the building. The design spectrum is shown in Figure 1. Concrete and steel characteristic strengths are 25 MPa and 420 MPa, respectively. Slab thickness for all floors is 140 mm and live load is 3.5 kN/m^2. Dimensions of the beams at the first four, the second four and the last four stories are 300x550, 300x500 and 300x450 mm2 respectively, whereas dimensions of the columns at the first four, the second four and the last four stories are 500x500, 450x450 and 400x400 mm2 respectively. There is no basement; height of the ground story is 4 m while the height of all other stories is 3.2 m.

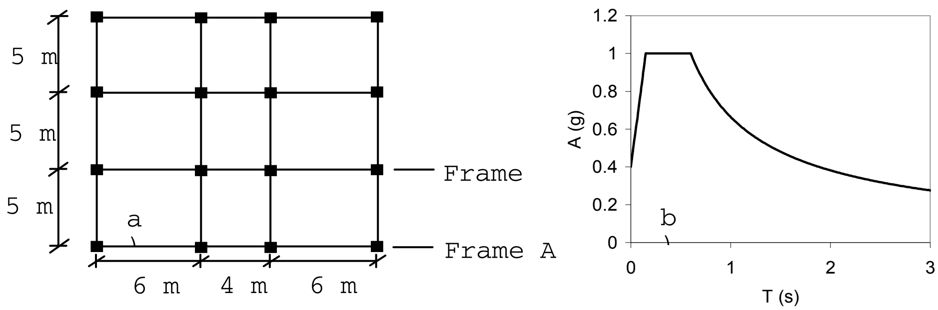

Figure 1. a) Typical plan of the 12 story building, b) design spectrum

Plane frame models consisting of Frames A and B are constructed for the analysis. They are analyzed by using the nonlinear analysis software Drain-2DX. Cracked section stiffness is employed for the initial linear segment of the moment-curvature relations. Gross moments of inertia are multiplied with 0.6 and 0.4 for the columns and beams, respectively, in order to represent cracking. Free vibration periods for the first three modes of the frame are 2.38, 0.86 and 0.50 seconds.

Strong ground motions

The building is analyzed under twelve different strong motion components. These ground motions are selected from a larger set in accordance with the FEMA 356 criteria limiting the applicability of conventional nonlinear static procedure. They produce significant higher mode effects on the investigated 12 story frame; hence FEMA 356 does not allow using standard NSP. The basic characteristics of the twelve ground motion components are presented in Table 1. The first nine contain a significant pulse whereas the last three are without a pulse. All ground motions were downloaded from the PEER strong motion database.

Table 1. Characteristics of strong ground motions

#	GM Code	Earthquake (Mw)	Station-Component	CD (km) [1]	Site Geol. [2]	PGA (g)	PGV (cm/s)	PGD (cm)	GM Type
1	BOL090	Duzce, 11/12/99 (7.1)	Bolu-090	12.0	D	0.822	62.1	13.6	Pulse
2	ERZ-EW	Erzincan, 03/13/92 (6.9)	Erzincan-EW	4.4	D	0.496	64.3	21.9	Pulse
3	H-E04140	Imp. Valley, 10/15/79 (6.5)	El Centro Array #4-140	7.1	D	0.485	37.4	20.1	Pulse
4	PRI090	Kobe, 01/16/95 (6.9)	Port Island (0 m)-090	3.3	E	0.278	54.2	24.9	Pulse
5	CLS090	Loma Prieta, 10/18/89 (7)	Corralitos-090	3.9	A	0.479	45.2	11.3	Pulse
6	LEX000	Loma Prieta, 10/18/89 (7)	Los Gat. - Lex. Dam-000	5.0	A	0.420	73.5	20.0	Pulse
7	SPV270	Northridge, 01/17/94 (6.7)	Sepulveda VA-270	8.9	D	0.753	84.5	18.7	Pulse
8	PCD254	San Fer., 02/09/71 (6.6)	Pacoima Dam-254	2.8	B	1.160	54.1	11.8	Pulse
9	CHY006-E	Chi-Chi, 09/20/99 (7.6)	CHY006-E	9.8	B	0.364	55.4	25.6	Pulse
10	BOL000	Duzce, 11/12/99 (7.1)	Bolu-000	12.0	D	0.728	56.4	23.1	Ordinary
11	ORR090	Northridge, 01/17/94 (6.7)	Cast.-Old Rdg Route-090	20.7	B	0.568	51.8	9.0	Ordinary
12	ORR360	Northridge, 01/17/94 (6.7)	Cast.-Old Rdg Route-360	20.7	B	0.514	52.0	15.3	Ordinary

Implementation of the GPA algorithm

The generalized pushover analysis algorithm is presented herein with an implementation to the 12 story building, subjected to the ground motion 7 in Table 1. The resulting force distributions $\mathbf{f}_j$ along height, obtained from Eq. (12) are presented in Figure 2. It can be observed that the second mode contribution is significant on the force distributions which produce maximum interstory drifts at all 12 stories.

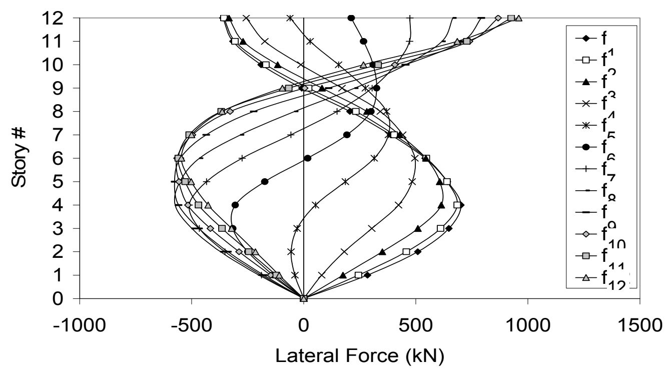

Figure 2. Generalized force distributions $\mathbf{f}_j$ for the 12 story building

Results

Beam plastic rotations and member internal forces are calculated respectively under the twelve ground motions in Table 1. Beam plastic rotations at member ends calculated with NRHA, GPA and PO-1 (conventional pushover analysis) are shown in Figure 3. Beam plastic rotations are calculated for each story, as the average value of all beam-end plastic rotations in that story. Plastic actions did not develop at the columns of the building under any of the twelve ground motion excitations except the ground story column bases.

The performance of GPA in predicting beam plastic rotations from NRHA is satisfactory. PO-1 is not able to capture plastic deformations in any of the beams at the upper five stories since first mode is not sufficient by itself for developing plastic rotations. GPA is quite successful in predicting the plastic beam deformations at both upper and the lower stories. Simultaneous combination of all modes in GPA leads to realistic estimations of plastic deformations. There are some unsuccessful cases at the lower stories however for GPA, such as GM2 and GM9, where higher mode contributions at the lower stories do not develop apparently during the nonlinear dynamic response history analysis contrary to the GPA prediction.

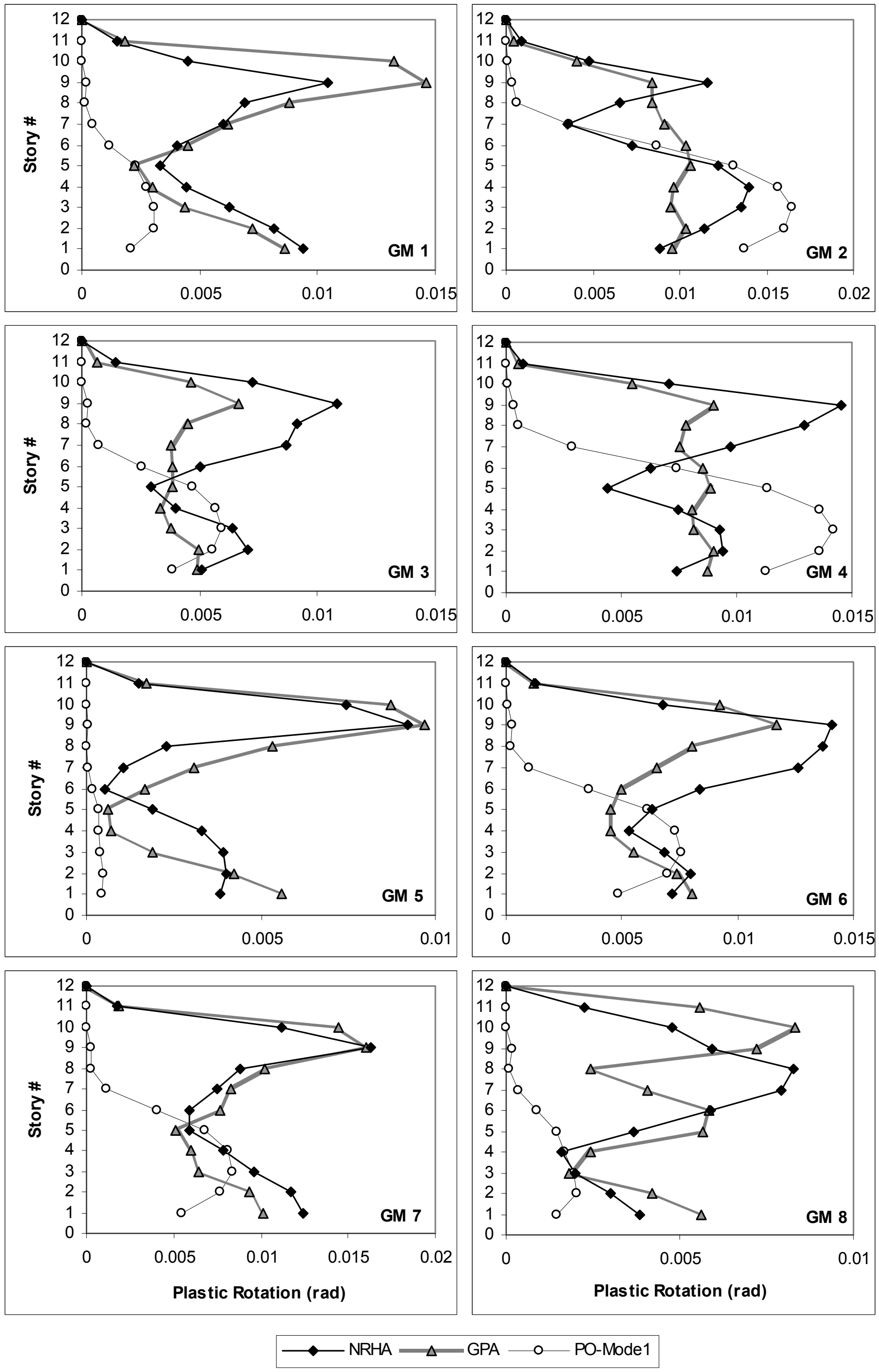

Figure 3. Beam plastic rotations

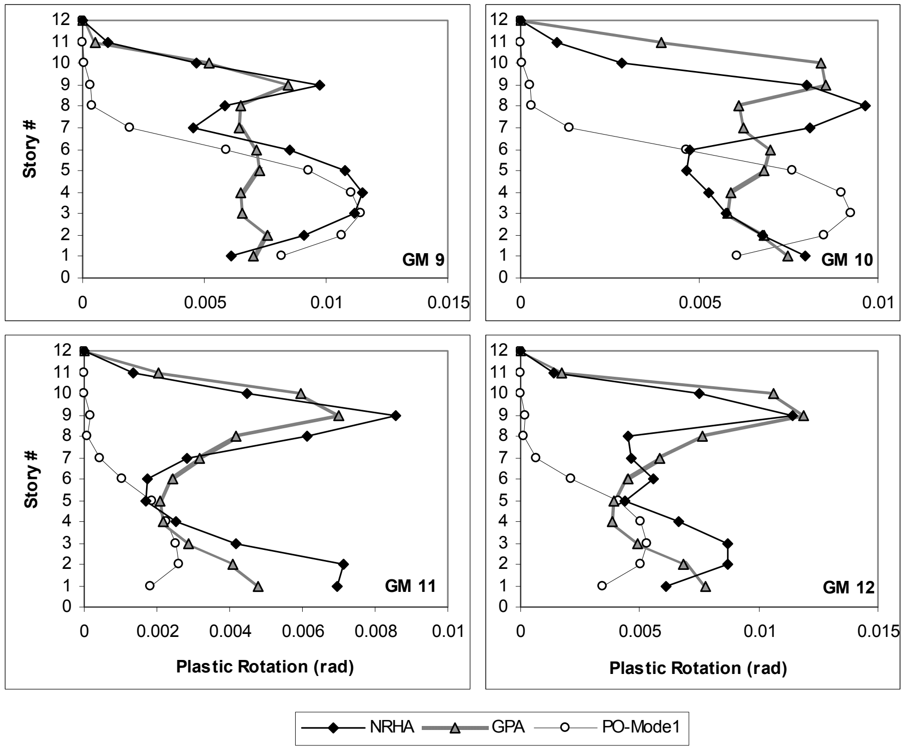

Figure 3 (continue). Beam plastic rotations

Conclusions

The generalized pushover analysis procedure presented in this study has three basic advantages when compared to the other multi-mode pushover procedures available in literature.

- GPA (like MPA) is not adaptive. It can be implemented conveniently by employing a general purpose nonlinear static analysis tool. There is no need for developing a special programming code.
- The computational effort required by GPA is much less compared to the adaptive pushover procedures.
- GPA does not suffer from the statistical combination of individually calculated inelastic modal responses. It activates all modes of a multi degree of freedom system simultaneously. Accordingly, all response parameters are obtained directly from a generalized pushover analysis at the associated target drift demand.

The number of pushover analysis required in GPA under a single ground excitation expressed by its response spectrum is equal to one plus the number of stories in symmetrical plan buildings (1+N), where N is the number of lateral dynamic degrees of freedom, or the number of stories. However 1+N pushover in GPA is only repetitive.

References

Antoniou, S., and R. Pinho, 2004. Development and verification of a displacement-based adaptive pushover procedure, *Journal of Earthquake Eng* **8** (5), 643-661.

Aydınoğlu, MN, 2003. An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation, *Bull Earthquake Eng* **1** (1), 3-36.

Chopra, AK., and R.K. Goel, 2002. A modal pushover analysis procedure for estimating seismic demands for buildings, *Earthquake Eng Str Dynamics* **31** (3), 561-582.

Gupta, B., and S.K. Kunnath, 2000. Adaptive spectra-based pushover procedure for seismic evaluation of structures, *Earthquake Spectra* **16** (2), 367-391.

Sasaki, F., Freeman, S., and T. Paret, 1998. Multi-mode pushover procedure (MMP). *Proc. Sixth U.S. NCEE*, Seattle, Washington.